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Dual bases in Temperley-Lieb algebras

Michael Brannan and Benoît Collins

Abstract.

This note is an announcement of the paper [BC16]. We derive a Laurent series expansion in d for the structure coefficients appearing in the dual basis corresponding to the Kauffman diagram basis of the Temperley-Lieb algebra $\mathrm{TL}_k(d)$, converging for all complex loop parameters d with $|d| > 2\cos\left(\frac{\pi}{k+1}\right)$. The coefficients appearing in each Laurent expansion are shown to have a natural combinatorial interpretation and their sign is explicitly understood. As an application, we solve a series of questions raised by Jones and improve substantially our understanding of the Jones Wenzl projection.

§1. Temperley-Lieb algebras, Markov traces, dual bases and a question of Jones

The *Temperley-Lieb algebras* form a very important class of finite-dimensional algebras, arising in a remarkable variety of mathematical and physical contexts including lattice models [TL71], knot theory [KL94], subfactors and planar algebras [JS97], quantum groups [Ban96, CFS95, Wor87b], and topological quantum computation [Abr08, Zha09, DRW16].

Definition 1. Given a complex number $d \in \mathbb{C}^*$ and a natural number $k \in \mathbb{N}$, the kth Temperley-Lieb algebra $\mathrm{TL}_k(d)$ (with loop parameter d) is the unital finite-dimensional complex associative algebra given by the finite set of generators $1, u_1, \ldots, u_k$ subject to the relations $u_i u_j = u_j u_i$ when $|i-j| \geq 2$, $u_i u_{i+1} u_i = u_i$, and $u_i^2 = du_i$.

With $k \in \mathbb{N}$ and $d \in \mathbb{C}^*$ fixed as above, we plot the set $[2k] = \{1, \ldots, 2k\}$ on a square clockwise with $\{1, \ldots, k\}$ on the top edge and $\{2k, \ldots, k+1\}$ on the bottom edge. If we connect these points by a non-crossing pairing $p \in NC_2(2k)$ (where $NC_2(2k)$ is the set of non-crossing

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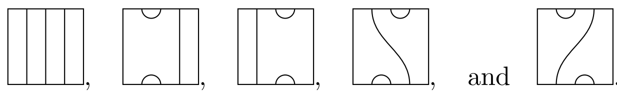
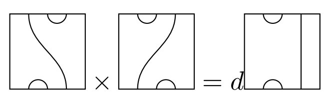
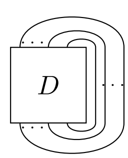
partitions on $\{1,\ldots,2k\}$ endowed with its natural order [NiSp06]), this results in a planar diagram D_p, called a *Temperley-Lieb diagram* (or *Kauffman diagram*). The collection of all such diagrams spans a basis for $\mathrm{TL}_k(d)$, known as the *Kauffman diagram basis*. For example, when $k=3$ there are $C_3=5$ Temperley-Lieb diagrams:



In this description of the Temperley-Lieb algebra, the product D_pD_q of diagrams D_p and D_q is obtained by first stacking diagram D_q on top of D_p, connecting the bottom row of k points on D_q to the top row of k points on D_p. The result is a new planar diagram, which may have a certain number c of internal loops. By removing these loops, we obtain a new diagram D_r for some $r \in NC_2(2k)$ (which is unique up to planar isotopy). The product D_pD_q is then defined to be d^cD_r. For example, we have



The *Markov trace* is the tracial linear functional $\mathrm{Tr} : \mathrm{TL}_k(d) \mapsto \mathbb{C}$ that sends a diagram $D \in \mathrm{TL}_k(d)$ to the *tracial closure of* D:



Specifically, we connect the k points on the top of D to the k points on the bottom of D as indicated in the above picture. The result is a system of loops in the plane. The number of resulting loops is denoted by #loops(D), and then we have

$$\mathrm{Tr}(D) = d^{\#\mathrm{loops}(D)}.$$

We refer to [KL94] for more details. The Temperley-Lieb algebra comes equipped with a natural transpose t which is a linear, antimultipicative map, obtained by replacing a Temperley-Lieb diagram D_p with its symmetric image D_p^t under an horizontal plane symmetry. Using the Markov trace and the transpose t, we can define a symmetric bilinear pairing $\langle\cdot,\cdot\rangle : \mathrm{TL}_k(d) \times \mathrm{TL}_k(d) \to \mathbb{C}$ given by

$$\langle D, D'\rangle = \mathrm{Tr}(D^tD') \qquad (D, D' \in \mathrm{TL}_k(d)).$$

This bilinear form turns out to be non-degenerate precisely when $\mathrm{TL}_k(d)$ is semisimple, and this is guaranteed to happen when $d \neq 2\cos(\frac{\pi}{n})$ for $n \neq 2, 3, 4, \ldots, k+1$. See for example [Wen87, Lic91, BC10].

