Programming Language APL, Extended
International Standards Organisation

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Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

In the field of information technology, ISO and IEC have established a joint technical committee, ISO/IEC JTC 1. Draft International Standards adopted by the joint technical committee are circulated to national bodies for voting. Publication as an International Standard requires approval by at least 75 % of the national bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO and IEC shall not be held responsible for identifying any or all such patent rights.

International Standard ISO/IEC 13751 was prepared by Joint Technical Committee ISO/IEC JTC 1, Information technology, Subcommittee SC 22, Programming languages, their environments and system software interfaces.

Annex A forms a normative part of this International Standard.

Introduction

APL stands for **A Programming Language**. It is a notation invented by K. E. Iverson in the late 1950s for the description of algorithms, and expanded on and made into the programming system *APL*\360 by Iverson and his colleagues Adin Falkoff, Larry Breed, Dick Lathwell, and Roger Moore in the mid-1960s.

This document, **Programming Language APL, Extended**, is a sequel to **Programming Language APL**, ISO 8485 (1989).

The principal differences that the reader will find here have to do with new features that have been added. These topics are:

without
greatest common divisor
least common multiple
duplicate
commute
table
join along first axis
mixed arrays
overbar in names
underbar in names
replicate
character grades
grades of arrays greater than rank one
unique
alpha as a name
omega as a name
ambivalent defined functions
event handling
n-wise reduction
complex arithmetic
left
right
function rank operator
defined operators
component file system

enclose
disclose
enlist
pick
depth
identical
each
first

An entry for each of these topics will be found in the index. Some new system commands have been added. Shared variable extensions have been added. Workspace Interchange Standard 2 is given, in which canonical representation vectors of type “E” are used to represent generalised arrays.

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Information technology — Programming languages, their environments and system software interfaces — Programming language Extended APL

1 Scope

This International Standard defines the programming language APL and the environment in which APL programs are executed. Its purpose is to facilitate interchange and promote portability of APL programs and programming skills. This International Standard specifies the syntax and semantics of APL programs and the characteristics of the environment in which APL programs are executed.

It also specifies requirements for conformance to this International Standard, including the publication of values and characteristics of implementation properties so that conforming implementations can be meaningfully compared.

This International Standard does not specify:

- implementation properties that are likely to vary with the particular equipment or operating system used;
- required values for implementation limits such as APL workspace size or numeric precision;
- the data structures used to represent APL objects;
- the facilities available through shared variables.

2 Normative References

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO/IEC 2382 - 15 : 1999 *Data processing – Vocabulary – Part 15: Programming languages.*

ISO/IEC 8485 : 1989 *Programming languages – APL* .

International Register of Coded Character Sets To Be Used With Escape Sequences, Registered character set 68. (http://www.itscj.ipsj.or.jp/ISO-IR/068.pdf)

3 Form of this International Standard

This International Standard is a formal model of an APL machine, specified as a collection of finite sets, diagrams, and evaluation sequences, and objects constructed from finite sets, diagrams, and evaluation sequences.

The finite sets are the **implementation-defined character-set**, the **implementation-defined** set of **numbers**, and the enumerated sets **array-type**, **class-names**, **keyboard-states**, **mode-names**, **required-character-set**, and **workspace-presence**.

Diagrams are directed graphs used to designate syntactic forms.

Evaluation sequences are formal procedures that operate on finite sets, diagrams, other evaluation sequences and objects defined in the International Standard.

Objects are entities consisting of enumerated set members and other objects. The objects are **list**, **array**, **defined-function**, **defined-operator**, **token**, **symbol**, **context**, **workspace**, **session**, **shared-variable**, and **system**.

Each object has attributes describing its state. The attributes of an array, for example, are its typical element, its shape, and its ravel.

Objects often have defined properties derived from their attributes. The rank of an array, for example, is the shape of the shape of the array.

3.1 Form of Definitions

Defined terms in this International Standard are always set in **bold** and indexed. The index entry begins with the page number of the definition followed by the page numbers of all references to the term. If the definition and a use of a term occur on the same page, that page number will occur twice in the Index.

The following terms occur throughout the document and are not cross-indexed: **character**, **content**, **class**, **item**, and **number**.

Note: *Terms in this document include both phrases such as* **implementation-parameter** *and words* *such as* **nil**.

Each definition in this International Standard takes one of four forms.

1. Regular definitions consist of the term being defined followed by a colon and the body of the definition. The term **Boolean** is defined in this way.
2. Members of an enumerated set are defined simply by being listed in the definition of the enumerated set. The term **nil** is defined in this way, as a member of the enumerated set **class-names**.
3. Diagrams are defined by directed graphs. The term **expression** is defined in this way.
4. Definitions of terms that designate evaluation sequences take the following form:

   – The term being defined, such as **scan**.
   – The forms that the evaluation mechanism used in the International Standard recognises as designating this term, such as Z ←**f** \\B.
   – An informal introduction indicating the purpose of the procedure. The informal introduction is considered commentary on the International Standard.
   – An evaluation sequence expressed in a formal, though English-like, language defined in the subsection **Evaluation Sequences**. A **conforming-implementation** is required to emulate the behaviour described in the evaluation sequence as modified by the **additional-requirements**, if any.
   – **Examples**, which show effects of the procedure specified by the evaluation sequence. Examples are considered commentary on the International Standard.
   – **Additional Requirements**, giving aspects of the behaviour required of this operation that cannot conveniently be expressed in the evaluation sequence.

3.2 Named Arrays in Examples

In the examples in this International Standard, APL identifiers beginning with N, such as N234, represent numeric arrays whose shape and content are specified by the digits in the identifier. Each digit in the identifier specifies an element of the shape vector; each element of the array, when broken down into digits, gives the index of that element.

For example,

Example:

```
      N4
1 2 3 4
      N23
11 12 13
21 22 23
```



```
      N234
111 112 113 114
121 122 123 124
131 132 133 134

211 212 213 214
221 222 223 224
231 232 233 234

      N234[2;3;1]
231
```

3.3 Notes

This International Standard contains notes that comment on the text of the International Standard, pointing out the significance of definitions, noting relationships between definitions, and otherwise making the text approachable. These notes are set in a different type style than the text of the International Standard, and are prefixed with the word "Note:". The following is an example of a note.

Note: *This is an example of a note.*

Notes never set requirements for conformance. They may suggest desired properties, but such suggestions are not mandatory for conformance.

3.4 Cross-References

The heading levels in this International Standard are chapter, section, and subsection. When cross-references are given, they are always to a subsection title. In the Index, subsections are treated like definitions: the page on which a subsection begins is always the first entry in the Index; subsequent page numbers in the index show where references are made to the subsection.

3.5 General Definitions

For the purpose of this International Standard, the definitions given in ISO/DIS 2382/15 shall apply, together with the following:

– **Program**: An application.

Note: *The term is used in this International Standard to include everything from an APL expression to a collection of workspaces communicating via shared variables.*

- **Implementation**: A combination of computer hardware and software that processes (APL) **programs**.

Note: *An implementation is an instance of the object* **system** *specified by this International Standard.*

- **Facility** (of an implementation): A unit of behaviour. Every **facility** is one of:

  - **Defined-Facility**: A **facility** fully specified in this International Standard and not designated **optional** or **implementation-defined**.
  - **Optional-Facility**: A **facility** fully specified in this International Standard and designated **optional**.
  - **Implementation-Defined-Facility**: A **facility** not fully specified by this International Standard that is designated **implementation-defined**.
  - **Consistent-Extension**: A **facility** not specified in this International Standard that, for a construct this International Standard specifies as producing an error, gives some effect other than signalling the specified error.

4 Compliance

4.1 Conforming Implementations

An APL **implementation** conforms to this International Standard if it meets the following requirements for both behaviour and documentation.

4.1.1 Required Behaviour for Conforming Implementations

A **conforming-implementation** shall provide all **defined-facilities** and **implementation-defined-facilities**. Each such **facility** shall behave as specified by this International Standard.

A **conforming-implementation** may provide **optional-facilities**. If provided, an **optional-facility** shall behave as specified by this International Standard. Attempted use of an **optional-facility** that is not provided shall cause the **conforming-implementation** to signal an error. A **conforming-implementation** shall not replace the error signalled by a missing **optional-facility** with other behaviour.

A **conforming-implementation** may provide **consistent-extensions**. The presence of a **consistent-extension** shall not affect the behaviour of a **conforming-program**.

A **conforming-implementation** shall use algorithms that produce results that are the same as those produced by the evaluation sequences. Mathematical function algorithms shall have at least the accuracy that the algorithms given in the evaluation sequences would produce.

Note: *The evaluation sequences used in this International Standard are intended to specify results, not implementation techniques.*

The errors produced by the absence of an **optional-facility** *cannot be replaced by* **consistent-extensions** *in a* **conforming-implementation***, since this would affect the behaviour of* **conforming-programs** *that use the* **optional-facility**.

4.1.2 Required Documentation for Conforming Implementations

A **conforming-implementation** shall provide a reference document that satisfies the following requirements for the documentation of **optional-facilities**, **implementation-defined-facilities**, and **consistent-extensions**:

4.1.2.1 Documentation of Optional-Facilities

A **conforming-implementation** shall document the presence or absence of each of the following **optional-facilities**:

– **Shared-Variable-Protocol**. A mechanism that permits one session to exchange data with other autonomous sessions.

– **Statement-Separator-Facility**. A mechanism for placing more than one statement on a line.

– **Trace-and-Stop-Control**. A mechanism that assists the testing and correction of defined functions.

– **Complex-Arithmetic-Facility**. A mechanism that permits the generation and manipulation of complex-arithmetic results.

4.1.2.2 Documentation of Implementation-Defined-Facilities

A **conforming-implementation** shall document the following aspects of **implementation-defined-facilities**:

– A description of the **character-set**. This shall include a table showing the correspondence between index positions in the **atomic-vector** and the members of the **required-character-set**. If the graphic symbols used in a **conforming-implementation** are dissimilar to those in **Table 1**, the correspondence between the graphic symbols used in the implementation and those in **Table 1** shall be given.

– A description of the **numbers**. This shall include a characterisation of the internal representation used for **numbers**.

– Descriptions of the characteristics of each **implementation-algorithm**.

– The value of each **implementation-parameter**.

– A description of each **internal-value-set**.

– A description of additional **event-types**.

4.1.2.3 Consistent Extensions

A **conforming-implementation** shall document all **consistent-extensions** it provides. The documentation shall clearly indicate that the use of a **consistent-extension** prevents a **program** from conforming with this International Standard.

Note: *Implementers of* **conforming-implementations** *should, in general, be wary of replacing* **limit-errors** *with* **consistent-extensions***, since these errors are the only safeguards a* **conforming-program** *has when attempting to operate in a* **conforming-implementation** *whose* **implementation-parameters** *are inadequate to support it.*

For example, if the **limit-error** *on* **identifier-length-limit** *were not signalled, a* **conforming-program** *with identifiers longer than the local* **identifier-length-limit** *could malfunction without warning.*

4.2 Conforming Programs

A **program** conforms to this International Standard if it meets the following requirements for both behaviour and documentation.

4.2.1 Required Behaviour for Conforming Programs

A **conforming-program** shall use only those **facilities** specified in this International Standard.

A **conforming-program** shall not use **consistent-extensions**.

A **conforming-program** shall not depend on the signalling of any error by a **conforming-implementation**.

Note: Conforming-programs *cannot depend on error behaviour, since* **consistent-extensions** *that replace errors are permitted in* **conforming-implementations***. A conforming program may make use of certain event handling features unreservedly (for example, ⎕ES), and others with some restrictions. Consider the following example:*

```
Z←'¯1' ⎕EA 'A ⎕SVO B'
Z←(0 1 2 ¯1)[0 1 2⍳Z]
```

This is conforming because it only depends on the values `0`*,* `1`*, and* `2`*, which have defined meanings when returned as the result of ⎕SVO. The first line can return anything, but the second line filters out all but* `0`*,* `1`*, and* `2`*, hence eliminating any subtle dependencies on error behaviour or consistent extensions.*

4.2.2 Required Documentation for Conforming Programs

A **conforming-program** shall document which **optional-facilities** it requires.

A **conforming-program** shall document any specific requirements it has for **implementation-parameters**.

Note: *A* **conforming-program** *may or may not work, and may or may not produce identical results on all* **conforming-implementations***, because of inherent dependencies on* **implementation-parameters** *or* **implementation-algorithms** *such as the algorithm used for matrix inversion.*

It is suggested that **conforming-programs** *provide documentation detailing the values of* **implementation-parameters** *they require so that their suitability for a given* **conforming-implementation** *can be determined readily.*

5 Definitions

5.1 Characters

– **Character-Set**: An **implementation-defined** finite set.

– **Character**: A member of the **implementation-defined** finite set **character-set**.

– **Required-Character-Set**: An enumerated set which is the union of a set of alphabetics with the sets of Numeric Characters and Special Characters designated in **Table 1**. The alphabetics may be any alphabet; this International Standard uses the Alphabetic Characters shown in **Table 1** to represent the dual-case Latin alphabet. The **character-set** provided by a **conforming-implementation** shall contain all members of the **required-character-set** which results from the choice of alphabet.

Additional Requirement:

Members of the **required-character-set** are represented in this International Standard by graphic symbols in a particular typeface, as shown in **Table 1**. The graphic symbols associated with the **required-character-set** in a **conforming-implementation** are **implementation-defined**.

Note: *There are conformance requirements associated with the* **required-character-set***. A* **conforming-implementation** *must publish, as part of its required documentation, a table giving the correspondence between the graphic symbols in* **Table 1** *and the* **atomic-vector***. Where the graphic symbols provided are not similar in appearance to those used in this International Standard, the correspondence between the graphic symbols in this International Standard and those provided by the implementation must also be provided.*

Some of the graphic symbols given in Table 1 are not used to designate APL constructs in this International Standard. They are present to provide all keys on common terminal keyboards with corresponding symbols.

The names in **Table 1** are not to be considered part of this International Standard.

