

$$\begin{pmatrix} 1 & x_2 & x_3^2 & x_4^3 & x_5^4 \\ x_1 & x_2^2 & x_3^3 & x_4^4 & x_5^5 \\ x_1^2 & x_2^3 & x_3^4 & x_4^5 & x_5^6 \\ x_1^3 & x_2^4 & x_3^5 & x_4^6 & x_5^7 \end{pmatrix} \sim \begin{pmatrix} 1 & x_3^2 & x_2 & x_4^3 & x_5^4 \\ x_1^2 & x_3^4 & x_2^3 & x_4^5 & x_5^6 \\ x_1 & x_3^3 & x_2^4 & x_4^4 & x_5^5 \\ x_1^3 & x_3^5 & x_2^6 & x_4^6 & x_5^7 \end{pmatrix} \begin{pmatrix} 1 & x_2 & x_3^2 & x_4^3 & x_5^4 \\ x_1 & x_2^2 & x_3^3 & x_4^4 & x_5^5 \\ x_1^2 & x_2^3 & x_3^4 & x_4^5 & x_5^6 \\ x_1^3 & x_2^4 & x_3^5 & x_4^6 & x_5^7 \\ x_1^4 & x_2^5 & x_3^6 & x_4^7 & x_5^8 \end{pmatrix} \sim \begin{pmatrix} 1 & x_3^2 & x_5^4 & x_2 & x_4^3 \\ x_1^2 & x_3^4 & x_5^6 & x_2^3 & x_4^5 \\ x_1^4 & x_3^6 & x_5^8 & x_2^5 & x_4^7 \\ x_1 & x_3^3 & x_5^5 & x_2^4 & x_4^4 \\ x_1^3 & x_3^5 & x_5^7 & x_2^6 & x_4^6 \end{pmatrix}$$

FIGURE 1. The rows and columns of $(x_i^{i+j-2})_{1 \leq i, j \leq n}$ are permuted to group the even monomials into two blocks.

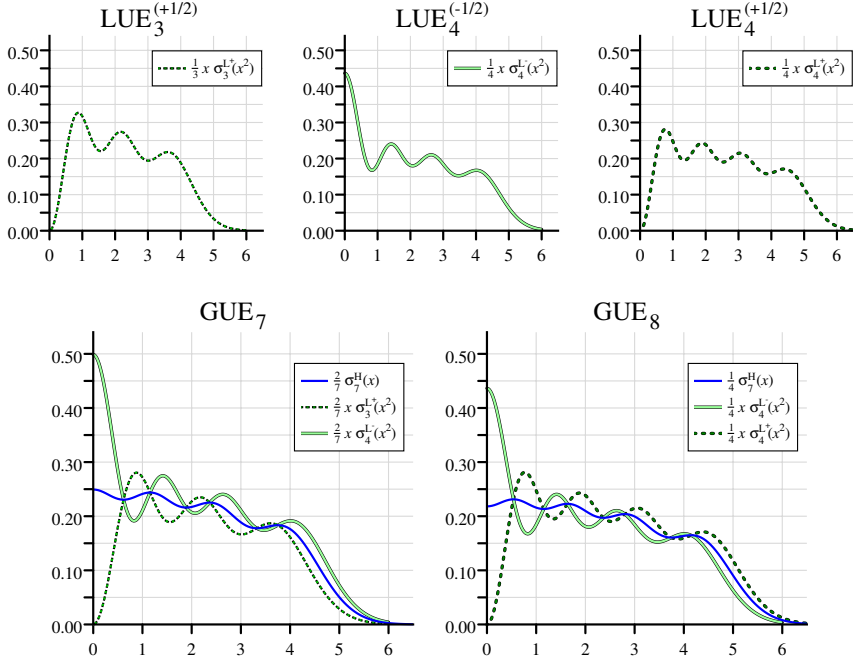


FIGURE 2. The bottom two plots give level densities for GUE singular values (blue) as weighted averages of the Laguerre level densities (green).

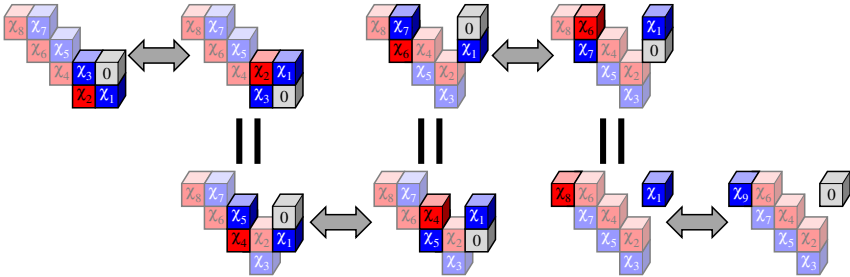


FIGURE 3. Four orthogonal transformations (gray arrows) act on two columns at a time to transform a matrix distributed as A_7 into one distributed as B_7 .