Given a finite-dimensional vector space E equipped with a non-degenerate bilinear form $\langle \cdot, \cdot \rangle$ and a linear basis $\mathcal{B} = \{x_1, \ldots, x_n\}$ for E, recall that the *dual basis* associated to $\mathcal{B}$ is the unique linear basis $\hat{\mathcal{B}} = \{\hat{x}_1, \ldots, \hat{x}_n\}$ of E with the property that

$$\langle x_i, \hat{x}_j \rangle = \delta_{ij}. \qquad (1 \leq i, j \leq n).$$

For $\mathrm{TL}_k(d)$ (with $k \in \mathbb{N}, d \in \mathbb{C} \backslash \{2\cos(\frac{\pi}{n})\}_{2 \leq n \leq k+1}$), equipped with its non-degenerate bilinear form induced by the Markov trace, we consider the canonical Kauffman diagram basis $\mathcal{B} = \{D_p\}_{p \in NC_2(2k)}$ and the corresponding dual basis $\hat{\mathcal{B}} = \{\hat{D}_p\}_{p \in NC_2(2k)}$. Within this non-degenerate regime, a fundamental problem of interest is to compute explicitly this dual basis. More precisely, Vaughan Jones asked us the following

Question 1. Does each Temperley-Lieb diagram D_q appear with non-zero coefficient in the expansion of each dual basis element $\hat{D}_p$?

In the sequel of this note, we describe a general result that answers this question as a byproduct.

§2. Main result

On the set of non-crossing pair partitions $NC_2(2k)$, we introduce a (non-symmetric) relation $p \to p'$ as follows.

Definition 2. Fix $k \geq 2$. Given two non-crossing pairings $p \neq p' \in NC_2(2k)$, we say that p' is a *non-crossing neighbor* of p (denoted by $p \to p'$), if there exists an interval block $\{t, t+1\} \in p$ and another pair block $\{x, y\} \in p$ with the property that

(1) The partition

$$p'' = \{t, t+1, x, y\} \cup \bigcup_{\substack{\{r,s\} \in p \\ \{r,s\} \neq \{t,t+1\}, \{x,y\}}} \{r, s\}$$

(obtained by merging the two blocks $\{t, t+1\}$ and $\{x, y\}$ into one and keeping all other blocks of p the same) is *non-crossing*, and

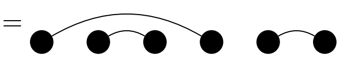
(2) $p' \leq p''$ is the *unique* element of $NC_2(2k)$ such that $p' \neq p$. Here $\leq$ refers to the refinement order on the lattice of partitions of $[2k]$.

In other words, given $p \in NC_2(2k)$, all non-crossing neighbors $p \to p'$ can be constructed via the following four-step algorithm:

(1) Select an interval $\{t, t+1\}$ of p.
(2) Find another pair-block $\{x, y\}$ of p with the property if we merge the two blocks $\{t, t+1\}$ and $\{x, y\}$, we produce a *non-crossing* partition $p'' \in NC(2k)$.
(3) The partition $p'' \in NC(2k)$ admits precisely two refinements contained in $NC_2(2k)$: We have the original pairing $p \leq p''$ that we started with, as well as one one other pairing $p' \leq p''$.
(4) Find this other pairing p' and declare $p \to p'$.

The above definition is understandably hard to digest, so let us further illustrate it with an example.

Example 1. Consider the pairing $p = \big\{\{1,4\},\{2,3\},\{5,6\}\big\} \in NC_2(6)$, which we depict using a typical non-crossing arch diagram:

(1) $p =$ 

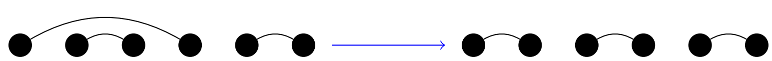
If we select the interval $\{5,6\} \in p$, our only choice is to merge $\{5,6\}$ with the pair $\{1,4\} \in p$ to produce the non-crossing partition $p'' = \big\{\{1,4,5,6\},\{2,3\}\big\}$. The unique $p' \neq p \in NC_2(6)$ that is a refinement of p'' is $p' = \big\{\{1,6\},\{2,3\},\{4,5\}\big\}$. Pictorially, we have

(2)

$p =$  $= p'$

If, on the other hand, we selected the interval $\{2,3\} \in p$ and follow the same procedure as above, we arrive at the only other non-crossing neighbor $p \to p'$:

(3)

$p =$  $= p'$

Given the notion of non-crossing neighbors, we next define an infinite directed graph $\mathcal{G} = (V_{\mathcal{G}}, E_{\mathcal{G}})$ as follows. The vertex set is given by

$$V_{\mathcal{G}} = \bigsqcup_{k \in \mathbb{N}_0} NC_2(2k) \times NC_2(2k),$$

where by convention we define $NC_2(0) \times NC_2(0) = \{(\emptyset, \emptyset)\}$. The set of directed edges $E_{\mathcal{G}} \subset V_{\mathcal{G}} \times V_{\mathcal{G}}$ is given by the following two rules.

