# Banach Algebras and Operator Algebras 2024

## Schedule

<table>
<tr><th></th><th>Monday<br>July 8</th><th>Tuesday<br>July 9</th><th>Wednesday<br>July 10</th><th>Thursday<br>July 11</th><th>Friday<br>July 12</th></tr>
<tr><td>9:30-10:30</td><td>Crann$^{e}$</td><td>Ghandehari$^{e}$</td><td>Losert$^{e}$</td><td>Turowska$^{e}$</td><td>Blecher$^{e}$</td></tr>
<tr><td>10:30-11:00</td><td>Fernandez Quero$^{e}$<br>Matsuda$^{r}$</td><td>Mathieu$^{e}$<br>Eleftherakis$^{r}$</td><td colspan="2">Coffee$^{c}$</td><td>Smith$^{e}$<br>Galindo$^{r}$</td></tr>
<tr><td>11:00-11:30</td><td colspan="2">Coffee$^{c}$</td><td rowspan="2">Daws$^{e}$</td><td rowspan="2">Soltan$^{e}$</td><td>Coffee$^{c}$</td></tr>
<tr><td>11:30-12:00</td><td rowspan="2">Anderson-<br>Sackaney$^{e}$</td><td rowspan="2">Yang$^{e}$</td><td rowspan="2">Dykema$^{e}$</td></tr>
<tr><td>12:00-12:30</td><td rowspan="2">Willis$^{e}$</td><td rowspan="2">Adamo$^{e}$</td></tr>
<tr><td>12:30-13:00</td><td colspan="2" rowspan="2">Lunch</td><td rowspan="3"></td><td rowspan="3">Lunch</td></tr>
<tr><td>13:00-14:45</td><td rowspan="4">Free<br><br>afternoon</td></tr>
<tr><td>14:45-15:15</td><td rowspan="2">Wiersma$^{e}$</td><td rowspan="2">Phillips$^{e}$</td><td rowspan="2">Choi$^{e}$</td></tr>
<tr><td>15:15-15:45</td><td>Schultz$^{e}$<br>Monfared$^{r}$</td></tr>
<tr><td>15:45-16:15</td><td colspan="2">Tea$^{c}$</td><td colspan="2">Tea$^{c}$</td></tr>
<tr><td>16:15-16:45</td><td>Georgescu$^{e}$<br>Hoefer$^{r}$</td><td>Rosbotham$^{e}$<br>Stokke$^{r}$</td><td rowspan="2">Voigt$^{e}$</td><td>Escolano$^{e}$<br>Chatzinikolaou$^{r}$</td></tr>
<tr><td>16:45-17:15</td><td>Marcoux$^{e}$<br>Naderi$^{r}$</td><td>Szczepanski$^{e}$<br>Rankin$^{r}$</td><td>Acuaviva$^{e}$<br>Racher$^{r}$</td></tr>
<tr><td>17:15-17:45</td><td>Paulsen$^{e}$</td><td>Samei$^{e}$</td><td>Lee$^{e}$</td><td></td></tr>
</table>

$e$ E2 1736, $c$ RCH 212, $r$ RCH 106 (we also have RCH 109, which people may use for discussion at any time)

# Plenary Lectures

**Maria Stella Adamo**, FAU Erlangen-Nürnberg

### Construction of Wightman fields for a class of 2D CFTs

We construct Wightman fields for a particular class of two-dimensional conformal quantum field theories (2d CFTs). These theories are described using Haag-Kasteler nets of operator algebras that verify a physically motivated set of axioms and are invariant under conformal transformations.

We take the case of full 2d CFTs with pointed representation categories, i.e., those with chiral components whose representation categories possess enough automorphisms, overviewing our construction for the U(1)-current example.

This talk is based on joint work with L. Giorgetti, Y. Tanimoto, CMP 2023.

**Benjamin Anderson-Sackaney**, University of Saskatchewan

### C*-Simplicity and related notions for quantum groups

A group is C*-simple if its reduced C*-algebra is simple. Two of the most historically important notions used for proving a group is C*-simple are Powers' averaging property and topologically free boundary actions. These techniques led to remarkable dynamical characterizations of C*-simplicity. Besides the establishment of many quantum examples via a quantum version of Powers' averaging property, little progress has been made on C*-simplicity for quantum groups. In this talk we will introduce the notion of a stably strong C*-faithful action of a discrete quantum group, which we note that in the case of a group acting on a topological boundary, is equivalent to topological freeness. We are able to use this notion to recover C*-simplicity of free unitary discrete quantum groups. We will also discuss boundary maps and their involvement with Powers's averaging property, C*-simplicity, and topological freeness.

This talk is based on joint work with Roland Vergnioux.

