Random Variables

- formally,

$$X : S \mapsto \mathbb{R}$$

- e.g.,

S	HH	HT	TH	TT
X	2	1	1	0

- in practice, just think "variables whose values are random, with different probabilities to take on different values"

Discrete versus Continuous

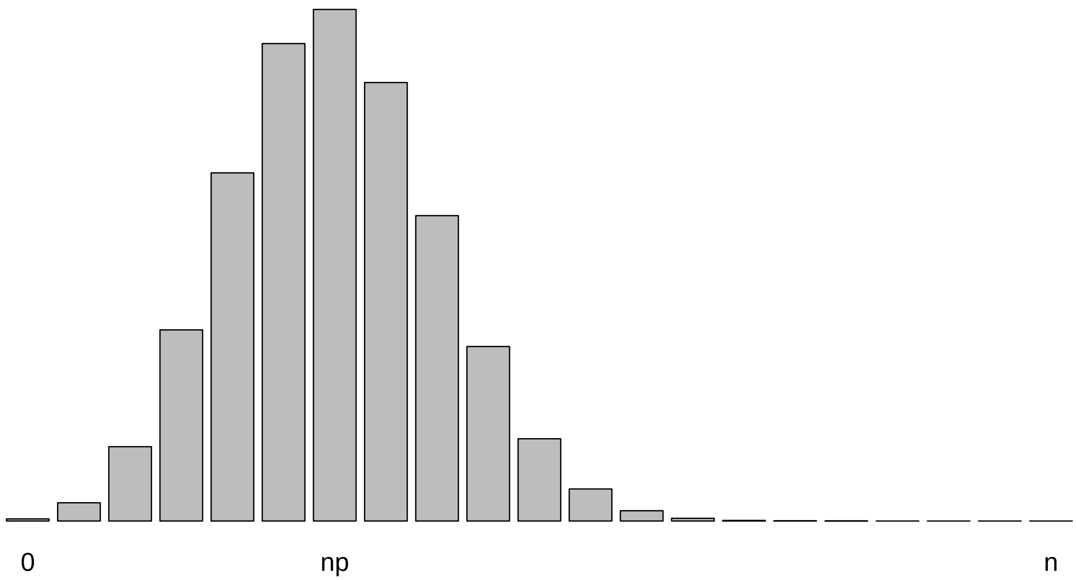
- discrete:
  - set of possible values countable, e.g., $\{x_1, ..., x_m\}$
  - but <u>can</u> be infinite

- continuous:
  - set of possible values uncountable, e.g., $(0, 1), [0, \infty), \mathbb{R}$

Example: Binomial(n, p)

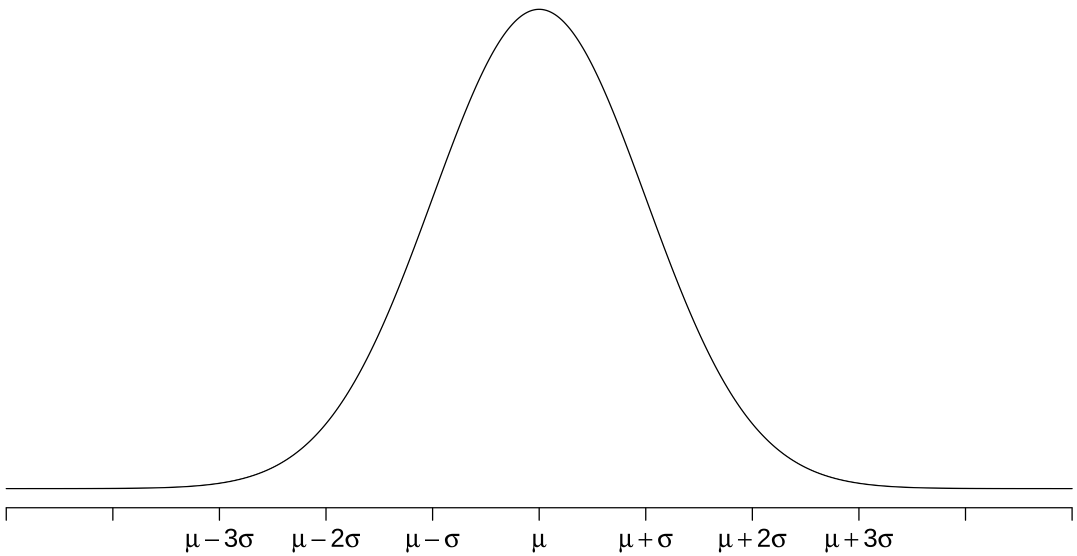
Description Number of "successes" from n independent binary trials, i.e., n coin flips:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x \in \{0, 1, 2, ..., n\}.$$



Example: $\mathbf{N}(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \quad x \in \mathbb{R}$$



Summation versus Integration

$$\mathbb{P}(X \in A) \quad = \quad \sum_{x \in A} f(x) \quad \underline{\text{or}} \quad \int_{x \in A} f(x) dx$$

depending on whether A is a countable or uncountable set

(can also be a "mix" sometimes but not our concern in this course)

Some Key Quantities

$$
\begin{aligned}
F(x) &= \mathbb{P}(X \leq x) \\
&= \sum_{t \leq x} f(t) \quad \underline{\text{or}} \quad \int_{-\infty}^{x} f(t)dt
\end{aligned}
$$

$$
\mathbb{E}(X) = \sum_{\text{all } x} xf(x) \quad \underline{\text{or}} \quad \int_{\text{all } x} xf(x)dx
$$

$$
\mathbb{V}\text{ar}(X) = \mathbb{E}\{[X - \mathbb{E}(X)]^2\}
$$

Remark Read Sections 2.3 + 2.4 on your own, which also highlights such properties as $\mathbb{E}(aX + b) = a\mathbb{E}(X) + b$ and $\mathbb{V}\text{ar}(aX + b) = a^2\mathbb{V}\text{ar}(X)$ if a, b are non-random.