Abstracts

Speaker: Amitabh Basu
UC Davis

Unique Minimal Liftings in Minimal Inequalities
Minimal valid inequalities are well understood for a relaxation of an MILP in tableau form where all the nonbasic variables are continuous; they are derived using the gauge function of maximal lattice-free convex sets. Recently a lot of work has been done to study lifting functions for the nonbasic *integer* variables starting from such minimal valid inequalities. We will present some new results regarding these lifting functions.

Speaker: Gerard Cornuéjols
Carnegie Mellon University

3-Slope Theorem for the Infinite Relaxation in the Plane
This talk is based on joint work by Gerard Cornuejols and Marco Molinaro. We consider the infinite relaxation of the corner polyhedron with two rows. For the single-row case, Gomory and Johnson proved in their seminal paper a sufficient condition for a minimal function to be extreme, the celebrated 2-Slope Theorem. Despite increased interest in understanding the multiple-row setting, no generalization of this theorem was known for this case. We present an extension of the 2-Slope Theorem for the case of two rows by showing that minimal 3-slope functions satisfying an additional regularity condition are extreme. Moreover, we show that this regularity condition is necessary, unveiling a structure which is only present in the multi-row setting.

Speaker: Claudia D'Ambrosio
DEIS, University of Bologna, Italy

Optimistic MILP Modeling of Non-linear Optimization Problems
We present a new piecewise linear approximation of non-linear optimization problems that can be seen as a generalization of classical triangulations. Intuitively, it is a generalization because it leaves more degrees of freedom to define any point as a convex combination of the samples. As an example, for the classical case of approximating a function of two variables, a convex combination of four points instead of only three is used. As a plane is defined by only three independent points, the approximation obtained by triangulation is uniquely determined, while different approximations are possible in our case. When embedded in a Mixed-Integer

Linear Programming (MILP) model, the choice among the different alternatives is guided by the objective function. The new approximation within an MILP requires a significant smaller number of additional binary variables, and allows the use of recent methods for representing such variables with a the logarithmic number of constraints. We show theoretical and computational evidence of the quality of the approximation and its impact within MILP models.
Joint work with Andrea Lodi, Silvano Martello, Riccardo Rovatti (DEIS, University of Bologna).

Speaker: Santanu Dey
Georgia Institute of Technology

Some Properties of Convex Hulls of Integer Points Contained in General Convex Sets
In this talk, we present properties of general closed convex sets that determine the closed-ness and polyhedrality of the convex hull of integer points contained in it. We first present necessary and sufficient conditions for the convex hull of integer points contained in a general convex set to be closed. This leads to useful results for special classes of convex sets such as pointed cones, strictly convex sets, and sets containing integer points in their interior. We then present sufficient conditions for the convex hull of integer points in general convex sets to be polyhedron. These sufficient conditions generalize the sufficient conditions given in Meyer (1974). Under a simple technical condition, we show that these sufficient conditions are also necessary conditions for the convex hull of integer points contained in general convex sets to be a polyhedron.

This is joint work with Diego A. Moran R.

Speaker: Sarah Drewes
University of California - Berkeley

Cover Inequalities for Mixed-01 Nonlinear Programming
We present how cover inequalities can be used in an outer approximation based LP/NLP based branch and bound approach for mixed 0-1 convex nonlinear programming problems. These algorithms derive assignments for the binary variables using linearizations of the nonlinear functions. We first show how covers can be identified based on the linearized constraints of the outer approximation. The corresponding cover inequalities can then be lifted to derive strong valid inequalities that tighten the continuous relaxation of the outer approximation. The potential of this approach is then discussed considering a class of mixed 0-1 second order cone programs for which a computational study is provided.
This is joint work with Alper Atamtrk.

Speaker: Samir Elhedhli
University of Waterloo

Generalized Branching based on an Interior-Point Approach
We propose, discuss, and test a new generalized branching approach for mixed integer programming based on the notion of analytic centres from interior-point methods. Unlike single variable branching, generalized branching uses multiple variables in the branching constraints. At each node, the polyhedron is first approximated using Dikin's ellipsoid which is centered at the analytic centre. The minimum-width disjunction is then found by solving a quadratic program. As solving a quadratic problem at each node of the branch-and-bound tree is impractical, we use a neighborhood search heuristic for its solution. We report computational results on hard mixed integer problems from the literature. We found that the proposed approach reduced the number of branching on the majority of the tested instances and explored on average 10% less nodes than CPLEX's strong branching.