**David Blecher**, University of Houston

NONCOMMUTATIVE FUNCTION THEORY IN OPERATOR ALGEBRAS

We begin by discussing the noncommutative (C*-algebraic) variant of peak interpolation sets in function theory, on which several profound results in that theory rely. We find appropriate 'quantized' versions of some of these classical facts. Through a delicate generalization of a theorem of Varopoulos, we show that, roughly speaking, sufficiently regular interpolation projections are peak precisely when their atomic parts are. As an application, we give alternative proofs and sharpenings of some recent peak-interpolation results of Davidson and Hartz for algebras on Hilbert function spaces. In another direction, given a convex subset of the state space, we study the associated Riesz projection. This is then applied to various important topics in noncommutative function theory, such as the F. & M. Riesz property, the existence of Lebesgue decompositions, the description of Henkin functionals, and Arveson's noncommutative Hardy spaces (maximal subdiagonal algebras). Our approach to noncommutative function theory is via replacing closed sets by closed projections. A common theme in the talk is noncommutative generalization of the profound role that 'A-null sets' play in function theory. Joint with Raphael Clouatre.

**Yemon Choi**, Lancaster University

HOMOLOGICAL INVARIANTS OF FOURIER ALGEBRAS

To each locally compact group $G$, one may associate its *Fourier algebra* $A(G)$; this is a Banach function algebra on $G$, whose norm encodes the group structure on $G$ and not just its topological structure. From a modern perspective one, one may view $A(G)$ as the $L^1$-convolution algebra of the quantum dual of $G$. It is therefore natural to study invariants of Banach algebras, specialized to the setting of Fourier algebras, to see how much information they remember about the group one started with.

Two possible invariants are: the amenability constant, and the alternating cohomology groups. In this talk I will give an overview of what is known for these invariants, attempting to motivate why they could be interesting to study for Fourier algebras. Time permitting, I will discuss some of my own recent contributions (`arXiv 2008.02226` and `2012.14413`) and suggest

avenues for further research.

**Jason Crann**, Carleton University

Self-testing in operator systems and the commuting operator framework

The area of self-testing aims to understand the structure of (bipartite) quantum correlations and to identify those correlations which admit a unique physical realization up to local isometric dilation. Aside from important applications in quantum information theory, self-testing techniques were crucial to the refutation of Connes' Embedding Problem. Motivated in part by this application, an operator-algebraic formulation of self-testing was recently introduced by Paddock et. al., where, among other things, they characterized finite-dimensional self-testing quantum correlations through a unique extension property of an associated state to the universal C*-algebra of bipartite positive operator-valued measures. Building on this work, we introduce a general operator system framework for self-testing which applies, in particular, to the recently introduced class of quantum no-signalling correlations. We also introduce a local isometric dilation order in the commuting operator framework and show that maximality in this order implies unique state extension to a pertinent universal C*-algebra. The converse holds for type I correlations. This is joint work with Ivan Todorov and Lyudmyla Turowska.

**Matthew Daws**, Lancaster University

Types of $L^1$ Banach and operator algebras

There has been much recent interest in $L^p$-operator algebras: closed subalgebras of operators on $L^p$ spaces. Interesting examples of such algebras, for example group algebras, seem to behave a little like C*-algebras, although often they can be more "rigid". In the case $p = 1$, things differ, as we are no longer in the world of dual Banach algebras, but also sometimes things simplify, for example, the action of the $L^1$ group algebra on the natural $L^1$ space is an isometry, and so there is no distinction between the $L^1$-operator algebra, and the familiar $L^1$-Banach algebra. I will discuss some related ideas through two classes of examples: analogues of the Cuntz algebra, and analogues of Hecke algebras. In both cases, we modify familiar Banach algebra

constructions, starting with semigroup algebras, to produce $L^1$-type algebras which are not quite $L^1$ operator algebras. While not overly sophisticated in construction, I hope to argue that these classes of examples might be worthy of further study.

**Ken Dykema**, Texas A&M University

Invariant subspaces and decompositions of operators in (or affiliated to) finite von Neumann algebras

A matrix $T$ of complex numbers can be understood by considering theeigenspaces of $T$ and using these to write $T$, via a similarity using an invertible matrix, in Jordan canonical form. We may regard this as a spectral decomposition of $T$. Now suppose $M$ is a von Neumann algebra having a (specified) tracial state and let $T$ be an element of $M$. Similar constructions in some cases can be performed using Haagerup-Schultz projections instead of eigenspaces. A key difference is that instead of spectral decompositions, we have decompositions according to Brown measure, which is in some sense a spectral distribution measure for elements of von Neumann algebra with a trace. The analogues of Jordan canonical forms are obtained via similarities by invertible elements of $M$. We can obtain such "Jordan canonical forms" for more elements, if we allow similarities by invertible elements in the algebra of unbounded operators affiliated to $M$.