(1) If $p, q, p', q' \in NC_2(2k)$, then $((p,q),(p',q')) \in E_{\mathcal{G}}$ if and only if

   (a) $p \to p'$ and $q = q'$, or
   (b) $q \to q'$ and $p = p'$
(2) If $p, q \in NC_2(2k)$ and $p', q' \in NC_2(2k-2)$, then $((p,q),(p',q')) \in E_{\mathcal{G}}$ if and only if there exists a common interval $\{t, t+1\} \in p, q$ and p', q' are the pairings obtained from p, q by removing this common interval.

We call $\mathcal{G}$ the *Weingarten graph*. In our paper [BC16], we prove that that every vertex in $\mathcal{G}$ is connected to $(\emptyset, \emptyset)$. Therefore, we can introduce the following definition.

Definition 3. Given $(p,q) \in NC_2(2k) \times NC_2(2k) \subset V_{\mathcal{G}}$, we denote by $L(p,q) \in \mathbb{N}_0$ the length of the geodesic (= shortest directed path) from (p,q) to $(\emptyset, \emptyset)$. This is an integer.

Next, we describe a connected subgraph $\mathcal{H}$ of the Weingarten graph $\mathcal{G}$. It has the same vertex set $V_{\mathcal{H}} = V_{\mathcal{G}}$, but fewer edges. More specifically, to each vertex (p,q) different from $(\emptyset, \emptyset)$, we chose one of the non-crossing partitions involved (let's say p for the sake of an example), one block $\{t, t+1\}$ of this non-crossing partition (p in our example) and we just keep the directed edges starting from (p,q) that are obtained with the help of this specific block $\{t, t+1\}$.

Definition 4. Fix a Weingarten subgraph $\mathcal{H} \subset \mathcal{G}$. For each vertex $(p,q) \in V_{\mathcal{H}}$ and each $r \in \mathbb{N}_0$, we denote by $m_r(p,q)$ the number of directed paths from (p,q) to $(\emptyset, \emptyset)$ of length $L(p,q) + 2r$ that are contained in $\mathcal{H}$.

With the above definitions, we are now able to state our main theorem

Theorem 2.1. *Let $\{D_p\}_{p \in NC_2(2k)} \subset TL_k(d)$ denote the Kauffman basis, and denote by $\{\hat{D}_p\}_{p \in NC_2(2k)}$ the corresponding dual basis with respect to the bilinear form $\langle \cdot, \cdot \rangle$ induced by the Markov trace. For each p, write $\hat{D}_p = \sum_q \mathrm{Wg}_d(p,q) D_q$, with $\mathrm{Wg}_d(p,q) \in \mathbb{C}$. Then the function $d \mapsto \mathrm{Wg}_d(p,q)$ has the following absolutely convergent Laurent series expansion*

(4)
$$\mathrm{Wg}_d(p,q) = (-1)^{|p \vee q| + k} \sum_{r \geq 0} m_r(p,q) d^{-L(p,q) - 2r} \qquad \Big(|d| > 2\cos\big(\frac{\pi}{k+1}\big)\Big),$$

where $L(p,q)$ and $(m_r(p,q))_{r \in \mathbb{N}_0}$ are the integer quantities defined previously.

This theorem has the following important corollary:

Theorem 2.2. *For generic loop parameters d, every coefficient in the diagram expansion of the dual basis (in particular, the Jones-Wenzl projection – c.f. Section 4) of $TL_k(d)$ is non-zero. More precisely, we have* $\mathrm{Wg}_d(p,q) \neq 0$ *when*

$$d \in \mathbb{R}\backslash\Big[-2\cos\big(\frac{\pi}{k+1}\big), 2\cos\big(\frac{\pi}{k+1}\big)\Big] \quad \textit{or} \quad |d|, \forall d \in \mathbb{C}, d \textit{large enough.}$$

§3. Relation to quantum groups

Our proof of Theorem 2.1 relies on connecting the problem of computing the values of the coefficients of each Temperley-Lieb diagram appearing in the expansion of a dual basis element in $\mathrm{TL}_k(d)$ to a seemingly different problem of computing polynomial integrals over a class of compact quantum groups, called *free orthogonal quantum groups.* Using a combinatorial tool called the *Weingarten calculus*, we are able to interpret generic coefficients of dual basis elements in terms of certain moments of coordinate functions over free orthogonal quantum groups taken with respect to the Haar integral. This new operator algebraic quantum group perspective has the advantage of revealing "hidden" algebraic relations between the structure coefficients of the dual basis.