(Joint work with J. Naoum-Sawaya)

Speaker: Willem-Jan van Hoeve
Carnegie Mellon University

Decision Diagrams for Combinatorial Optimization
Multivalued decision diagrams (MDDs) are layered directed acyclic graphs that can, in principle, compactly represent all solutions to a given problem. MDDs, and binary decision diagrams in particular, are best known as a technique for circuit verification and product configuration, but they have recently also been introduced as an attractive approach to combinatorial optimization. In this talk, I will describe how limited-width MDDs can serve as discrete relaxations (or restrictions) for combinatorial optimization problems. Specific example applications in the context of Constraint Programming and Integer Programming will illustrate this approach.
This talk is based on joint work with David Bergman, Andre A. Cire, Sam Hoda, and John N. Hooker.

Speaker: Adam Letchford
Lancaster University

Some unbounded convex sets arising in non-convex MIQP
We introduce a fundamental family of unbounded convex sets that arises in the context of non-convex mixed-integer quadratic programming. It is shown that any mixed-integer quadratic program with linear constraints can be reduced to the minimisation of a linear function over a set in the family. Some fundamental properties of these convex sets are derived, along with connections to some other well-studied convex sets, such as the positive semidefinite cone, the completely positive cone, and the boolean quadric polytope. Several classes of valid inequalities are also derived, some of which are shown to induce faces of maximal dimension.

This talk is based on joint work with Sam Burer, University of Iowa.

Speaker: Leo Liberti
Ecole Polytechnique

Don't break my orbits
Nature loves symmetry. Artists love symmetries. People love symmetry. Mathematicians and computer scientists also love symmetry, with the only exception of MIP people who always want to break it. Why? As a matter of fact, symmetry is of great help in simplifying optimization in a convex setting, but in the mixed-integer case it can trick enumeration because symmetric solutions are visited again and again. The usual approach to cope with this redundancy source is to destroy symmetry by introducing (artificial) conditions into the model, or by using a clever branching strategy such as isomorphism pruning or orbital branching. We will outline a different approach, that we call orbital shrinking, where additional (integer) variables expressing variable sums within each orbit are introduces and used to "encapsulate" model symmetry. The underlying idea here is that we see symmetry as a positive feature of our model, so we want to preserve it as much as possible, breaking it only as a last resort. This is work in progress, so we are not claiming any definite improvement on the state of the art yet.
Joint work with Matteo Fischetti.

Speaker: Marco Lübbecke
Aachen University

Towards Automatic Dantzig-Wolfe Decomposition
Dantzig-Wolfe decomposition in conjunction with column generation, embedded in a branch-and-bound algorithm, a.k.a. branch-and-price, has proven to be an efficient tool to solve well-structured integer programs. At the same time, to fully

exploit the embedded structure, the method is usually heavily tailored to the particular application. This fact may prevent non-specialists from applying column generation to their problems, and is one reason why most, if not all, implementations start from scratch, or from a vanilla framework (of which there are several). We report on our recent attempts to combine and extend well-known algorithms from the literature to overcome this defect. We show how to automatically detect exploitable structures in matrices from the MIPLIBs. When applying column generation to the identified decompositions, somewhat surprisingly, we are able to obtain very strong dual bounds from the respective linear relaxations. Our work is meant as a proof-of-concept and does not yet constitute a competitive solution approach to integer programs in general. However, once a good matrix structure (what does that mean?) is known, it may prove useful in a branch-and-bound algorithm in general, and we hint at some possible research directions.

Contributors: Michael Bastubbe, Martin Bergner, Alberto Caprara, Alberto Ceselli, Fabio Furini, Gerald Gamrath, Enrico Malaguti, Christian Puchert, Emiliano Traversi

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Speaker: Susan Margulies
Rice University

The Cunningham-Geelen Method in Practice:
Branch-decompositions and Integer Programming

Cunningham and Geelen (2007) describe an algorithm for solving the integer program $\max\{c^T x : Ax = b, x \geq 0, x \in \mathbb{Z}^n\}$, where $A \in \mathbb{Z}_{\geq 0}^{m \times n}, b \in \mathbb{Z}^m$, and $c \in \mathbb{Z}^n$, which utilizes a branch-decomposition of the matrix A and techniques from dynamic programming. In this talk, we report on the first implementation of the CG algorithm, and compare our results with the commercial integer programming software GUROBI. Using branch-decomposition trees produced by the heuristics developed by Ma et. al, and optimal trees produced by the algorithm designed by Hicks, we test both a memory-intensive and low-memory version of the CG algorithm on problem instances such as graph 3-coloring, set partition, market split and knapsack. We isolate a class of set partition instances where the CG algorithm runs twice as fast as GUROBI, and demonstrate that certain infeasible market split and knapsacks instances with width ≤ 6 range from running twice as fast as GUROBI, to running in a matter of minutes versus a matter of hours.
Joint work with J. Ma and I.V. Hycks

Speaker: Alexander Martin
University Erlangen-Nuremberg

A Scenario Tree-Based Decomposition for Solving Multistage Stochastic Programs with Application in Energy Production
Motivated by a power supply problem from energy design we discribe a multistage stochastic program. The structure of the program gives rise to a - to the best of our knowledge - new decomposition approach for mutlistage stochastic programs. We discuss the pros and cons of this approach and show its progress on realistic test instances from the energy design problem.