(Joint work with Amudahn Krishnaswamy-Usha.)

**Mahya Ghandehari**, University of Delaware

Characterizing directed singularities via continuous wavelet transforms

Let $H$ be a closed subgroup of $\mathrm{GL}_d(\mathbb{R})$, and consider the semidirect product group $G = \mathbb{R}^d \rtimes H$. Assuming that $H$ satisfies certain 'admissibility conditions', one can use the quasi-regular representation of $G$ together with a suitable vector (called a wavelet) to construct a generalized continuous wavelet transform. In this talk, we discuss the problem of characterizing the directed singularities of a tempered distribution in terms of its continuous wavelet transform based on a compactly supported wavelet. We show how the previously known necessary and sufficient criteria for band-limited

wavelets (i.e., the ones with compactly supported Fourier transforms) can be extended to the case of compactly supported wavelets. This talk is based on joint work with Hartmut Führ.

**Viktor Losert**, University of vienna

The centre of the bidual of Fourier algebras – compact groups

For the case of a compact group $G$, we talk about the question of strong Arens irregularity of the Fourier algebra $A(G)$, presenting various conditions and examples.

**N. Christopher Phillips**, University of Oregon

Incompressible Banach algebras

This is joint work with Bill Johnson.

We call a Banach algebra *isometrically incompressible* if whenever $B$ is another Banach algebra and $\varphi\colon A \to B$ is a contractive injective homomorphism, then $\varphi$ is isometric. We say that $A$ is *incompressible* if whenever $\varphi$ as above is bounded and injective, then $\varphi$ is bounded below. There are also a uniform version and a strengthening involving quotients.

The terminology is new, but the first results go back to about 1950: every C*-algebra is incompressible and isometrically incompressible, and whenever $E$ is a Banach space and $A \subset L(E)$ contains all finite rank operators, then $A$ is incompressible and isometrically incompressible. On the other hand, the algebra of $2 \times 2$ upper triangular matrices is not isometrically incompressible, and the disk algebra and the convolution algebra $l^1(\mathbb{Z})$ are not incompressible. Motivation for the current work is new results on incompressibility for Calkin algebras $L(E)/K(E)$, and the idea that isometric incompressibility of a Banach algebra is an indication that it is somehow "C* like".

In this talk, I will state the definitions carefully, give basic illustrative examples and nonexamples, and give some history. Then I will give some new results and open problems, including a previously unknown relation between isometric and uniform incompressibility, incompressibility for more Calkin algebras, and results, counterexamples, and open problems about when some

version of incompressibility for a Hilbert space operator algebra implies that it is C*-algebra.

**Piotr Sołtan**, University of Warsaw

Von Neumann algebras of discrete quantum groups, their invariants and the i.c.c. condition

I will describe invariants of compact quantum groups which are of similar nature to the invariants used in the classification of von Neumann algebras. These were used to study examples of compact quantum groups whose associated von Neumann algebras are factors of type III. A more systematic study of these invariants lead to a conjecture which has been shown to hold for some natural classes of compact quantum groups. One of these classes are compact quantum groups dual to discrete quantum groups satisfying a condition which for classical groups is equivalent to the i.c.c. condition.

**Lyudmila Turowska**, Chalmers University of Technology

No-signalling values of cooperative quantum games

Finding values, the optimal winning probability, of various non-local games over different strategies has been an important task in Quantum Information Theory and also for resolving the Connes Embedding Problem. In this talk I will discuss values of quantum games (games with quantum inputs and outputs), arising from the type hierarchy of quantum no-signalling correlations, establishing operator space tensor norm expressions for each of the correlation types. This is a joint work with Jason Crann, Rupert Levene and Ivan Todorov.

**Christian Voigt**, University of Glasgow

Beurling-Fourier algebras associated with $q$-deformations of compact semisimple Lie groups

Beurling-Fourier algebras are weighted versions of the Fourier algebra of a locally compact group. In the case of compact Lie groups the spectra of these commutative Banach algebras are closely related to the complexification of

the underlying group. This provides an interesting link between harmonic analysis and geometry. In this talk I will discuss Beurling-Fourier algebras for a certain class of compact quantum groups, namely those arising as standard deformations of compact semisimple Lie groups. It turns out that the resulting theory is again related to complexification, now interpreted in terms of the Drinfeld double construction. (This is joint work with Heon Lee)

**Matthew Wiersma**, University of Winnipeg

New tensor products of C*-algebras and characterization of type I C*-algebras as rigidly symmetric C*-algebras

For C*-algebras $A$ and $B$, we construct three related classes of cross-norm completions $A \otimes_\mu B$ of $A \odot B$. The first class of these constructions produces Banach algebras, the second Banach $*$-algebras, and the third produces C*-algebras. For certain discrete groups $G_1$ and $G_2$, our C*-algebraic construction produces $2^{\aleph_0}$ distinct C*-completions of $C_r^*(G_1) \odot C_r^*(G_2)$.