Alphabetic Characters

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
A̲ B̲ C̲ D̲ E̲ F̲ G̲ H̲ I̲ J̲ K̲ L̲ M̲ N̲ O̲ P̲ Q̲ R̲ S̲ T̲ U̲ V̲ W̲ X̲ Y̲ Z̲

Numeric Characters

0 1 2 3 4 5 6 7 8 9

Special Characters

⍺ alpha
↓ down arrow
← left arrow
→ right arrow
↑ up arrow
− bar
  blank
{ left brace
} right brace
[left bracket
] right bracket
∨ down caret
⍱ down caret tilde
< left caret
> right caret
∧ up caret
⍲ up caret tilde
○ circle
⍉ circle backslash
⊖ circle bar
⍟ circle star
⌽ circle stile
: colon
, comma
⍪ comma bar
∇ del
⍒ del stile
⍫ del tilde
∆ delta
⍋ delta stile
⍙ delta underbar
¨ diaeresis
⋄ diamond
÷ divide
$ dollar sign
. dot
∊ epsilon
= equal
≥ greater-than or equal
⍳ iota
∘ jot
≤ less-than or equal
× multiply
≠ not equal
¯ overbar
(left parenthesis
) right parenthesis
+ plus
⎕ quad
⌹ quad divide
? query
⍤ diaeresis jot
' quote
! quote dot
⍞ quote quad
⍴ rho
; semicolon
∪ down shoe
⊂ left shoe
⊃ right shoe
∩ up shoe
⍝ up shoe jot
/ slash
\ back slash
⌿ slash bar
⍀ back slash bar
⋆ star
| stile
⌊ down stile
⌈ up stile
⊥ up tack
⍎ up tack jot
⊢ right tack
⊣ left tack
⊤ down tack
⍕ down tack jot
∼ tilde
_ underbar
⍨ diaeresis tilde
⍵ omega
≡ equal underbar

Table 1: The Required Character Set

5.2 Numbers

– **Number-Set**: An **implementation-defined** finite set used to represent arithmetic quantities.

– **Number**: A member of the **number-set**.

Note: *The* **number-set** *is an abstraction used in this document to represent the floating-point arithmetic quantities of an arbitrary computer.*

5.2.1 Elementary Operations

Elementary-Operation: One of the following four **implementation-algorithms**.

– A **Plus** B.

Note: Plus *implements addition.*

– A **Minus** B.

Note: Minus *implements subtraction.*

– A **Times** B.

Note: Times *implements multiplication.*

– A **Divided-by** B.

Note: Divided-by *implements division.*

Note: *Each* **elementary-operation** *is a mapping from the Cartesian product of the* **number-set** *with itself back onto the union of the* **number-set** *and the* **metaclass error***, as described in the subsection* **Implementation-Algorithms***. It is assumed that members of the* **metaclass error** *are returned by* **elementary-operations** *when results cannot be represented as* **numbers***. In particular,* **exponent-overflow** *is returned when the result of an* **elementary-operation** *is too large in magnitude to be represented by a* **number***,* **exponent-underflow** *is returned when the result of an* **elementary-operation** *is non-zero but too small in magnitude to be represented by a* **number** *other than* **zero***, and* **domain-error** *is returned by an* **elementary-operation** *called with arguments for which its mathematical counterpart is undefined, such as* **one divided-by zero***.*

The treatment of **exponent-overflow** *and* **exponent-underflow** *is not specified by this International Standard, except as suggestions to the implementer. Implementers should avoid having* **exponent-overflow** *and* **exponent-underflow** *occur in the intermediate results of* **implementation-algorithms** *wherever possible. In any case,* **exponent-underflow** *should not cause a* **limit-error***.*

The fact that these fundamental algorithms are effectively undefined in the International Standard is intentional. A definition general enough to cover all known and possible floating-point systems was felt to be less useful than the requirement that the internal representation for **numbers** *in a* **conforming-implementation** *be described in the* **required-documentation** *for the implementation.*

5.2.2 Number Constants

Note: *The following* **numbers***, used in this International Standard, are defined here as terms to make clear the distinction between APL constant arrays represented as strings of APL digits, such as* `1`*, and members of the the* **implementation-defined** *finite set containing* **numbers***, such as* **one***.*

These terms are always set in **bold** *when they are used in this formal sense. They are not cross-indexed.*

- **Zero**: A **number** such that, for any **number** A, A **plus zero** is A.
- **One**: A **number** such that, for any **number** A, A **times one** is A.
- **Negative-One**: **Zero minus one**.
- **Two**: **One plus one**.
- **Three**: **Two plus one**.
- **Four**: **Three plus one**.
- **Five**: **Four plus one**.
- **Six**: **Five plus one**.
- **One-half**: **One divided-by two**.
- **Imaginary-One**: **Negative-One to-the-power one-half**.

5.2.3 Subsets of the Set of Numbers

Note: *The following subsets of the numbers are used to define the domains of operations.*

- **Boolean**: **zero** or **one**.

Note: *False is represented by* **zero***, true by* **one***.*

Note: *A* **counting-number** *is a member of the subset of the* **numbers** *accessible under a successor function.*

- **Positive-Counting-Number**: **one** or any **number** that can be generated by A **plus one**, where A is a **positive-counting-number**.
- **Negative-Counting-Number**: The **number negative-one** or any **number** that can be generated by N **plus negative-one**, where N is a **negative-counting-number**.
- **Nonnegative-Counting-Number**: A **positive-counting-number** or **zero**.

– **Counting-Number**: A **negative-counting-number** or a **nonnegative-counting-number**.

Note: *An* **integer** *is a member of the subset of the* **numbers** *accessible from* **zero** *and* **one** *through addition and subtraction.*

– **Positive-Integer**: A **positive-counting-number** or a **number** that can be generated by A **plus** B, where A and B are **positive-integers**.

– **Negative-Integer**: A **negative-counting-number** or a **number** that can be generated by A **plus** B, where A and B are **negative-integers**.

– **Nonnegative-Integer**: A **positive-integer** or **zero**.

– **Integer**: A **negative-integer** or a **nonnegative-integer**.

– **Positive-Number**: A **positive-integer**, or a **number** other than **zero** that can be generated by A **times** B, by A **divided-by** B, or by A **plus** B, where A and B are **positive-numbers**.

– **Negative-Number**: A **negative-integer**, or a **number** other than **zero** that can be generated by A **times** B, by A **divided-by** B, or by C **plus** D, where A is a **positive-number** and B, C and D are **negative-numbers**.

– **Nonnegative-Number**: A **positive-number** or **zero**.

– **Real-Number**: A **negative-number** or a **nonnegative-number**.

– **Complex-Number**: A **real-number**, or a **number** that can be generated by A **plus** (B **times imaginary-one**), where A and B are **real-numbers**.

– **Complex-Integer**: A **complex-number** that can be generated by A **plus** (**imaginary-one times** B), where A and B are **integers**.

– **Half-plane**: Two numbers are in the same **half-plane** if the signs of either their **real-parts** or **imaginary-parts** are nonzero and are equal.

– **Unit-square**: A **unit-square** contains all numbers for which the integral part of the real parts are the same and the integral parts of the imaginary parts are the same.

– **Arc**: The **arc** of a number is **zero** if the number is **zero**, and otherwise is the **imaginary-part** of the **natural-logarithm** of the number, and has a value greater than minus pi and less than or equal to pi.

– **Fraction**: A **number** lying in the rectangle of the **complex-plane** bounded by the two parallel lines passing through **zero** and **one** at an angle of 135 degrees with the real axis, and the two parallel lines passing through **one** and **negative-one** at an angle of forty-five degrees with the real axis, including the **numbers** on the lines going through **zero** and **negative-one**, but not including the **numbers** on the two lines through **one**.

– **First-Quadrant-Associate**: Either a **nonnegative-number**, or a **number** with both **real-part** and **imaginary-part positive-numbers**, and having the same **magnitude** as a given nonzero **number**, and separated from it by an angle of either 0, 90, 180, or 270 degrees.

Note: *The* **numbers** *are assumed to be identical to the* **complex-numbers**.

Example

Consider a normalised base-2 floating point number system with a two position exponent field and a two position mantissa field with the radix point between the digits. The decimal values representable with this system and the categories into which each falls are as follows:

Exponent in Base 2	Mantissa in Base 2	Value in Base 10	Boolean	Counting-Number	Integer
00	00	0	Y	Y	Y
00	10	1	Y	Y	Y
00	11	1.5			
01	10	2		Y	Y
01	11	3		Y	Y
10	10	4		Y	Y
10	11	6			Y
11	10	8			Y
11	11	12			Y

As can be seen in this example, **counting-numbers** form a dense subset of the **integers**; the largest **counting-number** is typically a power of the number system base.

A given number, such as a **counting-number**, may have several hardware representations. Except for their effect on system resources, these representations should be indistinguishable to a **conforming-program**.

5.2.4 Implementation Algorithms

An **Implementation-Algorithm** is an algorithm used in this International Standard whose behaviour is **implementation-defined**. The following implementation algorithms are used in this International Standard.

– **Conjugate**

– **Cosine**

– **Current-Time**

– **Deal**

– **Display**

– **Divided-by**

- **Exponential**
- **Function-Display**
- **Gamma-Function**
- **Greatest-Common-Divisor**
- **Hyperbolic-Cosine**
- **Hyperbolic-Sine**
- **Hyperbolic-Tangent**
- **Inverse-Cosine**
- **Inverse-Hyperbolic-Cosine**
- **Inverse-Hyperbolic-Sine**
- **Inverse-Hyperbolic-Tangent**
- **Inverse-Sine**
- **Inverse-Tangent**
- **Magnitude**
- **Matrix-Divide**
- **Minus**
- **Modulo**
- **Natural-Logarithm**
- **Next-Definition-Line**
- **Numeric-Input-Conversion**
- **Numeric-Output-Conversion**
- **Pi-Times**
- **Plus**
- **Produce-Canonical-Representation-Vector**
- **Pseudorandom-Number-Generator**
- **Read-Keyboard**
- **Sine**
- **Tangent**
- **Times**

– **Time-Stamp**

– **To-the-Power**

– **Trace-Display**

– **Typical-Element-For-Mixed**

The following **implementation-algorithms** take a **number** as an argument and return either a **number** or an **error**: **Cosine**, **Exponential**, **Gamma-Function**, **Hyperbolic-Cosine**, **Hyperbolic-Sine**, **Hyperbolic-Tangent**, **Inverse-Cosine**, **Inverse-Hyperbolic-Cosine**, **Inverse-Hyperbolic-Sine**, **Inverse-Hyperbolic-Tangent**, **Inverse-Sine**, **Inverse-Tangent**, **Magnitude**, **Natural-Logarithm**, **Pi-Times**, **Sine**, **Tangent**.

The following **implementation-algorithms** take two **numbers** as arguments and return either a **number** or an **error**: **Divided-by**, **Minus**, **Modulo**, **Plus**, **Times**, **To-the-Power**.

The properties of the remaining **implementation-algorithms** are described in the chapters in which they are used.

5.2.5 Defined Operations

– A **Equals** B: An operation that, for A and B **numbers**, returns **one** if A and B are the same **number**, and **zero** otherwise.

– **Direction** of A: An operation that, for A a **number**, returns **zero** if A is **zero**, and otherwise returns the **number** determined by the radial projection of A onto the **unit-circle**.

Note: *For A a* **real-number***, the* **direction** *of A is either* **negative-one** *or* **zero** *or* **one***.*

– A is **Greater-Than** B: An operation that, for A and B **real-numbers** returns **one** if A is a **positive-number** and B is a **negative-number**, returns **one** if A **minus** B is a **positive-number**, and returns **zero** otherwise.

– A is **Less-Than** B: An operation that, for A and B **real-numbers**, returns B **greater-than** A.

– **Negation** of A: An operation that, for any **number** A, returns **zero minus** A.

– **Magnitude** of A: An operation that, for A a **number**, returns the **non-negative real-number** determined by rotating A onto the **nonnegative real-axis**.

– **Open-Interval-Between** A and B: An operation that, for any **numbers** A and B, returns a subset of the **numbers** as follows: if A is not **greater-than** B, the set of all **numbers greater-than** A and **less-than** B; otherwise, the **open-interval-between** B and A.

– **Closed-Interval-Between** A and B: An operation that, for any **numbers** A and B, returns a subset of the **numbers** consisting of A, B, and the **open-interval-between** A and B.

– **Larger-Magnitude** of A and B: An operation that, for any **numbers** A and B, returns the **magnitude** of A if it is **greater-than** that of B, and the **magnitude** of B otherwise.

– **Distance-Between** A and B: An operation that, for any two **numbers** A and B, returns the **magnitude** of A **minus** B.

– A is **Tolerantly-Equal** to B **Within** C: An operation that, given three **numbers** A, B, and C, with C greater than or equal to zero, returns a **Boolean** Z determined as follows:
  If A **equals** B, then Z is **one**.
  If A and B are not in the same **half-plane**, then Z is **zero**.
  If the **distance-between** A and B is **less-than** or **equals** C **times** the **larger-magnitude** of A and B, then Z is **one**.
  Otherwise, Z is **zero**.

– **Tolerant-Floor** of A **Within** B: An operation that, for A a **number** and B a **nonnegative-number**, returns a **complex-integer** Z determined as follows:

  Let A be a member of the set of **numbers** in the **unit-square** at the **complex-integer** C, and let D be A **minus** C.
  If the sum of the real and imaginary parts of D is **tolerantly-less-than one within** B, then Z is C.
  Otherwise, if the **imaginary-part** of D is **greater-than** the **real-part** of D, then Z is C **plus imaginary-one**.
  Otherwise, Z is C **plus one**.

– A is **Integral-Within** B: An operation that, for a **number** A and a **positive-number** B, returns a **Boolean** Z determined as follows:
  Let C stand for the **negation** of A.
  Z is **one** if the **tolerant-floor** of C **within** B **equals** the **negation** of the **tolerant-floor** of A **within** B, and **zero** otherwise.