Speaker: Giacomo Nannicini
Carnegie Mellon University

How tight is the corner relaxation?
A study of the edge formulation of the maximum stable set problem.
The corner relaxation of a mixed-integer linear program is a central concept in cutting plane theory. In a recent paper Fischetti and Monaci provide an empirical assessment of the strength of the corner and other related relaxations on MIPLIB problems. In this work we study the bounds given by these relaxations in the special case of the edge formulation of the maximum stable set problem. Surprisingly, we can characterize the bounds exactly. Our study confirms the empirical results of Fischetti and Monaci. Furthermore, it shows that for the edge formulation of the maximum stable set problem, carefully choosing the basis of the LP relaxation from which cutting planes are derived is of fundamental importance.
Authors: Gerard Cornuéjols, Carla Michini, Giacomo Nannicini

Speaker: Michael Perregaard
FICO

Unifying reduce-and-split and lift-and-project for cutting and branching in Xpress
The two techniques of reduce-and-split and lift-and-project are complementary in the sense that reduce-and-split is good for identifying strong split disjunctions, whereas lift-and-project is good for evaluating the strength of such a disjunction. There has been interesting work done recently on trying to combining these two procedures for cut generation (ref. Balas et al., 2010). The talk here will be about how these tools are used in a unifying procedure within the Xpress solver to create both strong cuts and for selecting strong branching disjunctions.

Speaker: Sebastian Pokutta
Friedrich-Alexander-University of Erlangen-Nürnberg

Design and Verify: A New Scheme for Generating Cutting-Planes
A cutting-plane procedure for integer programming (IP) problems usually involves invoking a black-box procedure (such as the Gomory-Chvátal procedure) to *compute* a cutting-plane. In this paper, we describe an alternative paradigm of using the same cutting-plane black-box. This involves two steps. In the first step, we *design* an inequality $cx \leq d$, *independent* of the cutting-plane black-box. In the second step, we *verify* that the designed inequality is a valid inequality by verifying that the set $P \cap \{x \in \mathbb{R}^n \mid cx \geq d+1\} \cap \mathbb{Z}^n$ is empty using cutting-planes from the black-box. Here P is the feasible region of the linear-programming relaxation of the IP. We refer to the closure of all cutting-planes that can be verified to be valid using a specific cutting-plane black-box as the *verification closure* of the considered cutting-plane black-box. We will study these closures and show that they are strong, yielding many inequalities in only one application of the operator. However, we will also show that these closures are not unrealistically strong by providing lower bounds on the rank for certain families of polytopes.

This is a joint work with Santanu Dey.

Speaker: Oleg Prokopyev
University of Pittsburgh

Two-Stage Quadratic Integer Programs with Stochastic Right-Hand Sides
We consider a class of two-stage stochastic quadratic integer programs (SQIPs) where the uncertainty only appears in the second-stage right-hand sides. We propose a two-phase solution approach. The first phase constructs value functions of quadratic integer programs (QIPs) in both stages. The second phase solves the value function reformulation of the original problem using a global branch-and-bound algorithm or a level-set approach. We derive some basic properties of value functions of QIPs and utilize them in our algorithms. The developed method can solve very large instances SQIPs as measured by the size of the extensive form. We note that our approach is amenable to solve general two-stage SQIPs as long as the scenarios may be divided into relatively few groups that share the same objective functions and constraint matrices. The major limitation of our two-phase solution approach is the explicit storage of value functions in computer memory. This is why our computations are based on instances that have large number of columns and scenarios but relatively few rows. We also discuss possible research directions to overcome this limitation.
This is a joint work with Osman Ozaltin and Andrew Schaefer.

Speaker: Sebastian Sager
Universität Heidelberg

Decoupled Solution of Nonlinear Mixed-Integer Optimal Control Problems

In model-based nonlinear optimal control switching decisions that can be optimized often play an important role. Prominent examples of such hybrid systems are gear switches for transport vehicles, traffic lights, or on/off valves in engineering, [2]. Optimization algorithms need to take the discrete nature of the variables that model these switching decisions into account.