A Banach $*$-algebra $A$ is *symmetric* if the spectrum $\sigma(a^*a)$ is contained in $[0, \infty)$ for every $a \in A$ and *rigidly symmetric* if the Banach space tensor product $A \otimes_\gamma B$ is symmetric for every C*-algebra $B$. A theorem of Kügler asserts that every type I C*-algebra is rigidly symmetric. Leveraging our new constructions, we establish the converse of Kügler's theorem by showing for C*-algebras $A$ and $B$ that $A \otimes_\gamma B$ is symmetric if and only if $A$ or $B$ is type I. This settles a question of Leptin and Poguntke from 1979.

This is based on joint work with H.H. Lee and E. Samei.

**George Willis**, University of Newcastle NSW

Dynamics of automorphisms of locally compact groups

Each automorphism, $\alpha$, of a locally compact group, $G$, determines a dynamical system $(G, \mathbf{Z})$ with $\mathbf{Z}$ acting by iterates of $\alpha$. If $G$ is a Lie group, then the action of the linear transformation $\mathrm{ad}(\alpha)$ on the Lie algebra $\mathfrak{g}$ models the dynamics. This suffices to treat connected locally compact groups via the solution to Hilbert's $5^{\text{th}}$ problem and approximation by Lie groups.

In the case of totally disconnected locally compact (t.d.l.c.) groups, Lie alge-

bra methods apply to Lie groups over t.d.l.c. fields but these do not go close to approximating general t.d.l.c. groups. Much can be said about the action of $\alpha$ on t.d.l.c. $G$ all the same. There are strong parallels with the linear case, abstract models of important features exist and the techniques may be used to answer questions from the literature. The talk will survey these ideas.

**Dilian Yang**, University of Windsor

Higher rank Baumslag-Solitar semigroups

We propose a notion of higher rank Baumslag-Solitar semigroups and study their C*-algebras. In this talk, we focus on two special classes: one is related to products of odometers, and the other is related to Furstenberg's $\times p, \times q$ conjecture. For the former class, we characterize the factoriality of the associated von Neumann algebras and further determine their types; for the latter, we provide their canonical Cartan subalgebras.

# Contributed lectures

**Antonio Acuaviva**, Lancaster University

### Factorizations and minimality of the Calkin Algebra norm for $C(K)$-spaces

An algebra norm on an algebra $\mathcal{A}$ is minimal if the topology it generates is the smallest among all algebra-norm-induced topologies on $\mathcal{A}$. We investigate the minimality of the essential norm on the Calkin algebra for $C_0(K)$-spaces, where $K$ is a scattered, locally compact Hausdorff space. Examples of these spaces include ordinal intervals and Mrówka spaces.

**Alexandros Chatzinikolaou**, National and Kapodistrian University of Athens

### Operator systems, contextually and nonlocality

Nonlocality and contextuality are arguably some of the most interesting phenomena of quantum mechanics. Nonlocality has been studied extensively during the past decade via the setup of nonlocal games, uncovering connections between operator algebras and quantum information theory, among which being the equivalence of Connes's embedding problem with Tsirelson's problem. We show how operator algebraic constructions and methods can be used to describe several features of contextuality. This approach generalises many of the well known results in nonlocal games. Also, we provide equivalences of Connes's embedding problem in terms of tensor products of operator systems and C*-algebras arising from contextuality scenarios. The talk is based on a joint work with M. Anoussis and I. G. Todorov.

**George K. Eleftherakis**, University of Patras

### The similarity problem and hyperreflexivity of von Neumann algebras

We say that a C*-algebra $\mathcal{A}$ satisfies the similarity property (SP) if every bounded homomorphism $u\colon \mathcal{A} \to \mathcal{B}(H)$, where $H$ is a Hilbert space, is similar to a $*$-homomorphism and that a von Neumann algebra $\mathcal{A}$ satisfies the weak similarity property (WSP) if every w*-continuous, unital and bounded

homomorphism $u\colon \mathcal{A} \to \mathcal{B}(H)$, where $H$ is a Hilbert space, is similar to a $*$-homomorphism. We prove that a von Neumann algebra $\mathcal{A}$ satisfies WSP if and only if the algebras $\mathcal{A}'\bar{\otimes}\mathcal{B}(\ell^2(J))$ are hyperreflexive for all cardinals $J$. We also introduce the hypothesis (CHH): Every hyperreflexive separably acting von Neumann algebra is completely hyperreflexive. We show that under (CHH), all C*-algebras satisfy SP. Finally, we prove that the spatial tensor product of an injective von Neumann algebra and a von Neumann algebra satisfying WSP also satisfies WSP.