– A is a **Near-Integer**: An operation that, for a **number** A, returns **one** if A is **integral-within integer-tolerance**, and **zero** otherwise.

Note: *The foregoing definition contains a forward reference to* **integer-tolerance**.

– **Integer-Nearest-to** A: An operation that, for a **near-integer** A, returns the **tolerant-floor** of A **within integer-tolerance**.

– A is **Near-Boolean**: An operation that, for a **near-integer** A, returns **one** if the **integer-nearest-to** A is a **Boolean**, and **zero** otherwise.

Note: Near-integers *and* **near-Booleans** *include* **numbers** *whose* **magnitude** *is* **less-than integer-tolerance**. *The operation* **integer-nearest-to** *maps such numbers to* **zero**.

– A is **Real-Within** B: An operation that, for a **number** A and a **positive-number** B, returns a **Boolean** Z determined as follows:
  Let R stand for the **magnitude** of the **real-part** of A, and I for the **magnitude** of the **imaginary-part** of A.
  Z is **one** if I is **less-than-or-equal** either to B, or to R **times** B, and is **zero** otherwise.

– A is **Near-Real**: An operation that, for a **number** A, returns **one** if A is **real-within real-tolerance**, and **zero** otherwise.

5.3 Objects

Note: *The description of APL in this document is based on objects—abstract data structures that are described in terms of characters, numbers, and elementary set theory.*

Each object has a small set of named attributes that take on values that are characters, numbers, or other objects.

Each object has additionally a small set of operations that can be performed upon it.

All the objects in the description are defined here, as are all the attributes of each object. The operations are introduced as they are needed. All objects, attributes, and operations are cross-referenced in the index.

5.3.1 Lists

Index: A **nonnegative-counting-number** less than or equal to **index-limit**.

List: An object with the following attributes:

– **Index-Set:** A finite set I of **positive-counting-numbers** chosen so that for every subset of I, except the empty set, the cardinality of that subset is in I.

– **Value-Set:** A finite set in a specific correspondence with the **index-set** of the **list**.

Empty-List: A **list**, the cardinality of whose **index-set** is **zero**.

Nonempty-List: A **list**, the cardinality of whose **index-set** is **greater-than zero**.

Number-of-Items in L: The cardinality of the **index-set** of the **list** L.

Item X of L: An operation that, for X a member of the **index-set** of the **list** L, returns the member of the **value-set** of L associated with X by the correspondence between the **value-set** of L and the **index-set** of L.

Note: *This form of indexing is always in origin one.*

First-Item in L: An operation that for a **nonempty-list** L returns **item one** of L.

Last-Item in L: An operation that for a **nonempty-list** L, and for C the **number-of-items** in L, returns **item** C of L.

Rest-of L: An operation that, for a **nonempty-list** L whose **index-set** has cardinality C, returns a second **list** R whose **index-set** has cardinality C **minus one**, such that for each **item** J in the **index-set** of R, **item** J of R is **item** (J **plus one**) of L.

Product-of L: An operation defined on a **list** of **numbers** L as follows:

If the **number-of-items** in L is **zero**, **one**.
If the **number-of-items** in L is **one**, the **first-item** in L.
If the **number-of-items** in L is **greater-than one**, **first-item** in L **times** the **product-of** the **rest-of** L.

Prefix: A (possibly empty) **list** P is a **prefix** of a **list** L if the **index-set** of P is a subset of the **index-set** of L, and each **item** of P is the same as the corresponding **item** of L.

5.3.2 Arrays

Array-Type: An enumerated set containing the members **character**, **numeric**, and **mixed**.

Array: An object with the following attributes:

– **Shape-list:** A **list** of **nonnegative-counting-numbers**.

– **Ravel-list:** A **list**. Each **item** of the **list** is either a **character**, a **number**, or an **array**.

  The **number-of-items** in the **ravel-list** of an **array** A is the same as the **product-of** the **shape-list** of A.

– **Type:** A member of the enumerated set **array-type**.

  If the **type** of A is **character**, the **ravel-list** of A is a **list** of **characters**; if the **type** of A is **numeric**, the **ravel-list** of A is a **list** of **numbers**.

Sufficient-Type of L under T: the following operation on a **type** T and a **list** L.
If T is **character** or **numeric**, return T
else if L is a **nonempty-list**
if all items of L are **numbers**, return the **type numeric**
if all items of L are **characters**, return the **type character**
otherwise, return the **type mixed**
else return the **type** of the **typical-element** of L.

Note: *The foregoing definition is intended to provide for the cases where a mixed-array becomes a non-mixed array, and vice-versa.*

Rank of A: An operation that, for an **array** or **array-of-vectors** A, returns the **number-of-items** in the **shape-list** of A.

Simple: An **array** is **simple** if each item of its **ravel-list** is either a **character** or a **number**.

Scalar: An **array** whose **rank** is **zero**.

Simple-scalar: A **simple array** whose **rank** is **zero**.

Max-shape-of B An operation that, for B, an **array**s defined as follows:

Set $B1$ to the **first-thingy** of B.
Set $M1$ to $\rho B1$.
If the **count-of** B is **one**, return $M1$.
Set $B2$ to the **remainder-of** B.
Set $M2$ to the **max-shape-of** $B2$.
If the **shape-list** of $M1$ differs from the **shape-list** of $M2$, signal **rank-error**.
Return $M1 \lceil M2$

First-thingy in A: An opration that for A, an **array**, returns an **array** B, defined as follows:
If A is empty, set $B1$ to the **typical-element** of A.
Otherwise, set $B1$ to the **first-item** of the **ravel-list** of A.
If $B1$ is a **number** or a **character**, set B to an array, whose **ravel-list** contains the single item $B1$, and whose **shape-list** is empty.
Otherwise, set B to $B1$.

Numeric-Scalar with value I: For I a **number**, the **scalar** Z such that the **type** of Z is **numeric** and the **ravel-list** of Z is the **list** L such that the **number-of-items** in L is **one** and the **first-item** of L is I.

Count of A: For A an **array**, the **product-of** the **shape-list** of A.

Vector: An **array** whose **rank** is **one**.

Matrix: An **array** whose **rank** is **two**.

Length of A: For A a **vector**, the **count** of A.

Typical-Element of A: For A an **array**, if the **type** of A is **character**, the **character** blank; if the **type** of A is **numeric**, the number **zero**; if the **type** of A is **mixed**, the **typical-element** is determined by an **implementation-defined-algorithm**.

Empty: Of an **array**, the **count** of which is **zero**.

One-Element-Vector: A **vector** A is a **one-element-vector** if the **length** of A is **one**.

K is a **Valid-Axis** of A: An operation that, for A and K **arrays**, returns **one** if K is a **scalar** or **one-element-vector**, and the **first-item** in the **ravel-list** of K is a **near-integer**, the **integer-nearest-to** which is a member of the **index-set** of the **shape-list** of A; and returns **zero** otherwise.

Axis K of A: An operation that, for A an **array** and K a **valid-axis** of A, returns **item** K of the **shape-list** of A.

Array-of-vectors: An object with the following attributes:

– **Shape-list:** A **list** of **nonnegative-counting-numbers**.

– **Ravel-list:** Each **vector** V in the **ravel-list** of an **array-of-vectors** A has the **type** **sufficient-type** of the **ravel-list** of V under the **type** of A.

– **Type:** A member of the enumerated set **array-type**.

Note: *The term* **array-of-vectors** *is used only as an expository device.*

Along-Axis K of A: An operation that, for an **array** A with non-zero **rank** N, produces Z, an **array-of-vectors** of **rank** $N-1$ such that the **shape-list** of Z is the **shape-list** of A with **item** K omitted. Each **item** of the **ravel-list** of Z is a **vector** whose **length** is the same as **axis** K of A.

Note: *Some operations on vectors extend to arrays of greater rank in a manner similar to* **scalar-extension**. **Along-axis** *is an expository device used in the definition of these operations.*

Vector-Item I of A: An operation that, for an **array-of-vectors** A, returns **item** I of the **ravel-list** of A.

Ravel-Along-Axis K of A: An operation that, for an **array** A with non-zero **rank** N, produces an **array-of-vectors** Z such that the **shape-list** of Z is the **product-of** the **shape-list** of A with **item** K omitted, and the **ravel-list** of Z is the **ravel-list** of **along-axis** K of A.

First-Scalar in A: An operation that, for A a nonempty **array**, returns a **scalar** Z such that the **ravel-list** of Z is the **first-item** in the **ravel-list** of A and the **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of A.

Remainder-of A: An operation that, for A a non-empty **vector**, returns a **vector** Z such that the **length** of Z is **negative-one plus** the **length** of A, the **ravel-list** of Z is the **rest-of** the **ravel-list** of A and the **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of A.

Row I of A: An operation that, for A an **array** of **rank two** and I a **number**, returns **vector-item** I **along-axis two** of A.

Number-of-Rows in A: An operation that, for A an **array** of **rank two**, returns **item one** of the **shape-list** of A.

Integer-Array-Nearest-to A: An operation that, for A an **array** having the property that each **item** of the **ravel-list** of A is a **near-integer**, returns a **numeric array** Z such that the **shape-list** of Z is the same as the **shape-list** of A and each **item** of the **ravel-list** of Z is the **integer-nearest-to** the corresponding **item** of the **ravel-list** of A.

Boolean-Array-Nearest-to A: An operation that, for A an **array** having the property that each **item** of the **ravel-list** of A is a **near-Boolean**, returns the **integer-array-nearest-to** A.

5.3.3 Defined-Functions

Note: *A* **defined-function** *represents a function defined by a user. The attributes of a* **defined-function** *are given here. Operations are described in the* **Defined Functions** *chapter.* **Defined-operators** *are a subclass of* **defined functions**.

Defined-Function: An object with the following attributes:

- **Canonical-Representation:** A **character array** whose **rank** is **two**.
- **Stop-Vector:** A **numeric vector**.
- **Trace-Vector:** A **numeric vector**.

5.3.4 Tokens

Class-Names: A set containing the following members:

Assignment-Arrow
Axis-Error
Branch
Branch-Arrow
Character-Literal
Clear-State-Indicator
Colon
Command-Complete
Committed-Value
Complete-Index-List
Constant
Defined-Function
Defined-Function-Name
Defined-Dyadic-Operator
Defined-Dyadic-Operator-Name
Defined-Monadic-Operator
Defined-Monadic-Operator-Name
Definition-Error
Distinguished-Identifier
Domain-Error
Dyadic-Operator
Elided-Index-Marker
Escape
Implicit-Error
Incorrect-Command
Index-Error
Index-Separator
Interrupt
Label
Label-Name
Left-Argument-Name
Left-Axis-Bracket
Left-End-of-Statement
Left-Index-Bracket
Left-Operand-Name
Left-Parenthesis
Length-Error
Limit-Error
Local-Name
Monadic-Operator
Nil
Niladic-Defined-Function
Niladic-Defined-Function-Name
Niladic-System-Function-Name
Not-Copied
Not-Erased
Not-Found
Not-Saved
Numeric-Literal
Partial-Index-List
Primitive
Primitive-Function
Rank-Error
Result-Name
Right-Argument-Name
Right-Axis-Bracket
Right-End-of-Statement
Right-Index-Bracket
Right-Operand-Name
Right-Parenthesis
Semicolon
Shared-Variable
Shared-Variable-Name
Simple-Identifier
Small-Circle
Syntax-Error
System-Function-Name
System-Variable-Name
Unwind
Value-Error
Variable
Variable-Name

Token: An object with the following attributes:

– **Class:** A member of the set **Class-Names**.

– **Content:** A **number**, a **character**, or an object, according to the **class** of the **token**, as indicated in **Table 2**.

Note: **Tokens** *represent the internal objects manipulated by an implementation of APL.*

The **class** *of a* **token** *is a straightforward indication of the sort of object it represents. A* **token** *of* **class nil**, *for example, is used to return a* **value-error** *from a defined function that does not set its result name. Note that* **class-names** *is an enumerated set. This means that* **nil** *has no definition other than its literal appearance on this page. Its significance, like the significance of APL characters in this document, lies in the use of its name in evaluation sequences.*

The **content** *of a* **token** *varies with the* **class** *of the* **token**. *If specified, the content is a* **character**, *a* **list** *of* **characters**, *a* **list** *of* **numbers**, *an* **array**, *an* **index-list**, *a* **shared-variable**, *or a* **defined-function**.

5.3.4.1 Metaclasses

Note: *Metaclasses are subsets of the enumerated set* **class-names**. *They are used to shorten evaluation sequences by abstracting a sequence of tests on the* **class** *of a* **token** *into a single test for membership in a metaclass. "If B is an* **error**," *is equivalent to "If the* **class** *of B is in the metaclass* **error**."

– **Metaclass**: A subset of the enumerated set **class-names**.

– **Identifier**: A metaclass containing **simple-identifier** and **distinguished-identifier**.

– **Literal**: A metaclass containing **character-literal** and **numeric-literal**.

– **Lexical-Unit**: A metaclass containing **primitive**, **literal**, and **identifier**.

– **Value**: A metaclass containing **committed-value** and **constant**.

– **Delimiter**: A metaclass containing **primitive-function**, **branch-arrow**, **assignment-arrow**, **left-end-of-statement**, **right-end-of-statement**, **left-index-bracket**, **right-index-bracket**, **elided-index-marker**, **left-axis-bracket**, **right-axis-bracket**, **left-parenthesis**, **right-parenthesis**, **small-circle**, and **semicolon**.

– **Defined-Name**: A metaclass containing **shared-variable-name**, **variable-name**, **defined-function-name**, **defined-dyadic-operator-name**, **defined-monadic-operator-name**, **niladic-defined-function-name**, **result-name**, **left-argument-name**, **right-argument-name**, **label-name**, and **local-name**.

– **Defined-Operator**: A metaclass containing **defined-dyadic-operator** and **defined-monadic-operator**.

– **System-Name**: A metaclass containing **system-variable-name**, **system-function-name**, and **niladic-system-function-name**.

– **Classified-Name**: A metaclass containing the members of **system-name** and **defined-name**.