Mixed-integer optimal control problems (MIOCPs) include features related to different mathematical disciplines. Hence, it is not surprising that distinct approaches have been proposed to analyze and solve them. We present an approach based on direct methods for optimal control, a partial convexification, relaxation, adaptive control grids, and tailored rounding schemes, [5, 1]. This allows to decouple the MIOCP into one NLP and one MILP. We will discuss efficient rounding schemes to solve the structured MILP. We will show how these polynomial-time rounding schemes yield ϵ-optimal solutions for the nonlinear MIOCP, where ϵ depends on the control grid size, [3]. We will also discuss the efficient treatment of explicit constraints on the switching decisions on the MILP level, [4], and the treatment of mathematical programs with vanishing constraints.

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Speaker: Domenico Salvagnin
University of Padova

Three ideas for the Quadratic Assignment Problem
We recently started the ambitious project of solving to proven optimality (at least, some of) the largest "esc" instances of the famous quadratic assignment problem. These are extremely hard instances that remained unsolved—even allowing for a tremendous computing power—by using all previous techniques from the literature. During this challenging task we tested a number of ideas, and found that three of them were particularly useful and qualified as a breakthrough for our approach. This talk is about describing these ideas and the status of our attempt. So far our method was able to solve, in a matter of seconds or minutes on a single PC, all the easy cases (all esc16* plus esc32e and esc32g). The three previously-unsolved esc32c, esc32d and esc64a were solved in less than half an hour, in total, on a single PC. We also report the solution of the previously-unsolved tai64c, again within very reasonable computing time. We are currently attacking the remaining cases, including "the big fish" esc128 for which we found an improved lower bound of 56 out of 64 (the previous best lower bound was 2).
Joint work with Matteo Fischetti and Michele Monaci.

Speaker: Laura Sanita
Ecole Polythechnique Federale de Lausanne

Set covering with ordered replacement
We consider set covering problems where the underlying set system satisfies a particular replacement property w.r.t. a given partial order on the elements: Whenever a set is in the set system then a set stemming from it via the replacement of an element by a smaller element is also in the set system. Many variants of Bin Packing that have appeared in the literature are such set covering problems with ordered replacement. We provide a rigorous account on the additive and multiplicative integrality gap and approximability of set covering with ordered replacement. In particular we provide a polylogarithmic upper bound on the additive integrality gap that also yields a polynomial time additive approximation algorithm if the linear programming relaxation can be efficiently solved.
Joint work with: Friedrich Eisenbrand, Naonori Kakimura, Thomas Rothvo.

Speaker: Levent Tuncel
University of Waterloo

Mixed-Integer Programming approaches to Revenue Management
I will discuss various mathematical optimization models for revenue management. The focus of the talk will be on valid inequalities as well as computational and theoretical worst-case performance, complexity analysis of families of heuristic algorithms.
This talk is based on joint work with Demirtas, Hui, Myklebust, Sharpe and Shioda.

Speaker: Francois Vanderbeck
University of Bordeaux

Column Generation for Extended Formulations
Working in an extended variable space allows one to develop tight reformulations for mixed integer programs. However, the size of the extended formulation grows rapidly too large for a direct treatment by a MIP-solver. Then, one can use projection tools and derive valid inequalities for the original formulation, or consider an approximate extended formulation (f.i. by aggregating variables). Both approaches result in outer approximations of the intended extended formulation. An alternative is to work with inner approximations defined and improved by generating dynamically the variables of the extended formulation. It assumes that the extended formulation stems from a decomposition principle: a subproblem admits an extended formulation from which the original problem extended formulation is derived. Then, one can implement column generation for this extended formulation by transposing the equivalent procedure for the Dantzig-Wolfe reformulation. Pricing subproblem solutions are expressed in the variables of the extended formulation and added to the current restricted version of the extended formulation along with the subproblem constraints that are active for the subproblem solution. Such column-and-row generation procedure is reviewed and analysed herein. We compare numerically a direct handling of the extended formulation, a standard column generation approach, and the column-and-row generation procedure, highlighting a key benefit of the latter: lifting pricing problem solutions in the space of the extended formulation permits their recombination into new subproblem solutions and results in faster convergence.
Joint work with R. Sadykov

Speaker: Robert Weismantel
ETH Zurich

About optimization of nonlinear functions over integer points in polyhedra

This talk deals with the problem of optimizing nonlinear functions over the lattice points in a polyhedral set. We present families of polynomial time algorithms for special cases of the general problem. Each such algorithm makes use of combinatorial, algebraic or geometric properties of the underlying problem. Of particular importance for obtaining complexity results is the oracle presenting the nonlinear objective function. A particular focus of the talk concerns the special case when the nonlinear objective function is convex.