**Gerrardo M. Escolano**, University of Granada

Lie-Trotter formulae in Jordan-Banach algebras & applications

We establish some Lie–Trotter formulae for unital complex Jordan–Banach algebras, showing that for each couple of elements $a, b$ in a unital complex Jordan–Banach algebra $\mathfrak{A}$ the identities

$$\lim_{n\to\infty} \left(e^{\frac{a}{n}} \circ e^{\frac{b}{n}}\right)^n = e^{a+b}, \ \lim_{n\to\infty} \left(U_{e^{\frac{a}{n}}}\left(e^{\frac{b}{n}}\right)\right)^n = e^{2a+b}, \text{ and}$$

$$\lim_{n\to\infty} \left(U_{e^{\frac{a}{n}},e^{\frac{c}{n}}}\left(e^{\frac{b}{n}}\right)\right)^n = e^{a+b+c}$$

hold. These formulae are employed in the study of spectral-valued (non-necessarily linear) functionals $f : \mathfrak{A} \to \mathbb{C}$ satisfying $f(U_x(y)) = U_{f(x)}f(y)$, for all $x, y \in \mathfrak{A}$. We prove that for any such a functional $f$, there exists a unique continuous (Jordan-)multiplicative linear functional $\psi\colon \mathfrak{A} \to \mathbb{C}$ such that $f(x) = \psi(x)$, for every $x$ in the connected component of set of all invertible elements of $\mathfrak{A}$ containing the unit element. If we additionally assume that $\mathfrak{A}$ is a JB$^*$-algebra and $f$ is continuous, then $f$ is a linear multiplicative functional on $\mathfrak{A}$. The new conclusions are appropriate Jordan versions of results by Maouche, Brits, Mabrouk, Shulz, and Touré.

**Adriana Fernández Quero**, University of Iowa

Rigidity results for group von Neumann algebras with diffuse center

We introduce the first examples of groups $G$ with infinite center which in a natural sense are completely recognizable from their von Neumann algebras,

$\mathcal{L}(G)$. Specifically, assume that $G = A \times W$, where $A$ is an infinite abelian group and $W$ is an ICC wreath-like product group with property (T) and trivial abelianization. Then whenever $H$ is an *arbitrary* group such that $\mathcal{L}(G)$ is $*$-isomorphic to $\mathcal{L}(H)$, via an *arbitrary* $*$-isomorphism preserving the canonical traces, it must be the case that $H = B \times H_0$ where $B$ is infinite abelian and $H_0$ is isomorphic to $W$. Moreover, we completely describe the $*$-isomorphism between $\mathcal{L}(G)$ and $\mathcal{L}(H)$. This yields new applications to the classification of group C*-algebras, including examples of non-amenable groups which are recoverable from their reduced C*-algebras but not from their von Neumann algebras.

**Jorge Galindo**, IMAC, University Jaume I

Two problems on the Fourier algebra related to almost periodic functions

In this talk, we will consider two results on the group algebra $L^1(G)$ of a compact group $G$ that do not seem to have a proved counterpart for the Fourier algebra $A(\Gamma)$ of a discrete group $\Gamma$. Both involve almost periodic functions and are related to the construction of Riesz and Rosenthal sets. These questions arose in a joint quest with Mahmoud Filali and Reza Esmailvandi to find ideals of $A(\Gamma)$ that are Arens regular but not reflexive. Although we will leave the former results unproved we will describe an alternative way to obtain the latter.

**Magdalena Georgescu**

Nuclear dimension of twisted groupoid C*-algebras

Groupoid models of C*-algebras provide an avenue for exploring properties of a C*-algebra by identifying relevant features at the groupoid level. One property of interest is finite nuclear dimension, which plays a pivotal role in the classification program.

In 2017, Guentner, Willet and Yu introduced the concept of dynamic asymptotic dimension for a locally compact, Hausdorff and étale groupoid $\mathcal{G}$. If the groupoid $\mathcal{G}$ is principal, has dynamic asymptotic dimension $d$ and the unit space of $\mathcal{G}$ has topological covering dimension $N$ then the nuclear dimension of the reduced C*-algebra of $\mathcal{G}$ is bounded above by $(N+1)(d+1)-1$.

This talk is based on joint work with Kristin Courtney, Anna Duwenig, Astrid an Huef and Maria Grazia Viola. We extended the Guentner, Willet and Yu result to a twist over a locally compact, Hausdorff, étale and principal groupoid $\mathcal{G}$. I will discuss our approach to the proof, and related results for non-principal groupoids.