– **Syntactic-Unit**: A metaclass containing the members of **classified-name** and **delimiter**.

Class-Name	Content
Assignment-Arrow	
Axis-Error	
Branch	An array
Branch-Arrow	
Character-Literal	A list of characters
Clear-State-Indicator	
Colon	
Command-Complete	
Committed-Value	An array
Complete-Index-List	An index-list
Constant	An array
Defined-Function	A defined-function
Defined-Function-Name	A list of characters
Defined-Dyadic-Operator	A defined-function
Defined-Dyadic-Operator-Name	A list of characters
Defined-Monadic-Operator	A defined-function
Defined-Monadic-Operator-Name	A list of characters
Definition-Error	
Distinguished-Identifier	A list of characters
Domain-Error	
Dyadic-Operator	A character
Elided-Index-Marker	
Escape	
Implicit-Error	
Incorrect-Command	
Index-Error	
Index-Separator	
Interrupt	
Label	An array
Label-Name	A list of characters
Left-Argument-Name	A list of characters
Left-Axis-Bracket	
Left-End-of-Statement	
Left-Index-Bracket	
Left-Operand-Name	A list of characters
Left-Parenthesis	
Length-Error	
Limit-Error	

Table 2: Relationship between Class-Name and Content

Class-Name	Content
Local-Name	A list of characters
Monadic-Operator	A character
Nil	
Niladic-Defined-Function	A defined-function
Niladic-Defined-Function-Name	A list of characters
Niladic-System-Function-Name	A list of characters
Not-Copied	
Not-Erased	
Not-Found	
Not-Saved	
Numeric-Literal	A list of numbers
Partial-Index-List	An index-list
Primitive	A character
Primitive-Function	A character
Rank-Error	
Result-Name	A list of characters
Right-Argument-Name	A list of characters
Right-Axis-Bracket	
Right-End-of-Statement	
Right-Index-Bracket	
Right-Operand-Name	A list of characters
Right-Parenthesis	
Semicolon	
Shared-Variable	A shared-variable
Shared-Variable-Name	A list of characters
Simple-Identifier	A list of characters
Small-Circle	
Syntax-Error	
System-Function-Name	A list of characters
System-Variable-Name	A list of characters
Unwind	
Value-Error	
Variable	An array
Variable-Name	A list of characters

Table 2: (Continued)

- **Error**: A metaclass containing **axis-error**, **domain-error**, **implicit-error**, **index-error**, **length-error**, **limit-error**, **rank-error**, **syntax-error**, **value-error**, and **interrupt**.

Note: *This* **metaclass** *includes only errors that occur in the evaluation of APL statements.*

- **Report**: A metaclass containing **incorrect-command**, **not-copied**, **not-erased**, **not-found**, and **not-saved**.
- **Exception**: A metaclass containing **branch**, **escape**, **clear-state-indicator**, **unwind**, and the members of **error** and **report**.
- **Result**: A metaclass containing **nil** and the members of **exception** and **value**.

5.3.4.2 Index-List

- **Index-List**: A (possibly empty) **list** consisting of **tokens** whose **class** is either **constant** or **elided-index-marker**.

5.3.5 Symbols

Symbol: An object with the following attributes:

- **Name:** A **list** of **characters**.
- **Referent-List:** A **list** of **tokens**.

5.3.6 Contexts

- **Mode-Names**: An enumerated set containing the members **immediate-execution**, **execute**, **function-definition**, **quad-input**, and **defined-function**.

Context: An object with the following attributes:

- **Mode:** An element of the enumerated set **mode-names**.
- **Stack:** A **list** of **tokens**.
- **Current-Line:** A **list** of **characters**.
- **Current-Statement:** A **list** of **tokens**.
- **Current-Function:** If **mode** is **defined-function**, a **defined-function**; otherwise undefined.
- **Current-Line-Number:** If **mode** is **defined-function**, an **index**; otherwise undefined.

5.3.7 Workspaces

Note: *The workspace is the basic organisational unit in an APL system. A workspace contains data, programs, execution status, and environmental information.*

– **Workspace-Presence**: An enumerated set containing **absent** and **present**.

Workspace: An object with the following attributes:

– **Owner:** A **user-identification**.

– **Workspace-Name:** A **list** of **characters**.

– **Symbol-Table:** A **list** of all **symbols** whose **names** are distinct.

– **State-Indicator:** A **list** of **contexts**.

– **Existential-Property:** A member of the enumerated set **workspace-presence**.

Clear-Workspace: A **workspace** with the following values:

– **Owner**: **this-owner**.

– **Workspace-Name**: The **clear-workspace-identifier**.

– **Symbol-Table**: A **list** of all **symbols** whose **names** are distinct and whose **referent-lists** each consist of the **list** whose only member is **nil**.

– **State-Indicator**: An empty **list** of **contexts**.

– **Existential-Property**: **absent**.

5.3.8 Sessions

Note: *A user interacts with an APL system through a session, an abstraction that represents a hypothetical machine capable of carrying out the evaluation sequences in the International Standard.*

– **Session-Identification**: Either a **number** or a **list** of **characters**, depending upon the **implementation-parameter session-identification-type**.

– **User-Identification**: Either a **number** or a **list** of **characters**, depending upon the **implementation-parameter user-identification-type**.

– **Keyboard-States**: An enumerated set containing the members **open-keyboard** and **locked-keyboard**.

Session: An object with the following attributes:

– **Active-Workspace:** A **workspace**.

– **This-Session:** A **session-identification**.

– **This-Owner:** A **user-identification**.

– **Attention-Flag:** A member of the enumerated set **Boolean**.

– **Keyboard-State:** A member of the enumerated set **keyboard-states**.

– **Current-Prompt:** A **character vector**.

– **Quote-Quad-Prompt:** A **character vector**.

– **Event-Time:** A **nonnegative-number**.

– **Event-Message:** A **character array** giving, in an **implementation-defined** form, the **event-report** and any other information the implementation deems helpful in locating the error's source.

– **Event-Type:** A two-element integer vector corresponding to the most recent event. Defined values are:

  `0 0` — No error

  `0 1` — Undefined event

  There may be additional **implementation-defined** values as well as values supplied by a progarm via ⎕ES.

– **Current-Context**: The **first-item** in the **state-indicator** of the **active-workspace**.

– **Current-Stack**: The **stack** of the **current-context**.

– **Symbol-Named-By** T: The **symbol** in the **symbol-table** of the **active-workspace** whose **name** is the same as the **content** of the **token** T.

– **Current-Referent** of T: The **first-item** in the **referent-list** of the **symbol-named-by** T.

– **Current-Class** of T: The **class** of the **current-referent** of T.

– **Current-Content** of T: The **content** of the **current-referent** of T.

– **Comparison-Tolerance**: The **current-content** of ⎕CT.

– **Random-Link**: The **current-content** of ⎕RL.

– **Print-Precision**: The **current-content** of ⎕*PP*.

– **Index-Origin**: The **current-content** of ⎕*IO*.

– **Latent-Expression**: The **current-content** of ⎕*LX*.

System-Parameter: Any of **comparison-tolerance**, **random-link**, **print-precision**, **index-origin**, or **latent-expression**.

The initial values of **system-parameters** in a **clear-workspace** are **implementation-defined**.

5.3.9 Shared-Variables

Note: *A* **shared-variable** *is a variable shared between two* **sessions**.

Shared Variables are an **optional-facility**.

Shared-Variable: An object with the following attributes:

– **Session-A:** A **session-identification**.

– **Session-A-Active:** A **Boolean**.

– **Session-A-ACV:** A **Boolean vector** of **length four**.

Note: *ACV stands for access control vector.*

– **Session-B:** A **session-identification**.

– **Session-B-Active:** A **Boolean**.

– **Session-B-ACV:** A **Boolean vector** of **length four**.

– **Shared-Name:** An **identifier**.

– **Shared-Value:** A **token**, either a **constant** or **nil**.

– **State:** An **integer**, either **zero**, **one**, or **two**.

– **Session-A-Event:** A **Boolean**.

– **Session-B-Event:** A **Boolean**.

Note: *The operations in the chapter* **Shared Variables** *use the* **shared-variable** *attributes as follows:*

– **Session-A***: The* **session-identification** *of the first* **session** *to offer the* **shared-variable**.

- **Session-A-Active**: *A* **Boolean**; **one** *if* **session-A** *is currently sharing this* **shared-variable**, **zero** *if not.*
- **Session-A-ACV**: *The contribution of* **session-A** *to the ACV.*
- **Session-B**: *The* **session-identification** *of the* **session** *with which* **session-A** *offered to share the* **shared-variable**; *this may be the* **general-offer** *while* **state** *is* **one**.
- **Session-B-Active**: *A* **Boolean**; **one** *if* **session-B** *is currently sharing this* **shared-variable**, **zero** *if not.*
- **Session-B-ACV**: *The contribution of* **session-B** *to the ACV.*
- **Shared-Name**: *An* **identifier**, *the name* **session-A** *designated for this* **shared-variable**.
- **Shared-Value**: *A* **token** *of class* **constant** *or* **nil**; *the current value of this* **shared-variable**.
- **State**: *An* **integer**, *either* **zero**, **one**, *or* **two**, *used in combination with the ACV to determine which operations must be delayed.*

5.3.10 Systems

Note: *A* **system** *represents a set of active APL users, a library, and, optionally, a shared variable facility.*

System: An object with the following attributes:

- **Library:** A **list** of **workspaces** in which each combination of possible values for the attributes **owner** and **workspace-name** occurs exactly once.
- **Active-Users:** A **list** of **sessions**.
- **Shared-Variable-List:** A **list** of **shared-variables**.
- **Implementation-Parameters:** The following quantities, referred to in this International Standard by name, whose values are **implementation-defined**:
  - **Atomic-Vector**: An **implementation-defined character vector** containing every member of the **required-character-set** exactly once.
  - **Initial-Comparison-Tolerance**: A member of the **internal-value-set** for **comparison-tolerance** that is the value of **comparison-tolerance** in a **clear-workspace**.
  - **Initial-Index-Origin**: A member of the **internal-value-set** for **index-origin** that is the value of **index-origin** in a **clear-workspace**.
  - **Initial-Latent-Expression**: A member of the **internal-value-set** for **latent-expression** that is the value of **latent-expression** in a **clear-workspace**.
  - **Initial-Print-Precision**: A member of the **internal-value-set** for **print-precision** that is the value of **print-precision** in a **clear-workspace**.
  - **Initial-Random-Link**: A member of the **internal-value-set** for **random-link** that is the value of **random-link** in a **clear-workspace**.

– **Initial-Event-Message**: The **empty-event-message**.
– **Reduction-Style**: One of the two symbolics **Enclose-Reduction-Style** or **Insert-Reduction-Style**, indicating which of the two styles of reduction the system uses.
– **Initial-Event-Type**: The two-element integer vector `(0,0)`.
– **Clear-Workspace-Identifier**: A **list** of **characters**.
– **Positive-Number-Limit**: The **real-number greater-than** all other **real-numbers**.
– **Negative-Number-Limit**: The **real-number less-than** all other **real-numbers**.
– **Positive-Counting-Number-Limit**: The **counting-number** greater than all other **counting-numbers**.
– **Negative-Counting-Number-Limit**: The **counting-number less-than** all other **counting-numbers**.
– **Index-Limit**: The **index** greater than all other indices. This value specifies the maximum value of any **item** of the **shape-list** of any **array**, ignoring storage limitations.
– **Count-Limit**: An **index** not greater than **index-limit** that specifies the maximum value for the **count** of an array, ignoring storage limitations.
– **Rank-Limit**: An **index** not greater than **count-limit** that specifies the maximum value for the **rank** of any array, ignoring storage limitations.
– **Workspace-Name-Length-Limit**: A **positive-counting-number** that specifies the maximum number of **characters** in a **workspace-name**.
– **Identifier-Length-Limit**: A **positive-counting-number** not greater than **count-limit** that specifies the maximum number of **characters** in an **identifier**.
– **Quote-Quad-Output-Limit**: A **nonnegative-counting-number** that specifies the maximum number of **characters** that can be used in a prompt set by **Quote Quad Output**.
– **Comparison-Tolerance-Limit**: The largest **real-number** permitted by the implementation for the system parameter **comparison-tolerance**.
– **Integer-Tolerance**: A **real-number** not greater than **comparison-tolerance-limit** used to determine whether a given number is to be considered integral.
– **Real-Tolerance**: A **nonnegative-number** not greater than **comparison-tolerance-limit** used to determine whether a given number is to be considered real.
– **Full-Print-Precision**: The smallest **positive-counting-number** which, when used as the value of **print-precision**, causes **numeric-output-conversion** to produce a unique **numeric-scalar-literal** for every **number**.
– **Print-Precision-Limit**: The largest **positive-counting-number** permitted by the implementation for the system parameter **print-precision**.

Note: Print-precision-limit *must be at least* **full-print-precision** *if every unique* **number** *is to have a unique decimal representation.*

– **Exponent-Field-Width**: A **positive-counting-number** giving the number of places, including sign and trailing blanks, used for the exponent field in the result of **dyadic format**.

– **Session-Identification-Type**: A member of the enumerated set **array-type**, either **character** or **numeric**.

– **User-Identification-Type**: A member of the enumerated set **array-type**, either **character** or **numeric**.

– **Indent-Prompt**: A **list** of **characters** used to indicate to the user that the **session** is in **immediate-execution** mode.

– **Quad-Prompt**: A **list** of **characters** used to indicate to the user that the **session** is in **quad-input** mode.

– **Function-Definition-Prompt**: A **list** of **characters** used to indicate to the user that the **session** is in **function-definition** mode.

– **Line-Limit**: A **positive-counting-number** that specifies the maximum number of lines permitted in a defined function, ignoring storage limitations.

– **Definition-Line-Limit**: A **positive-number** that specifies the maximum value of a line number in **function-definition** mode.

– **General-Offer**: A reserved **session-identification** used to indicate that the offerer of a **shared-variable** is willing to share the proffered variable with any other **session**. This **implementation-parameter** is required only if the **optional-facility shared-variable-protocol** is present.