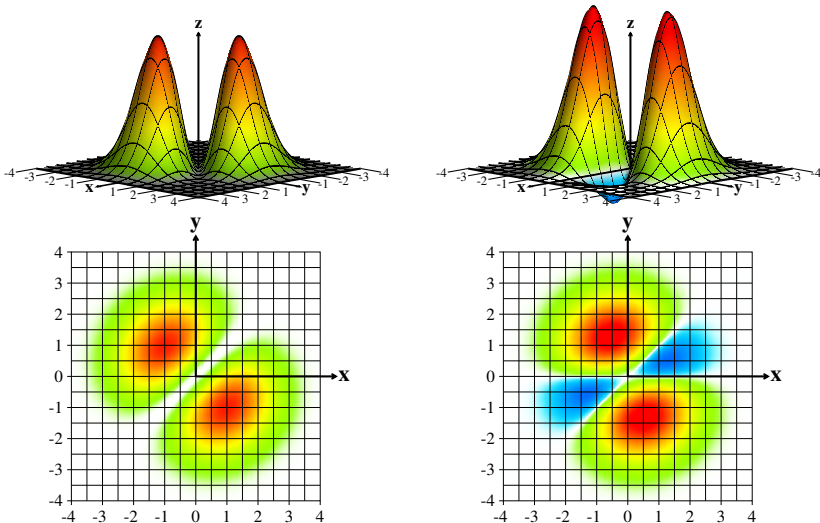


FIGURE 4. The density $p_2^H(x, y)$ (left) and signed measure $\mu_2^H(x, y)$ (right).

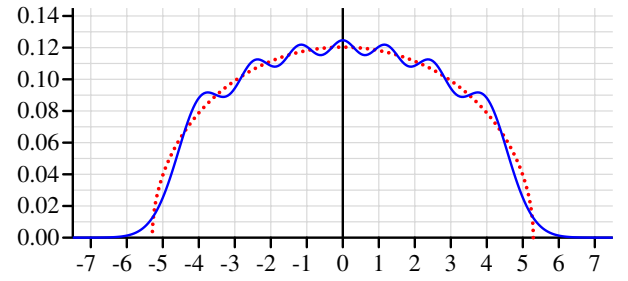


FIGURE 5. A semi-ellipse, $y = \frac{1}{\sqrt{7}\pi} \sqrt{1 - \frac{x^2}{4.7}}$, is approximated by the level density function for the 7×7 GUE, $\frac{1}{7} \sigma_7(x) = \frac{1}{7\sqrt{2\pi}} \sum_{k=0}^6 \frac{H_k(x)^2}{k!} e^{-x^2/2}$.

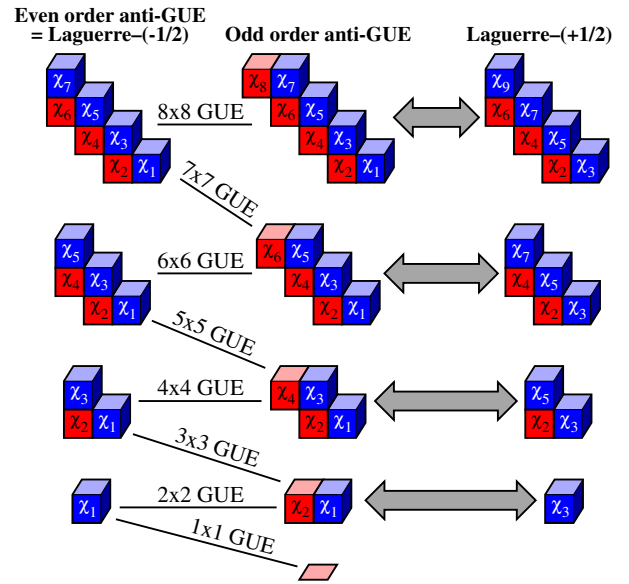


FIGURE 6. Each GUE takes its singular values from two independent blocks corresponding to an even and an odd order anti-GUE. Each odd-order anti-GUEs (second column) has an alternate representation when considered as a Laguerre-(+1/2) matrix (third column).

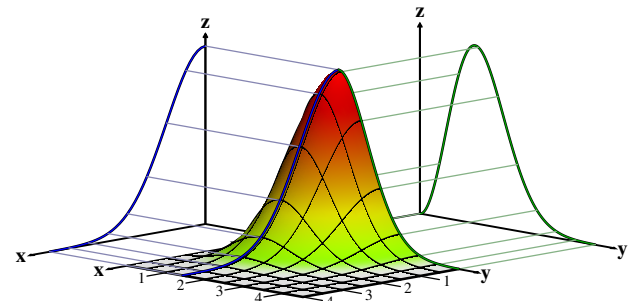


FIGURE 7. The function $g(x, y) = \frac{e}{2} y^2 e^{-x^2/2 - y^2/2}$ satisfies the relation $g(x, y) = g(x, \sqrt{2}) \times g(0, y)$. The factors correspond to the fact that the singular values of the 2×2 GUE have the same distribution as independently distributed χ_1 and χ_3 random variables.