**Gage Hoefer**, University of Delaware

Transfer of strategies for quantum games

The study of non-local games, rooted in Bell's celebrated work from the '60s, has over the past few decades shown the benefits and limitations of quantum entanglement. Interest in quantum non-local games – where input and output sets of questions and answers are replaced by input and output states – has been particularly growing in the last few years, due to significant potential applications in the study of operator algebras. Utilizing the simulation paradigm in information theory (amongst other tools), in this talk I introduce the necessary setup for the comparison and transference of strategies of quantum games, which are sufficiently "similar". I investigate game strategy transport and the existence of strategies for quantum games using an operator system approach. Specifically focusing on perfect strategies for quantum games of different types, I also briefly discuss applications of strategy transport between games. This is partially based on previous joint work with Ivan G. Todorov (in `arXiv:2211.04851` and `arXiv:2311.06355`), and current solo work.

**Hun Hee Lee**, Seoul National University

Analytic subalgebras of Beurling-Fourier algebras and complexification of Lie groups

The Gelfand spectrum of a Fourier algebra can be identified with the underlying group. Giselsson/Turowska recently established that the Gelfand spectrum of a weighted Fourier algebra (called a Beurling-Fourier algebra) on a connected Lie group G can be realized inside a (abstract) complexification of G when the weight is coming from an abelian subgroup. In this paper, we take one step further to show that the elements in the aforementioned Gelfand spectrum can be viewed as evaluations on specific points of a

complexification G when restricted to a particular dense subalgebra, which we call an "analytic subalgebra". We first introduce an analytic subalgebra allowing a "local" solution for general connected Lie groups. We will demonstrate that a "global" solution is also possible for connected, simply connected and nilpotent Lie groups through a different choice of an analytic subalgebra. Finally, we examine the case of the $ax + b$-group as an example of a non-nilpotent, non-unimodular Lie group with a "global" solution.

**Laurent Marcoux**, University of Waterloo

Operator theory problems in Banach and Operator algebras

Over the years, there have been a number of problems in (single) operator theory which have been resolved in that setting, but which have interesting analogues when recast in the setting of Banach and Operator algebras. In this talk, we shall provide background on these problems, and discuss several subproblems arising from them.

**Junichiro Matsuda**, University of Waterloo

On the degree of regular quantum graphs

The degree of a regular finite quantum graph is defined as an eigenvalue of the adjacency matrix, which may not be an integer in general. However, in the tracial case, we obtained that the degree is indeed an integer. The clue is a characterization of regularity in terms of the operator bimodule associated with the quantum graph. This is a joint work with Matthew Kennedy and Larissa Kroell.

**Martin Mathieu**, Queen's University Belfast

Answering Jafarian's question

In joint work with Francois Schulz (Johannesburg), we establish Jafarian's 2009 conjecture that every additive spectrum preserving mapping from a von Neumann algebra onto a semisimple Banach algebra is a Jordan isomorphism.

**Mehdi Monfared**, University of Windsor

On algebras with locally convex topologies admitting Arens products in their second topological duals

We identify the exact conditions required for the existence of Arens products on the second duals of algebras with locally convex topologies. We show that for algebras admitting Arens products, the identity $WAP(A) = A^*$ is equivalent to Arens regularity (Pym's criterion). We show that on any infinite dimensional normed algebra $(A, \|\cdot\|)$, there exist uncountably many locally convex topologies $\tau$ compatible with the duality $\langle A, A^*\rangle$, such that $(A, \tau)$ admits Arens products in its second dual. Furthermore, if $(A, \|\cdot\|)$ is Arens regular, strongly Arens irregular or extremely non-Arens regular, then there are uncountably many locally convex topologies $\tau$ on $A$ for which $(A, \tau)$ has the same properties.

The talk is based on a joint work with M. Filali.

**Fouad Naderi**, University of Manitoba

Non-commutative Lebesgue decomposition of non-commutative measures

A positive non-commutative (NC) measure is a positive linear functional on the free disk operator system which is generated by a $d$-tuple of non-commuting isometries. By introducing the hybrid forms, their Cauchy transforms, and techniques from NC reproducing kernel Hilbert spaces (RKHS), we construct a natural Lebesgue decomposition for any positive NC measure against any other such measure. Our work extends the Jury-Martin decomposition, which originally decomposes positive NC measures against the standard NC Lebesgue measure. In fact, we give a more generalized definition of absolute continuity and singularity, which reduces to their definition when the splitting measure is the standard NC Lebesgue measure. This generalized definition makes it possible to extend Jury-Martin theory for any splitting NC measure, and it recovers their decomposition when the splitting NC measure is the Lebesgue one. Our work implies a Lebesgue decomposition for representations of the Cuntz-Toeplitz C*-algebra. Furthermore, our RKHS method gives a new proof of the classical Lebesgue decomposition when applied to the classical one dimensional setting, i.e. $d = 1$.