Any action that would cause a limit specified by an implementation parameter to be exceeded shall signal a **limit-error**.

5.4 Evaluation Sequences

The evaluation sequences that define APL operations in the remainder of this International Standard are written in English in the imperative mood. The English phrases used in evaluation sequences are restricted to the set specified in this subsection.

Indention is used in evaluation sequences to indicate scope, typically of the consequent of an implication.

For example, in the evaluation sequence fragment below, the indented text is evaluated only if both A and B are **vectors**; the “otherwise” clause is evaluated only if at least one of A or B is not a **vector**.

...
If A is a **vector** and B is a **vector**,
    If A is **empty** and ...
    ...
Otherwise, return ...

Expressions in APL are used in evaluation sequences. A given evaluation sequence uses only APL operations that have been specified earlier in the International Standard. Where indices are generated or used by APL expressions in evaluation sequences, they are evaluated with origin **one**. The APL relational operations are never used in evaluation sequences unless they are qualified with the value to be used for **comparison-tolerance**.

5.4.1 Evaluation Sequence Phrases

Note: *The following phrases are used in evaluation sequences. They are not set in* **bold** *type nor cross-referenced when employed in evaluation sequences.*

- **For all** A, C: An evaluation sequence phrase used to specify that the action or condition specified by the consequent C is to be performed or checked for every value in the antecedent A.

  Example:
  ...
  For all I in the **index-set** of A,
  Set **item** I of the **ravel-list** of A to **zero**.
  ...

- **For form** F**,** C:

- **For pattern** F**,** C:

  An evaluation sequence phrase used to specify that the actions listed in the consequent C are to be performed only if the pattern or form F is the one that caused this evaluation sequence to be selected.

  Example:
  For form A ⌽B
  If B is a **scalar** ...
  ...
  For form A ⊖B
  If ...

- **If** A**,** C: An evaluation sequence phrase used to specify that the actions listed in the consequent C are to be performed only if the value of the antecedent A is **one**.

- **If** T is an **mc**, C: For T a **token** and **mc** a **metaclass**, an evaluation sequence phrase equivalent to “If the **class** of T is in the **metaclass mc**, C.”

- **Let** A **stand for** B: An evaluation sequence phrase used to indicate that the name A is to be an abbreviation for the phrase B in subsequent evaluation sequence lines.

- **Otherwise**, C: An evaluation sequence phrase used to indicate that the actions listed in consequent C are to be performed only if the antecedent in the immediately preceding **if** phrase was **zero**. If a consequent is an indented paragraph, the immediately preceding **if** statement is the one at the same level of indention as the **otherwise** phrase.

- **Repeat**: An evaluation sequence phrase used to indicate that the block of text indented after the **repeat** is to be executed repeatedly until a **return** or **signal** phrase is encountered. The end of a repeated block is indicated by the parenthetic remark “(End of repeated block).”

- **Return** X: An evaluation sequence phrase used to specify that evaluation of this evaluation sequence is to stop and that a **token** is to be returned to the caller of the evaluation sequence. If X is a **token**, then X is returned; if X is an **array**, a **token** of class **constant** and content X is returned.

- **Set** A **to** B: An evaluation sequence phrase used to specify that the referent of A is to be assigned the value B.

- **Signal** X: An evaluation sequence phrase used to specify that evaluation of this evaluation sequence is to stop and that a **token** whose **class** is X, where X is an **error**, is to be returned.

- **Using** O**,** C: An evaluation sequence phrase used to indicate that the consequent C is to be evaluated against the specific object O. This construct is used, for example, in the description of shared variables to indicate which shared variable is to be changed.

- **Wait until**: An evaluation sequence phrase that indicates that the **session** is waiting for a condition to hold before continuing.

5.4.2 Diagrams

Note: Diagrams *are used in this International Standard to indicate permissible sequences of* **characters** *or of* **tokens**.

- **Character-Diagram**: A graph that designates a subset of the set of all **lists** of **characters**.

- **Token-Diagram**: A graph that designates a subset of the set of all **lists** of **tokens**.

- **Thread** D **with** A: An evaluation sequence phrase used to indicate that a search is to be made for a route through the diagram D that corresponds to the list or part of a list A. A **character-diagram** is **threaded** with a **list** of **characters** by setting a list-cursor to the first item in the **list**, and a diagram-cursor to the arrow-tail ►——, in the diagram, then progressing along paths in the diagram to the arrow-head ——►. At a junction, the diagram-cursor may enter the alternate path only if it can do so by making an acute-angle turn from its current direction. It may not back up into the alternate path.

  For the diagram-cursor to advance along a path labelled with a graphic symbol such as ÷, that graphic symbol must be pointed to by the list-cursor, which then also advances. For the diagram-cursor to advance along a path labelled with a diagram name, that diagram must itself be threaded, using the same list-cursor and a new diagram-cursor. If in either case the diagram-cursor cannot advance, the diagram-cursor is set back to the previous junction, the list-cursor is set back correspondingly, and a new path is tried; or if the diagram-cursor is now at the arrow-tail, the diagram cannot be threaded with the list.

  If a route is found through a diagram for a given **list** of **characters**, it is unique. In this case, the diagram can be **rethreaded**, following the same route without entering blind alleys. For example, characters are collected into identifier tokens through actions performed while a **character-diagram** is being **rethreaded**.

Token-diagrams are **threaded** exactly like **character-diagrams**, except that the items pointed to by the list-cursor are **tokens** and are matched either by their **class** or by both their **class** and their **content**.

- **Rethread** D **with** A: An evaluation sequence phrase used to indicate that the route found by a previous **threaded** phrase is to be threaded again, in order to effect certain actions through **when** phrases.
- **When** A**,** C: An evaluation sequence phrase used during a **rethread** phrase to indicate that when the antecedent A is true, the consequent C is to be performed.
- A **Matches** D: An operation that, for diagram D and **list** A, is true if A is a member of the set of **lists** designated by D, and false otherwise.

  A **list** matches a diagram if it can be **threaded** in such a way that there are no items remaining in the **list** when the final exit path is taken.

Note: *The question of whether a* **list matches** *a diagram is different from the question of whether a* **list** *can* **thread** *a diagram, since there will normally be items left over in a* **list** *once a diagram has been* **threaded***—in collecting the digits in a number, for example, the* **digit** *diagram removes only one digit at a time from a* **list** *of* **characters***.*

5.5 Other Terms

- **Side-effect**: Any effect an operation has other than returning a result.
- **Atomic**: The property of an operation with **side-effects** to produce its **side-effects** only if it completes without error.

Note: *For example, the specification of items 3, 4, and 5 of* A *in the APL line*

```
B ←A[3 4 5] ←2
```

is a side effect, since the result placed in B *is the scalar* 2*; further, since* **indexed assignment** *is specified to be* **atomic***, no change to* A *will occur if the indexed assignment fails with, for example, an index error.*

6 Syntax and Evaluation

6.1 Introduction

Note: *This chapter specifies the rules for evaluating lines. These rules are used by the subsections* **execute***,* **quad input***,* **immediate-execution***, and* **defined-function-control***.*

The rules are described in three subsections, **evaluate-line***,* **evaluate-statement***, and* **reduce-statement***.*

The data structures and procedures used to describe syntax and evaluation in this International Standard are strictly expository devices; they are not intended to represent required or desirable implementation techniques. The order of reporting errors implied by the diagram-matching actions in the model is not required.

6.1.1 Evaluate-Line

Form: **Evaluate-Line**

Informal Description: Evaluate-line is the principal procedure in the evaluation of APL lines. It decomposes the **list** of **characters** named **current-line** into **lexical-units** according to the **character-diagram** named **line**.

Some diagrams referred to in **line** have their exit paths flagged with asterisks. Once the diagram **line** has been threaded, the rethreading pass is used to gather certain character sequences (diagrams ending in two asterisks), and either create tokens (diagrams ending in one asterisk) or discard the character sequences gathered (diagrams ending in three asterisks).

Evaluate-line calls **evaluate-statement** to convert the **lexical-units** into a **result**.

Note that the **result** can be a **constant** (from, for example, 2+2), **committed-value** (from *A*←1), **nil** (⍎''), **branch** (⍎'→3'), **escape** (→), **unwind** (→), **error** (2−), **clear-state-indicator** (from)*SIC*) or **interrupt** (from **signal-interrupt**).

Note: *The handling of these result classes by the different callers of* **evaluate-line** *specifies the treatment of exceptions, and should be analysed carefully.*

Evaluate-line is called from evaluation sequences in the subsections **defined-function-control**, **execute**, **execute-alternate**, **immediate-execution**, and **quad-input**. If the **optional-facility statement-separator-facility** is implemented, **evaluate-line** evaluates successive statements in **current-line** beginning with the leftmost statement and continuing until evaluation produces an **exception** or the rightmost statement has been evaluated.

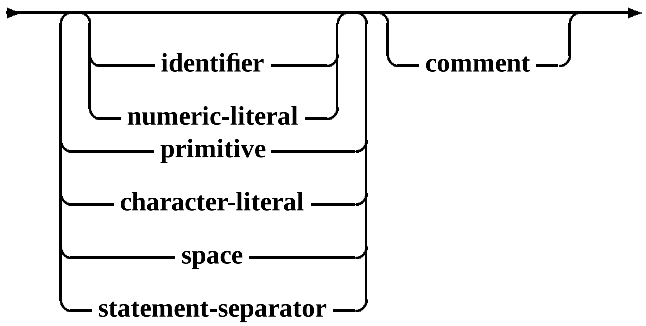
Evaluation Sequence:

Set C to the **empty-list** of **characters**.
Set **current-statement** to the **empty-list** of **tokens**.
Thread **line** with **current-line**; if **current-line** does not match **line**, signal **syntax-error**.
Rethread **line** with **current-line**, taking the following actions:
  When a **character-diagram** ending in '-***→' is recognised, set C to the **empty-list** of **characters**.
  When a **character-diagram** ending in '—**→' is recognised, append to C as a new last **item** the **character** just passed.
  When a **character-diagram** ending in '——*→' is recognised,
    Append to **current-statement** as a new last **item** a **token** with **class** given by the name of the **character-diagram** and with **content** C.
    Set C to the **empty-list** of **characters**.
  When the **optional-facility statement-separator-facility** is implemented and the **character-diagram statement-separator** is recognised,
    Set Z to **evaluate-statement**.
    If Z is an **exception**, return Z.
    If Z is a **constant**, **display** Z.
    Set **current-statement** to the **empty-list** of **tokens**.
    Set C to the **empty-list** of **characters**.
  When the **character-diagram line** is matched,
    Set Z to **evaluate-statement**.
    Return Z.

6.1.2 Character-Diagrams

The diagrams in this subsection are **character-diagrams**: the APL Graphic Symbols such as ∇ in these **character-diagrams** match corresponding **characters** in the **required-character-set**.