Very roughly, the problem of computing polynomial integrals over this class of quantum groups is encoded in a family of functions indexed by pairs of non-crossing pairings called *Weingarten functions*, which turn out to be exactly the coefficients $\mathrm{Wg}_d(p,q)$, when viewed as functions of $d \in \mathbb{C}$. With regards to the asymptotics of the Weingarten function, estimates were given [BCS12, CS11] in an attempt to isolate the order and the value of the leading term in the $\frac{1}{d}$-expansion of $\mathrm{Wg}_d(p,q)$. The best among these prior works was Theorem 4.6 in [CS11], which isolates the leading non-zero term in $\mathrm{Wg}_d(p,q)$ for certain pairs of pairings (p,q). On the other hand, it is clear that the Laurent series expansion for $\mathrm{Wg}_d(p,q)$ in Theorem A provides the first explicit description of the leading term for *all* possible pairs (p,q). In fact, in some cases, the leading order of $\mathrm{Wg}_d(p,q)$ that one might anticipate based on an examination of Theorem 4.6 in [CS11] can differ from the true value given by Theorem A. See [BC16, Example 3].

§4. Historical remarks and application to Jones-Wenzl projections

In this section we make some connections between our work and prior works on the computation of special elements of $\mathrm{TL}_k(d)$ called

Jones-Wenzl projections. We begin with a definition/theorem describing these objects. See [Wen87, KL94] for details.

Definition 5. Let $k \in \mathbb{N}$ and $d \in \mathbb{C}\backslash\{2\cos(\frac{\pi}{n})\}_{2\leq n\leq k+1}$ be as above. Then there exists a unique non-zero idempotent $q_k \in \mathrm{TL}_k(d)$, called the *Jones-Wenzl projection*, with the property that

$$u_i q_k = q_k u_i = 0 \qquad (i = 1, \ldots, k-1). \tag{5}$$

The Jones-Wenzl projections are certain "highest weight" idempotents $q_k \in \mathrm{TL}_k(d)$, and are key to the structure and applications of Temperley-Lieb algebras in representation theory, operator algebras and mathematical physics. Despite the importance of the Jones-Wenzl projections, remarkably very little is known about these idempotents beyond the fundamental Wenzl recursion formula [Wen87] and its various generalizations and extensions. See for example [FK97, Mor15, Ocn02].

The problem of explicitly determining the expansion of q_k in terms of the Kauffman basis $\{D_p\}_{p\in NC_2(2k)}$, as well as the problem of determining when the structure coefficients appearing in this expansion vanish arose in many contexts: from subfactor theory and representation theory [Ocn02], to topological quantum computation [DRW16, Problem 3.15]. Over the years, some progress on these problems has been made. Perhaps the most notable is the announcement of a closed formula for the coefficients of the Jones-Wenzl projection q_k by Ocneanu [Ocn02] which solves both problems in the affirmative, at least for real-valued loop parameters. This formula of Ocneanu was later verified in certain special cases by Reznikoff [Rez02, Rez07]. Another complementary approach to the computation of the coefficients of q_k was developed independently by Morrison [Mor15] and Frenkel-Khovanov [FK97].

The above problems concerning the Jones-Wenzl projection q_k are in fact just special cases of the more general problem of computing structure coefficients of the dual diagram basis $\{\hat{D}_p\}_{p\in NC_2(2k)}$ that we address with Theorem 2.1. Indeed, one has the $q_k = \frac{\hat{D}_{\mathbf{1}}}{\langle \hat{D}_{\mathbf{1}}, \hat{D}_{\mathbf{1}}\rangle}$, where $\mathbf{1} \in NC_2(2k)$ is the pairing corresponding to the unit of $\mathrm{TL}_k(d)$. See [BC16, Lemma 2.1] for details.

Specializing Theorem 2.1 to the case of the Jones-Wenzl projection q_k, we see that it confirms the non-zero coefficients result of Ocneanu [Ocn02], and complements the previous works of Morrison [Mor15, Proposition 5.1] and Frenkel-Khovanov [FK97]. With regards to the more general problem of computing the coefficients of *arbitrary* dual basis elements in $\mathrm{TL}_k(d)$, we note that essentially no prior progress seems to have been made.

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Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368, USA
E-mail address: mbrannan@math.tamu.edu

Department of Mathematics, Kyoto University, and CNRS
E-mail address: collins@math.kyoto-u.ac.jp