**Vern Paulsen**, University of Waterloo

Completely bounded norms of $k$-positive maps

We study the problem of how large the cb-norm of a unital $k$-positive (UkP) map between two operator systems can be. We prove that when we fix an operator system as either the domain or range of such maps, then the asymptomatic behavior as $k$ tends to infinity of the supremum of the cb-norm of all UkP maps characterizes whether or not the operator system has the lifting property or is exact. Even when the domain or range is a matrix algebra these values were not known.

**Gerhard Racher**, University of Salzburg

Compactness via the Fourier algebra?

Using only the Fourier algebra we show the following very small part of the theorem of Shtern-Baggett-Wang; e.g. [S.P. Wang, Math. Ann., 1975]. Let $G$ be a separable l.c. group such that "$\widehat{G}$" is discrete. If $G$ is amenable or "IN", then $G$ is compact.

**Finlay Rankin**, Carleton University

Quantum automorphisms of commuting squares

Banica defined a compact quantum group of automorphisms for an inclusion of finite-dimensional C*-algebras and determined its representation theory in certain cases. We generalize Banica's work and assign a compact quantum group of automorphisms to a nondegenerate commuting square consisting of finite-dimensional C*-algebras and show that it can be realized as a generalized Drinfeld double. Finally, we discuss the representation theory in special cases.

**Michael Rosbotham**, University of Maine

Trace-class Operators on Hilbert Modules and the Haagerup Tensor Product

In 2021, Stern and van Suijlekom used frames to define and study the trace-

class, and other Schatten classes, for operators on Hilbert modules. They showed that, when the modules are over a commutative C*-algebra, many key features carry over from the Hilbert space case. In this talk, I discuss joint work with Tyrone Crisp where we show that the space of trace-class operators on a Hilbert module over a commutative C*-algebra is completely isometrically isomorphic to a Haagerup tensor product of the module with its operator-theoretic adjoint. This generalises a well-known property of Hilbert spaces.

**Ebrahim Samei**, University of Saskatchewan

Amenable actions of groups with rapid decay on stationary spaces

Let $G$ be a countable discrete group, and let $\mu$ be a probability measure on $G$ with finite (Shannon) entropy. We initiate the study of several related concepts associate to a probability measure $\mu$ and exploit their relations. First, we look at the concept of *Lyapunov exponent* of $\mu$ with respect to weights on $G$ and build a framework that connects it to the entropy of $\mu$ in $G$. This is done by introducing a generalization of Avez entropy, taking into account the given weight, and investigating in details their relations together as well as to the actions of $G$ on measurable stationary spaces.

As a byproduct of our techniques, we show that if $G$ has rapid decay w.r.t. a length function $L$ and $\mu$ has a finite logarithm moment (w.r.t. $L$), the weak containment of the representation $\pi_X$ of $G$ on a $\mu$-stationary space $(X, \xi)$ implies that

$$h(G, \mu) = h_\mu(X, \xi),$$

where $h_\mu(X, \xi)$ is the Furstenberg entropy of $(X, \xi)$. This allows us to characterize amenable action of $(G, \mu)$ on stationary spaces: $(X, \xi)$ is an amenable $(G, \mu)$-space if and only if it is a measure-preserving extension of the Poisson boundary of $(G, \mu)$. In particular, if $(X, \xi)$ is a boundary, then $\pi_X$ is weakly contained in $\lambda_G$ if and only if $(X, \xi)$ coincides with the Furstenberg-Poisson boundary of $(G, \mu)$. Hence the action of $G$ on a proper $\mu$-boundary of $G$ is not amenable. This extends the results of Nevo, Zimmer, and others on many hyperbolic like groups.

This is a join work with Benjamin Anderson-Sackaney, Tim de Laat, and

Matthew Wiersma.

**Francois Schulz**, University of Johannesburg

### Banach algebra mappings preserving the invertibility of linear pencils

Denote by $M_n(\mathbb{C})$ the Banach algebra of all complex $n \times n$ matrices. Marcus and Purves showed in 1959 that if a linear map $\varphi : M_n(\mathbb{C}) \to M_n(\mathbb{C})$ preserves invertibility (that is, $\varphi(x)$ is invertible whenever $x$ is invertible), then there exist invertible $u, v \in M_n(\mathbb{C})$ such that

$$\varphi(x) = uxv \text{ for all } x \in M_n(\mathbb{C}) \quad \text{or} \quad \varphi(x) = ux^t v \text{ for all } x \in M_n(\mathbb{C}), \tag{1}$$

where $x^t$ denotes the transpose of $x$. Let $A$ and $B$ be complex unital Banach algebras. In this talk we shall study pairs of surjective mappings $\varphi, \psi : A \to B$ which together preserve the invertibility of linear pencils in both directions; that is, which satisfy the condition that for any $x, y \in A$ and $\lambda \in \mathbb{C}$,

$$\lambda x + y \text{ is invertible in } A \iff \lambda\varphi(x) + \psi(y) \text{ is invertible in } B. \tag{2}$$