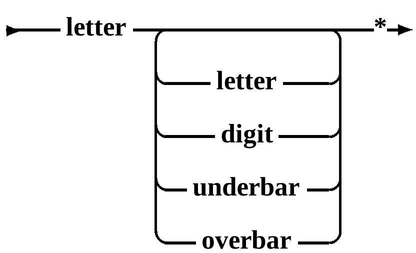
Line



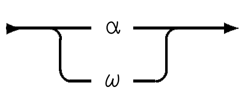
Identifier

Simple-identifier

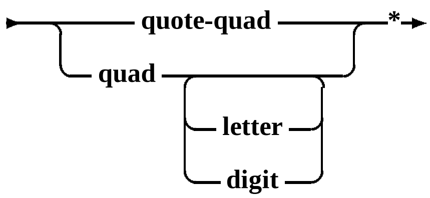
Literal-identifier



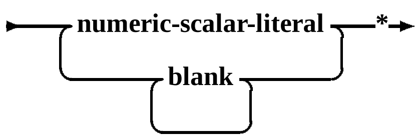
Direct-identifier



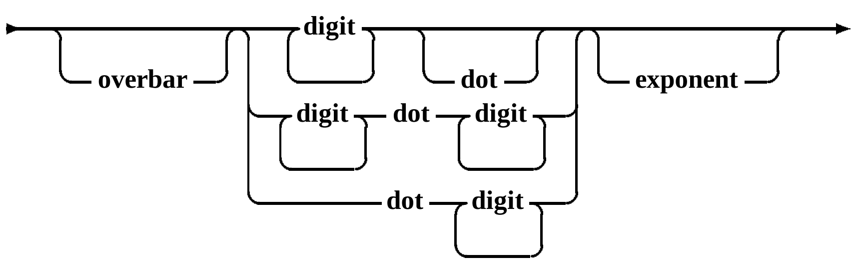
Distinguished-identifier



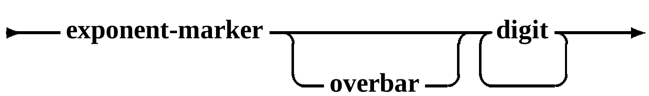
Numeric-Literal



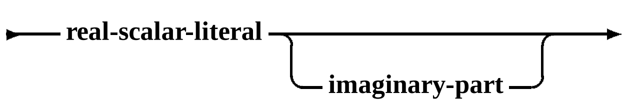
Real-scalar-literal



Exponent

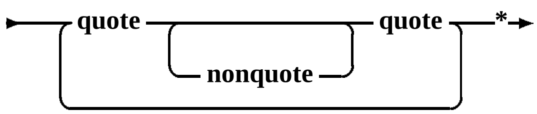


Numeric-scalar-literal

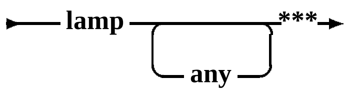


Imaginary-part

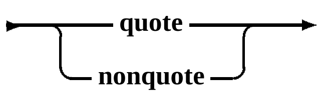
Character-literal



Comment



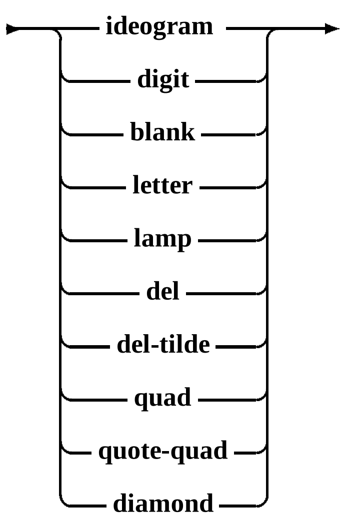
Any



Primitive

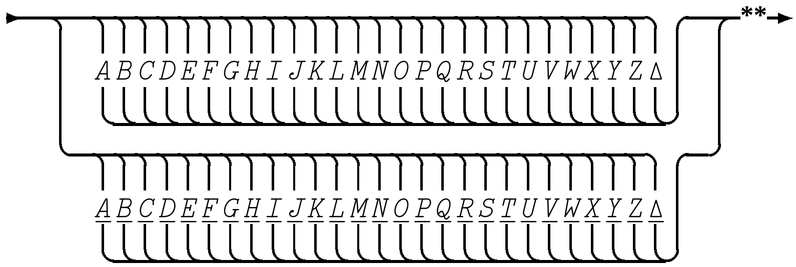
Space

Nonquote

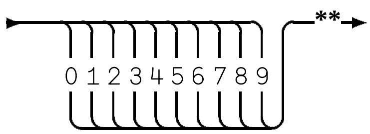


Statement-separator

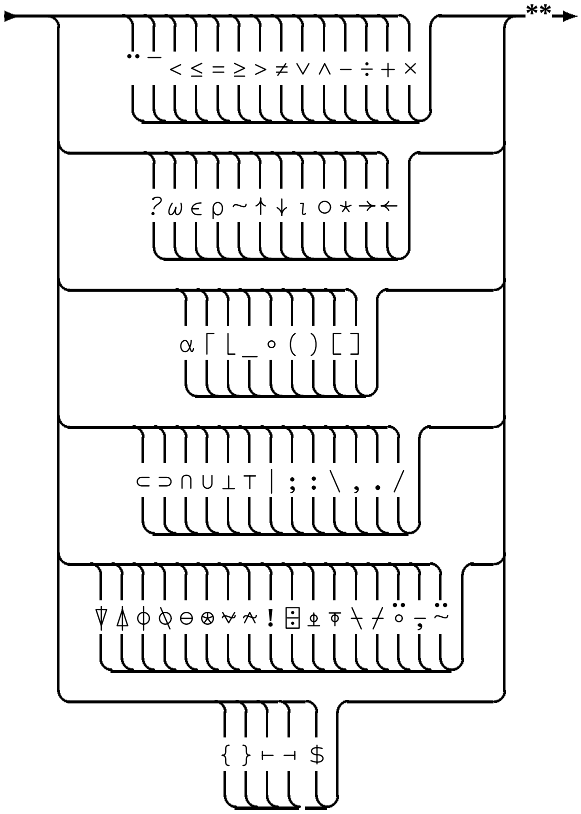
Letter



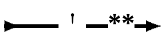
Digit



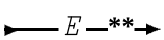
Ideogram



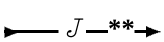
Quote



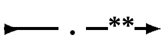
Exponent-marker



Complex-marker



Dot



Underbar

```
►—— _ —**→
```

Overbar

```
►—— ¯ —**→
```

Blank

```
►——   —**→
```

Del

```
►—— ∇ —**→
```

Del-tilde

```
►—— ⍫ —**→
```

Lamp

```
►—— ⍝ —**→
```

Quad

```
►—— ⎕ —**→
```

Quote-quad

```
►—— ⍞ —**→
```

Diamond

```
►—— ⋄ —**→
```

Example

The example introduced in this subsection is continued through the syntax analysis portion of this International Standard.

After **evaluate-line** has processed the **current-line**

```
ABC←FN ⎕⌽[1+0] DEF[1;5 6]×3.45E4,⍴'ABC' ⍝COMMENT
```


current-statement looks like this:

Identification	**Content**	**Class**
*T*01	'*ABC*'	**simple-identifier**
*T*02	'←'	**primitive**
*T*03	'*FN*'	**simple-identifier**
*T*04	'⎕'	**distinguished-identifier**
*T*05	'ϕ'	**primitive**
*T*06	'['	**primitive**
*T*07	'1'	**numeric-literal**
*T*08	'+'	**primitive**
*T*09	'0'	**numeric-literal**
*T*10	']'	**primitive**
*T*11	'*DEF*'	**simple-identifier**
*T*12	'['	**primitive**
*T*13	'1'	**numeric-literal**
*T*14	';'	**primitive**
*T*15	'5 6'	**numeric-literal**
*T*16	']'	**primitive**
*T*17	'×'	**primitive**
*T*18	'3.45*E*4'	**numeric-literal**
*T*19	','	**primitive**
*T*20	'ρ'	**primitive**
*T*21	''*ABC*''	**character-literal**

Each **token** has a **content**, which is a **list** of **characters**, and a **class**, which is in the metaclass **lexical-unit**. The **tokens** produced are numbered *T*01 to *T*21 for later reference. Note that comments and blanks between tokens are discarded during this tokenisation process.

Note: *As the diagram* **line** *shows, identifiers and numeric literals are separated by one or more spaces, character-literals, or primitives.* 1 *ABC*'*A*' *is a line,* 1*ABC*'*A*' *is not. No such separation rule applies to primitive function symbols.* 1ρ'*AB*' *is a line.*

The sequence 3+.4 *is parsed* **numeric-literal**, **primitive**, **numeric-literal** *but the sequence* 3+.× *is parsed* **numeric-literal**, **primitive**, **primitive**, **primitive**.

A comment may appear at the end of a line or alone on a line.

The **statement-separator-facility** is an **optional-facility**.

Parsing an **identifier token** signals a **limit-error** if the **number-of-items** in the **list** of **characters** is greater than **identifier-length-limit**.

6.1.3 Evaluate-Statement

Evaluate-Statement

Informal Description: Evaluate-statement is performed on **current-statement**, a **list** of

tokens found in the **current-context**.

It uses **bind-token-class** to convert **identifiers** to **classified-names** and **constants**.

It uses **literal-conversion** to convert **literals** to **constants**.

It uses the **token-diagram** named **statement** to verify that the statement is properly formed and to resolve certain ambiguous **tokens**. For example, the **token** containing] is resolved to either **right-axis-bracket** or **right-index-bracket**.

It calls the evaluation sequence in the subsection **reduce-statement** to convert the **current-statement** to a **result**, which it returns to its caller.

Evaluate-statement is called by **evaluate-line**.

Evaluation Sequence:

For every **index** I in the **index-set** of **current-statement**,
  Let T stand for **item** I of **current-statement**.
  If T is an **identifier**, set Q to **bind-token-class** of T
  If T is a **literal**, set Q to the **literal-conversion** of T.
  If T is a **primitive**, set Q to T.
  If Q is an **exception**, return Q.
  Otherwise, set T to Q.
Thread **statement** with **current-statement**; if **statement** cannot be matched, signal **syntax-error**.
Rethread **statement** with **current-statement**, taking the following action:
  When any **token-diagram** ending in '—*→' is threaded,
    Replace the **token** in **current-statement** that matched the diagram with a **token** having the same **content**, but having a **class** given by the name of the **token-diagram**.
Append to **current-statement** as a new first **item** a **left-end-of-statement token**.
Append to **current-statement** as a new last **item** a **right-end-of-statement token**.
Set Z to **reduce-statement**.
If **mode** is **defined-function** and **current-line-number** is in **current-trace-vector**, set Z to **trace-display** of Z.
Return Z.

Additional Requirement:

This International Standard permits two orders of evaluation for expressions in brackets, as follows:

The syntax evaluator used here distinguishes axis brackets from index brackets in order to classify a function immediately to the right as monadic or dyadic. For example, the evaluation of ϕ[ρX]+ X is specified to proceed by first calling monadic plus, then evaluating monadic rho.

The permitted alternative is to evaluate any expression in brackets before determining whether a function immediately to the right is monadic or dyadic. In the example given, this alternative behaviour would evaluate monadic rho before discovering that plus is used monadically.

Note: *The distinction can be observed only through side effects.*

6.1.4 Bind-Token-Class

Bind-Token-Class of *T*

Informal Description: Bind-token-class is used to bind each **identifier** in **current-statement** to its current **syntactic-unit**; if the **class** of a **token** changes between this point and the time the **token** is used (as it would for example in *F* ⎕*EX*'*F*', assuming *F* were initially a **defined-function**), the change will be detected and reported as a **syntax-error** in the appropriate **phrase-evaluator**.

This prebinding limits the International Standard to defining the meaning of statements only when the syntax class of their tokens does not change in mid-statement. **Conforming-implementations** may, of course, relax these rules. **Conforming-programs** must abide by them.

Evaluation Sequence:

Let **f** stand for the **content** of *T*.
If *T* is a **simple-identifier**,
If the **current-class** of *T* is
defined-monadic-operator, return a **token** of **class defined-monadic-operator-name** and **content f**.
defined-dyadic-operator, return a **token** of **class defined-dyadic-operator-name** and **content f**.
defined-function, return a **token** of **class defined-function-name** and **content f**.
niladic-defined-function, return a **token** of **class niladic-defined-function-name** and **content f**.
nil or **variable**, return a **token** of **class variable-name** and **content f**.
shared-variable, return a **token** of **class shared-variable-name** and **content f**.
label, return a **token** of **class constant** and **content** the **current-content** of *T*.

Note: *Other cases cannot occur.*
If *T* is a **distinguished-identifier**,
If both forms *Z* ←**f** and *Z* ←**f** ←*B* occur in the **form-table**, return a **token** of **class system-variable-name** and **content f**.
If the form *Z* ←**f** occurs in the **form-table**, but the form *Z* ←**f** ←*B* does not, return a **token** of **class niladic-system-function-name** and **content f**.
If either form *Z* ←**f** *B* or form *Z* ←*A* **f** *B* occurs in the **form-table**, return a **token** of **class system-function-name** with **content f**.
Otherwise, signal **syntax-error**.

6.1.5 Literal-Conversion

Literal-Conversion of T.

Informal Description: **Literal-conversion** converts T, a **token** of **class literal**, to a **token** of **class constant**. The **content** of such a **token** is converted from a **list** of **characters** to an **array**.

Evaluation Sequence:

If T is a **character-literal**, generate Z, a **character vector** such that the **ravel-list** of Z is the **content** of T with the initial and final quotes removed, and each pair of adjacent quotes in T replaced by a single quote.
If T is a **numeric-literal**, generate Z, a **numeric vector** such that the **ravel-list** of Z is a **list** of **numbers**, each of which is obtained by calling the **implementation-algorithm numeric-input-conversion** for the corresponding **numeric-scalar-literal** in the **numeric-literal**.
If the **length** of Z is **greater-than one**, return Z.
Otherwise, return **first-scalar** in Z.

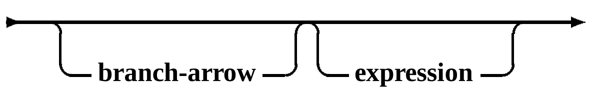
Note: *A quote character is represented in a* **character-literal** *by two adjacent quote characters. A single character between quotes is a scalar. All other cases are* **character** *vector literals. For example, the* **character** *literal* `''` *is the empty* **character** *vector literal and the character literal* `''''` *is the* **character** *scalar "quote".*

A numeric literal containing a single number is a scalar. A numeric literal containing two or more numbers is a vector.

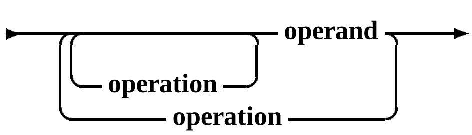
6.1.6 Statement-Analysis Token-Diagrams

Paths containing **ideograms** such as ⊛ in these **token-diagrams** match **tokens** whose **class** is **primitive** and whose **content** is the ideogram. Paths containing the word **token**, such as **shared-variable-name token** in **operand**, match **tokens** with the given **class**.

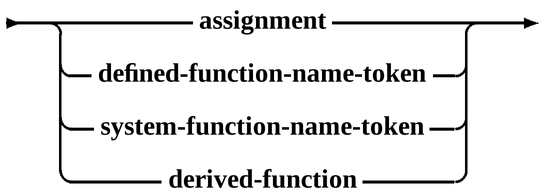
Statement



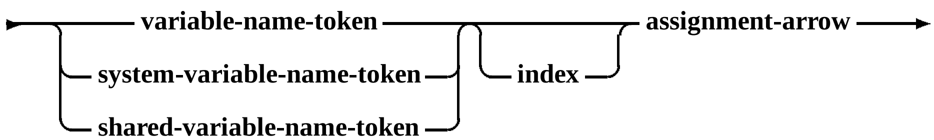
Expression



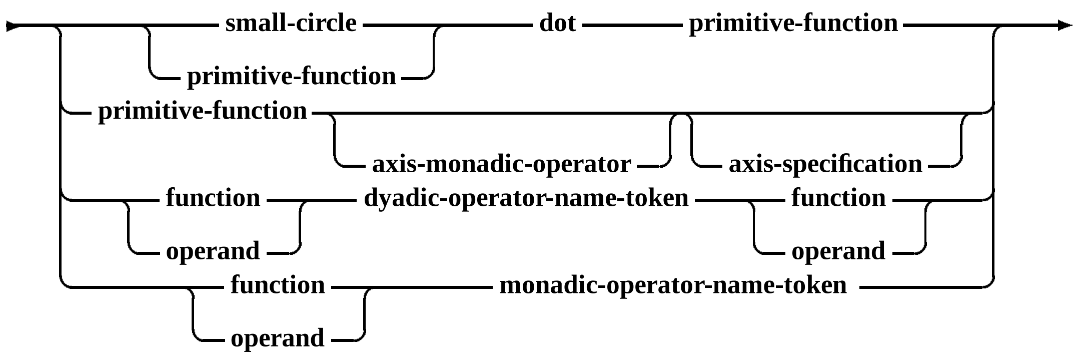
Operation



Assignment

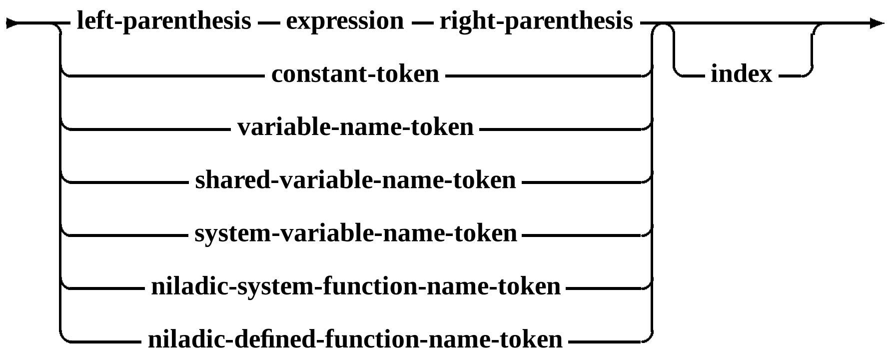


Derived-Function



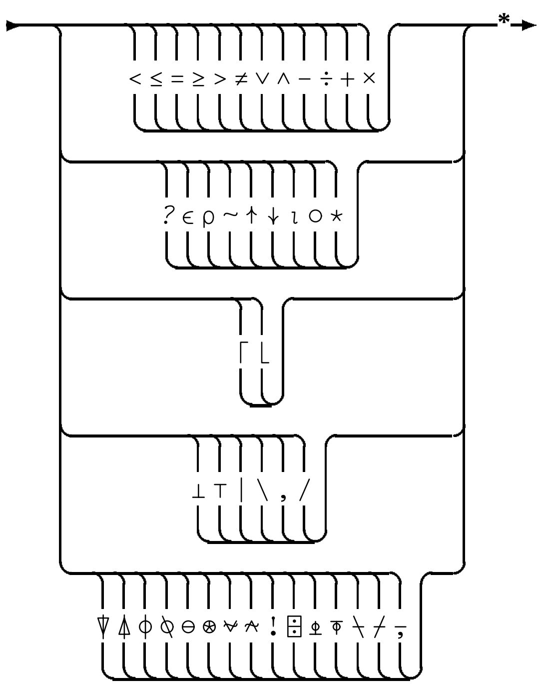
Axis-Specification

Operand

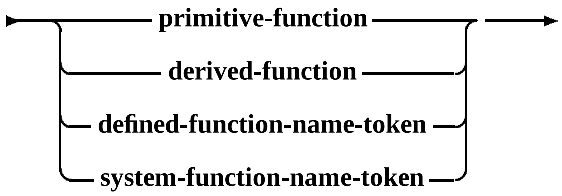


Index

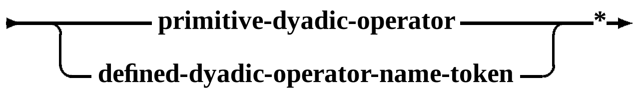
Primitive-Function



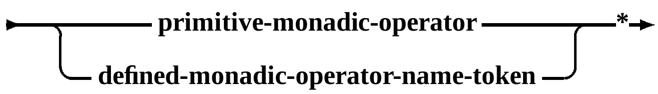
Function



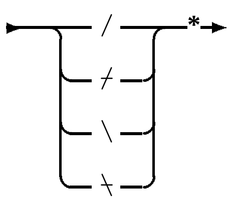
Dyadic-Operator



Monadic-Operator



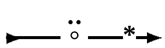
Axis-Monadic-Operator



Primitive-Monadic-Operator

Primitive-Dyadic-Operator

Diaeresis-Jot



Diaeresis-Tilde

►—— ⍨ ——*—►

Left-Parenthesis

►—— (——*—►

Right-Parenthesis

►——) ——*—►

Left-Axis-Bracket

►—— [——*—►

Right-Axis-Bracket

►——] ——*—►

Branch-Arrow

►—— → ——*—►

Assignment-Arrow

►—— ← ——*—►

Left-Index-Bracket

►—— [——*—►

Right-Index-Bracket

►——] ——*—►

Index-Separator

►—— ; ——*—►

Small-Circle

►—— ∘ ——*—►

Example

From the **list** of **characters** in **current-line**,

```
ABC←FN ⎕⌽[1+0] DEF[1;5 6]×3.45E4,⍴'ABC' ⍝COMMENT
```

evaluate-line generated a **list** of **tokens** and stored it in **current-statement**. Here, **evaluate-statement** has replaced that **list** with a new **list** of **tokens**.

Identification	Content	Class
*T*00		**left-end-of-statement**
*T*01	'*ABC*'	**variable-name**
*T*02		**assignment-arrow**
*T*03	'*FN*'	**defined-function-name**
*T*04	'⎕'	**system-variable-name**
*T*05	'⌽'	**primitive-function**
*T*06		**left-axis-bracket**
*T*07	1	**constant**
*T*08	'+'	**primitive-function**
*T*09	0	**constant**
*T*10		**right-axis-bracket**
*T*11	'*DEF*'	**variable-name**
*T*12		**left-index-bracket**
*T*13	1	**constant**
*T*14		**index-separator**
*T*15	5 6	**constant**
*T*16		**right-index-bracket**
*T*17	'×'	**primitive-function**
*T*18	34500	**constant**
*T*19	','	**primitive-function**
*T*20	'⍴'	**primitive-function**
*T*21	'*ABC*'	**constant**
*T*22		**right-end-of-statement**

The new **list**, shown above, begins with a **left-end-of-statement token** and ends with a **right-end-of-statement token**. Old **identifier tokens** have a new **class** and the same **content**; old **literal tokens** are now **constants** whose **content** is an appropriate **array**; old **primitive** tokens are now either **primitive-functions** whose **content** is the **character** identifying the primitive function, or are grouping signs such as **right-axis-bracket** with no specified **content**.

6.2 Reduce-Statement

Reduce-Statement

Informal Description: **Reduce-statement** converts **current-statement**, a **list** of **syntactic-units**, to a **result** by decomposing it into shorter lists of **syntactic-units** called **phrases**, then calling procedures termed **phrase-evaluators** to convert these phrases into **tokens**.

The letters Z, K, and J in this evaluation sequence refer to the graphic symbols found in the **resultant-prefix** column of **Table 3**.

Evaluation Sequence:

Set **current-stack** to the **empty-list** of **tokens**.
Repeat:
  Find the first entry in the **phrase-table** whose **pattern** matches a **prefix** of **current-stack**.
  If there is no matching entry,
    If **current-statement** is empty, signal **syntax-error**.
    Otherwise,
      Remove the last **token** from **current-statement**.
      Append it to **current-stack** as a new first **item**.
  If there is a matching entry,
    Let **p** stand for the **prefix** of the **current-stack** that matched the entry.
    Let **r** stand for the **resultant-prefix** of the entry.
    Let **s** stand for the **phrase-evaluator** of the entry.
    Call **s** and set **y** to the **token** it returns.
    If **s** is **process-end-of-statement**, return **y**.
    If **y** is an **exception**, return **y**.
    Otherwise, replace **p** with **r** in which **y** has been substituted for Z, K or J according to whether **y** is a **result**, a **complete-index-list** or a **partial-index-list**.
(End of repeated block)

The graphic symbols in the **Pattern** and **Resultant-Prefix** columns of **Table 3** designate lists of **syntactic-units**. Each graphic symbol matches **tokens** of the designated **classes** or **metaclasses**.

Note: *The graphic symbols employed are chosen to be suggestive of the* **list** *of* **characters** *that give rise to such phrases.*

The **build-index-list** *entries are called with B bound to a* **value** *and I bound to a* **partial-index-list***; they return either another* **partial-index-list** *(J) or a* **complete-index-list** *(K). Brackets are not passed as arguments or returned by* **build-index-list***; they are preserved by* **reduce-statement** *to make the* **patterns** *more obvious.*

Example

This example is continued from **evaluate-statement**.

The line being evaluated is

```
ABC ← FN ⎕ ϕ [ 1 + 0 ] DEF [ 1 ; 56 ] × 34500  , ρ 'ABC'
↑   ↑ ↑  ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  ↑ ↑ ↑ ↑  ↑ ↑ ↑      ↑ ↑ ↑
T   T T  T T T T T T T T  T T T T  T T T      T T T
0   0 0  0 0 0 0 0 0 1 1  1 1 1 1  1 1 1      1 2 2
1   2 3  4 5 6 7 8 9 0 1  2 3 4 5  6 7 8      9 0 1
```

*T*00 is **left-end-of-statement**, and *T*22 is **right-end-of-statement**.

The **tokens** *T*00 through *T*22 initially form the columns of **Figure 1**. The rows of the figure show the actions taken because of the pattern or lack of pattern in **current-stack**.

Consider step 5: the line fragment **,** ρ '*ABC*' matches the phrase *X F B* as follows: *X* matches **token** *T*19 because **,** is a **primitive-function**. *F* matches **token** *T*20 because ρ is a **primitive-function**. *B* matches **token** *T*21 because *ABC* is a **constant**. **Token** *T*22, **right-end-of-statement**, is not considered because the pattern *X F B* is concerned with only the first three **tokens** in **current-stack**.

The **phrase-evaluator** associated with *X F B* is **evaluate-monadic-function**. This **phrase-evaluator**, seeing that ρ is a primitive function, searches the **form-table** for *Z* ←ρ*B*, and calls the corresponding evaluation sequence, **shape**. Shape returns a **constant**, the one-element-vector containing **three**.

This becomes *Z* in the **resultant-prefix** column for *X F B*. **Current-stack** now holds three **tokens**: (**primitive-function**; **,**), (**constant**; **,** 3) and (**right-end-of-statement**;). Since no entry in the **phrase-table** has an entry whose **pattern** matches a **prefix** of **current-stack**, the **token** *T*18 (**constant**; 34500) is added to **current-stack**. A **prefix** of **current-stack** now matches a pattern (*A F B* matches (**constant**; 34500), (**primitive-function**; **,**), (**constant**; **,** 3)), so the corresponding **phrase-evaluator** (**evaluate-dyadic-function**) is called.

Pattern	Phrase Evaluator	Resultant-Prefix
(*B*)	**Remove-Parentheses**	*Z*
N	**Evaluate-Niladic-Function**	*Z*
X F B	**Evaluate-Monadic-Function**	*X Z*
X F [*C*] *B*	**Evaluate-Monadic-Function**	*X Z*
X F M B	**Evaluate-Monadic-Operator**	*X Z*
X F M [*C*] *B*	**Evaluate-Monadic-Operator**	*X Z*
A F M B	**Evaluate-Monadic-Operator**	*X Z*
A F M [*C*] *B*	**Evaluate-Monadic-Operator**	*X Z*
A F B	**Evaluate-Dyadic-Function**	*Z*
A F [*C*] *B*	**Evaluate-Dyadic-Function**	*Z*
X F D G B	**Evaluate-Dyadic-Operator**	*Z*
A F D G B	**Evaluate-Dyadic-Operator**	*Z*
A ∘*D G B*	**Evaluate-Dyadic-Operator**	*Z*
A [*K*]	**Evaluate-Indexed-Reference**	*Z*
V [*K*] ←*B*	**Evaluate-Indexed-Assignment**	*Z*
V ←*B*	**Evaluate-Assignment**	*Z*
V	**Evaluate-Variable**	*Z*
]	**Build-Index-List**	*J*]
; *I*	**Build-Index-List**	*J*
; *B I*	**Build-Index-List**	*J*
[*I*	**Build-Index-List**	[*K*
[*B I*	**Build-Index-List**	[*K*
L R	**Process-End-of-Statement**	–
L B R	**Process-End-of-Statement**	–
L →*B R*	**Process-End-of-Statement**	–
L →*R*	**Process-End-of-Statement**	–

Legend:

A , *B* , *Z* match **result**.

D matches **dyadic-operator**.

F , *G* match **defined-function-name**, **primitive-function**, or **system-function-name**.

I , *J* match **partial-index-list**.

C , *K* match **complete-index-list**.

L matches **left-end-of-statement**.

M matches **monadic-operator**.

N matches **niladic-defined-function-name** or **niladic-system-function-name**.

R matches **right-end-of-statement**.

V matches **variable-name**, **system-variable-name**, or **shared-variable-name**.

X matches **assignment-arrow**, **branch-arrow**, **defined-function-name**, **index-separator**, **left-axis-bracket**, **left-end-of-statement**, **left-index-bracket**, **left-parenthesis**, **primitive-function**, **system-function-name**, or **right-axis-bracket**.

(matches **left-parenthesis**.

) matches **right-parenthesis**.

[matches **left-axis-bracket** or **left-index-bracket**.

] matches **right-axis-bracket** or **right-index-bracket**.

∘ matches **small-circle**.

; matches **index-separator**.

← matches **assignment-arrow**.

→ matches **branch-arrow**.