It should be noted that linearity does not form part of our hypothesis. Our work is motivated by a recent paper of Costara, [2020, Linear Alg. Appl.], which deals with the case where $\varphi, \psi$ satisfy (2) and $A = B = M_n(\mathbb{C})$. Remarkably, if only *one* of the mappings in this situation is either surjective or continuous, then $\varphi = \psi$ and $\varphi$ takes one of the forms in (1). Our main aim in this talk is to discuss a version of this result in the infinite-dimensional setting; in particular, one valid for the Banach algebra of all bounded linear operators acting on a Banach space $X$, denoted by $\mathcal{L}(X)$. Specifically, we show that if a pair of surjective mappings $\varphi, \psi : A \to B$ satisfy (2), where $A$ is semisimple with an essential socle, then $\varphi(x) = \psi(x) = uJ(x)$ for all $x \in A$, with $u$ invertible in $B$ and $J : A \to B$ a Jordan isomorphism. As a consequence, this then yields a classification (similar to (1)) for these mapping pairs between $\mathcal{L}(X)$ and $\mathcal{L}(Y)$.

**James Smith**, Lancaster University

Orthogonal bases: operator algebra structure and Arens regularity thereof

It is well known that $\ell_p$ is a Banach algebra in the pointwise product, and since the 70's it has been known this algebra is isomorphic to a subalgebra of bounded operators on a Hilbert space. We explore how unconditional bases offer a nice way to generalise this discussion, where some rather exotic Banach spaces can be realised as operator algebras.

**Ross Stokke**, University of Winnipeg

Homomorphisms of spine algebras

An old problem, solved in the abelian case by Paul Cohen in 1960, asks for a description of all homomorphisms $\varphi : A(G) \to B(H)$, where $A(G)$ and $B(H)$ are the Fourier and Fourier–Stieltjes algebras of locally compact groups $G$ and $H$, respectively. For non-abelian groups, M. Ilie, N. Spronk, M. Daws and H.L. Pham have, among others, made significant contributions to this problem. One can also ask for a description of the homomorphisms $\varphi : A \to B(H)$ where $A$ is some other closed translation-invariant subalgebra of $B(G)$. We will briefly discuss aspects of this general problem, noting that its difficulty is significantly impacted by the complexity of the Gelfand spectrum, $\Delta(A)$, of $A$. The spine subalgebra of $B(G)$, $A^*(G)$, was introduced by J. Inoue and J.L. Taylor in the abelian case and later defined for any $G$ by Ilie and Spronk; its spectrum is typically of intermediate complexity between the spectra of $A(G)$ and $B(G)$. After reviewing and generalizing the definition of the spine, we will describe a characterization of all (completely) positive and all (completely) contractive homomorphisms $\varphi : A \to B(H)$ where $A$ is a spine algebra. This is joint work with Nico Spronk and Aasaimani Thamizhazhagan.

**Tomasz Szczepanski**, University of Alberta

On the chain of commuting operators and Lomonosov's Invariant Subspace Theorem

In 1973 V. Lomonosov proved that if $T$ is an operator on a complex Banach

space $X$ such that it commutes with a non-scalar operator $S$ and $S$ commutes with a non-zero compact operator $K$, then $T$ has an invariant subspace. We say $T$ is a Lomonosov operator if it satisfies the assumptions of Lomonosov's Theorem.

Initially it wasn't clear if there are operators that are not of Lomonosov type. The negative answer to this question came from the construction of the first counterexamples of operators without invariant subspaces (P. Enflo (1970's), C.Read (1985,1986)). Another interesting question was if perhaps every operator that has an invariant subspace is a Lomonosov operator. It turns out the answer to this question is also negative (D. Hadwin, E. Nordgren, H. Radjavi and P. Rosenthal, (1980)).

What is interesting, in 2000 V. Troitsky showed that the chain of commuting operators in Lomonosov's Theorem cannot be extended to four operators (i.e. connecting an operator $T$ to a non-zero compact operator via a chain of four operators does not guarantee existence of an invariant subspace of $T$). This observation started the study of chains of commuting operators. In this talk we show that the non-Lomonosov operator from the 1980 paper can be connected via a chain of four operators to a compact operator. We also show that this is true for a certain class of operators including lattice isomorphisms on $\ell_p$ spaces. The question of the existence of operators that require a chain of more than four operators connecting them with a non-zero compact operator remains open.