Table 3: The Phrase Table.


```
 1 GET NEXT TOKEN T22                                        ()
 2 GET NEXT TOKEN T21                                       (R)
 3 GET NEXT TOKEN T20                                     (B R)
 4 GET NEXT TOKEN T19                                   (F B R)
 5 EVALUATE MONADIC FUNCTION                         (<X F B>R)
 6 GET NEXT TOKEN T18                                (<X Z>  R)
 7 EVALUATE DYADIC FUNCTION                        (<A F B>  R)
 8 GET NEXT TOKEN T17                                (Z      R)
 9 GET NEXT TOKEN T16                              ( F B     R)
10 BUILD INDEX LIST                             (<]> F B     R)
11 GET NEXT TOKEN T15                          (J ]  F B     R)
12 GET NEXT TOKEN T14                        (B I ]  F B     R)
13 BUILD INDEX LIST                       (<; B I>]  F B     R)
14 GET NEXT TOKEN T13                       (J    ]  F B     R)
15 GET NEXT TOKEN T12                    (B I    ]   F B     R)
16 BUILD INDEX LIST                   (<[ B I>    ]   F B     R)
17 GET NEXT TOKEN T11                   ([ K      ]   F B     R)
18 EVALUATE VARIABLE                (<V>[ K      ]   F B     R)
19 EVALUATE INDEXED REFERENCE       (<A [ K      ]>  F B     R)
20 EVALUATE DYADIC FUNCTION         (<A              F B>    R)
21 GET NEXT TOKEN T10                (Z                      R)
22 BUILD INDEX LIST               (<]>B                      R)
23 GET NEXT TOKEN T09            (I ] B                      R)
24 GET NEXT TOKEN T08          (B I ] B                      R)
25 GET NEXT TOKEN T07       (F B I ] B                       R)
26 EVALUATE DYADIC       (<A F B>I ] B    FUNCTION           R)
27 GET NEXT                (Z    I ] B    TOKEN T06          R)
28 BUILD INDEX          (<[ B    I>] B    LIST               R)
29 GET NEXT              ([ C      ] B    TOKEN T05          R)
30 GET NEXT            (F [ C      ] B    TOKEN T04          R)
31 EVALUATE         (<V>F [ C      ] B    VARIABLE           R)
32 EVALUATE         (<A F [ C      ] B>   DYADIC FUNCTION    R)
33 GET NEXT          (Z                   TOKEN T03          R)
34                  (F B      GET NEXT TOKEN T02             R)
35               (<X F B>    EVALUATE MONADIC FUNCTION       R)
36                (X Z       GET NEXT TOKEN T01              R)
37             (<V ← B>      EVALUATE ASSIGNMENT             R)
38              (Z           GET NEXT TOKEN T00              R)
39 *         (<L B           END OF STATEMENT                R>)
```

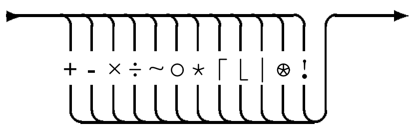
Figure 1: Statement Evaluation.

6.3 The Phrase Evaluators

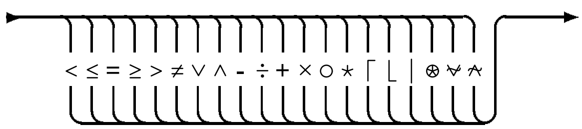
Informal Description: Each **phrase-evaluator** takes a **phrase** and reduces it to a single **token**. The **form-table** used by the **phrase-evaluators** is given as **Table 4**.

6.3.1 Diagrams

Primitive-Monadic-Scalar-Function



Primitive-Dyadic-Scalar-Function



6.3.2 Remove-Parentheses

Pattern (*B*)

Evaluation Sequence:

If *B* is **nil**, signal **value-error**.
If *B* is a **branch**, signal **value-error**.
Otherwise, return *B*.

Note: (⍎'') *and* (⍎'→3') *fail with a* **value-error**. (*A*←3) *and* (⍎'*A*←3') *do not display, since the* **token** *returned by* **remove-parentheses** *is a* **committed-value**, *not a* **constant**; *for the same reason,* (⎕←3) *displays the value* **three** *only once.* (→3) *fails with a* **syntax-error** *in* **evaluate-statement**.

(→) *also fails in* **evaluate-statement**, *but* (⍎'→') *succeeds;* **evaluate-statement** *threads* **line** *successfully, and* **remove-parentheses** *receives and returns an* **escape token**.

6.3.3 Evaluate-Niladic-Function

Pattern *N*

Evaluation Sequence:

Let **n** stand for the **content** of *N*.
If *N* is a **niladic-defined-function-name**,
  If the **current-class** of *N* is **niladic-defined-function**,
    Search the **form-table** for *Z ←DFN*.
    Call the corresponding evaluation sequence, passing **n** as the value of *DFN*.
    Return the **token** it returns.
  Otherwise, signal **syntax-error**.
If *N* is a **niladic-system-function-name**,
  Search the **form-table** for *Z ←***n**.
  If it is not found, signal **syntax-error**.
  Otherwise, call the corresponding evaluation sequence.
  Return the **token** it returns.

Note: *This phrase evaluator checks for a change in syntax class.*

6.3.4 Evaluate-Monadic-Function

Pattern *X F B*

Pattern *X F[C] B*

Evaluation Sequence:

If *B* is not a **value**, signal **value-error**.
Let **f** stand for the **content** of *F*.
For pattern *X F B*
  If *F* is a **defined-function-name**,
    If the **current-class** of *F* is **defined-function**,
      Search the **form-table** for *Z ←DFN B*.
      Call the corresponding evaluation sequence, passing **f** as the value of *DFN*.
      Return the **token** it returns.
    Otherwise, signal **syntax-error**.
  If *F* is a **primitive-function** or a **system-function-name**,
    If *F* matches **primitive-monadic-scalar-function** and *B* is not a **scalar**, perform **monadic-scalar-extension** as follows:
      Return a **numeric array** *Z* such that the **shape-list** of *Z* is the **shape-list** of *B* and for all *I* in the **index-set** of the **ravel-list** of *Z*, **item** *I* of the **ravel-list** of *Z* is **f** (**item** *I* of the **ravel-list** of *B*).
    Otherwise,
      Search the **form-table** for *Z ←***f** *B*.
      If it is not found, signal **valence-error**.
      Otherwise, call the corresponding evaluation sequence.
      Return the **token** it returns.
For pattern *X F[C] B*

If *F* is not a **primitive-function**, signal **syntax-error**.
If **index-origin** is **nil**, signal **implicit-error**.
Let *C*1 stand for the **first-item** in the **index-list** *C*.
If *C*1 is not a **scalar** or **one-element-vector**, signal **axis-error**.
If any **item** of the **ravel-list** of *C*1 is not a **number**, signal **axis-error**.
Set *K* to *C*1 **plus** (**one minus index-origin**).
Search the **form-table** for *Z* ←**f**[*K*] *B*.
If it is not found, signal **syntax-error**.
Otherwise, call the corresponding evaluation sequence and return the **token** it returns.

Additional Requirement:

The order in which the individual items of *Z* are produced during **monadic-scalar-extension** is not specified by this International Standard.

If any call of **f** signals an error during **monadic-scalar-extension**, that error is returned as the result of **evaluate-monadic-function**.

Note: *This phrase evaluator checks for a change in syntax class.*

6.3.5 Evaluate-Monadic-Operator

Pattern *X F M B*

Pattern *A F M B*

Pattern *X F M*[*C*] *B*

Pattern *A F M*[*C*] *B*

Evaluation Sequence:

If *B* is not a **value**, signal **value-error**.
Let **m** stand for the **content** of *M*.
Let **f** stand for the **content** of *F*.
For pattern *X F M B*
If *M* is a **primitive-monadic-operator**
Search the **form-table** for *Z*←**f m** *B*
If it is not found, signal **syntax-error**.
Otherwise, call the corresponding evaluation sequence, passing token *F* as the value of **f**.
Return the **token** it returns.
Otherwise,
Search the **form-table** for *Z*←**f** *DFN B*.
If it is not found, signal **syntax-error**.

Otherwise, call the corresponding evaluation sequence, passing **token** F as the value of **f** and the **token** DFN as the value of **m**.
Return the **token** it returns.
For pattern $A\ F\ M\ B$
If A is not a **value**, signal **value-error**.
If **m** is a **primitive-monadic-operator**
Search the **form-table** for $Z{\leftarrow}A$ **f** **m** B.
If it is not found, signal **syntax-error**.
Call the corresponding evaluation sequence, passing **token** F as the value of **f**.
Return the **token** it returns.
Otherwise,
Search the **form-table** for $Z{\leftarrow}A\ F\ DFN\ B$.
If it is not found, signal **syntax-error**.
Call the corresponding evaluation sequence, passing **token** F as the value of **f** and the **token** DFN as the value of **m**.
Return the **token** it returns.
For pattern $X\ F\ M[C]\ B$ or $A\ F\ M[C]\ B$
If M is not an **axis-operator**, signal **syntax-error**
If F is not a **primitive-function**, signal **syntax-error**.
If **index-origin** is **nil**, signal **implicit-error**.
Let $C1$ stand for the **first-item** in the **index-list** C.
If $C1$ is not a **scalar** or **one-element-vector**, signal **axis-error**.
If any **item** of the **ravel-list** of $C1$ is not a **number**, signal **axis-error**.
Set K to $C1$ **plus** (**one minus index-origin**).
For pattern $X\ F\ M[C]\ B$
Search the **form-table** for $Z\ \leftarrow$**f m**$[K]\ B$.
If it is not found, signal **syntax-error**.
Otherwise, call the corresponding evaluation sequence, passing **token** F as the value of **f**.
Return the **token** it returns.
For pattern $A\ F\ M[C]\ B$
If A is not a **value**, signal **value-error**
Search the **form-table** for $Z\ \leftarrow\ A$ **f m**$[K]\ B$.
If it is not found, signal **syntax-error**.
Otherwise, call the corresponding evaluation sequence, passing **token** F as the value of **f**.
Return the **token** it returns.

6.3.6 Evaluate-Dyadic-Function

Pattern $A\ F\ B$

Pattern $A\ F[C]\ B$

Evaluation Sequence:

If A is not a **value**, signal **value-error**.
If B is not a **value**, signal **value-error**.
Let **f** stand for the **content** of F.
For pattern `A F B`
  If F is a **defined-function-name**,
    If the **current-class** of F is **defined-function**,
      Search the **form-table** for `Z ←A DFN B`.
      Call the corresponding evaluation sequence, passing **f** as the value of DFN.
      Return the **token** it returns.
    Otherwise, signal **syntax-error**.
  If F is a **primitive-function** or a **system-function-name**,
    If F matches **primitive-dyadic-scalar-function** and A and B are not both **scalars**, perform **dyadic-scalar-extension** as follows:
      If the **rank** of A differs from the **rank** of B,
        If A is a **scalar** or **one-element-vector** and B is not a **scalar**, return `((ρB)ρA)` **f** `B`.
        If B is a **scalar** or **one-element-vector**, return A **f** `(ρA)ρB`.
        Otherwise, signal **rank-error**.
      If the **shape-list** of A differs from the **shape-list** of B, signal **length-error**.
      Return Z, an array such that the **shape-list** of Z is the same as the **shape-list** of A, the **type** of Z is **numeric**, and the **ravel-list** of Z is such that, for all I in the **index-set** of the **ravel-list** of Z, **item** I of the **ravel-list** of Z is (**item** I of the **ravel-list** of A) **f** (**item** I of the **ravel-list** of B).
    Otherwise, search the **form-table** for `Z ←A`**f**`B`.
    If it is not found, signal **valence-error**.
    Call the corresponding evaluation sequence and return the **token** it returns.
For pattern `A F[C] B`
  If F is not a **primitive-function**, signal **syntax-error**.
  If **index-origin** is **nil**, signal **implicit-error**.
  Let `C1` stand for the **first-item** in the **index-list** C.
  If `C1` is not a **scalar** or **one-element-vector**, signal **axis-error**.
  If any **item** of the **ravel-list** of `C1` is not a **number**, signal **axis-error**.
  Set K to `C1` **plus** (**one minus index-origin**).
  Search the **form-table** for `Z ←A` **f**`[K] B`.
  If it is not found, signal **syntax-error**.
  Otherwise, call the corresponding evaluation sequence, passing **token** F as the value of **f**.
  Return the **token** it returns.

Additional Requirement:

There is an intentional forward reference to **shape** and **reshape** in the description of **dyadic-scalar-extension**.

The order in which the individual items of Z are produced during **dyadic-scalar-extension** is not specified by this International Standard.

If any call of **f** signals an error during **dyadic-scalar-extension**, that error is returned as the result of **evaluate-dyadic-function**.

Note: Dyadic-scalar-extension *is intentionally stricter than it is in existing systems. For example,* `(1 1⍴1)+⍳3` *signals a* **rank-error** *and* `1 2 + ,1` *signals a* **length-error**.

This phrase evaluator checks for a change in syntax class.

An ambivalent function must be called either monadically or dyadically; hence, during syntax analysis and execution of a line, there is never a reference to an ambivalent function as any given instance is either monadic or dyadic.

6.3.7 Evaluate-Dyadic-Operator

Pattern *X F D G B*

Pattern *A F D G B*

Pattern *A* ∘ *D G B*

Evaluation Sequence:

If *B* is not a **value**, signal **value-error**.
Let **d** stand for the **content** of *D*.
Let **f** stand for the **content** of *F*.
Let **g** stand for the **content** of *G*.
For pattern *X F D G B*
  If *D* is a **primitive-dyadic-operator**
    Search the **form-table** for *Z* ← **f d g** *B*.
    If it is not found, signal **syntax-error**.
    If it is found, call the corresponding evaluation sequence, passing **token f** as the value of *F* and **token g** as the value of *G*.
    Return the **token** it returns.
  Otherwise,
    Search the **form-table** for *Z* ← **f** *DFN* **g** *B*.
    If it is not found, signal **syntax-error**.
    If it is found, call the corresponding evaluation sequence, passing **token f** as the value of *F*, **token g** as the value of *G*, and **token** *DFN* as the value of **d**.
    Return the **token** it returns.
For pattern *A F D G B*
  If *A* is not a **value**, signal **value-error**.
  If **d** is a **primitive-dyadic-operator**,
    Search the **form-table** for *Z* ← *A* **f d g** *B*.
    If it is not found, signal **syntax-error**.
    If it is found, call the corresponding evaluation sequence, passing **token f** as the value of *F* and **token g** as the value of *G*.
    Return the **token** it returns.
  Otherwise,

Search the **form-table** for $Z \leftarrow A$ **f** DFN **g** B.
If it is not found, signal **syntax-error**.
If it is found, call the corresponding evaluation sequence, passing **token f** as the value of F and **token g** as the value of G, and **token** DFN as the value of **d**.
Return the **token** it returns.

For pattern $A \circ D\ G\ B$
If D is not **dot**, signal **syntax-error**.
Otherwise,
Search the **form-table** for $Z \leftarrow A \circ$ **d g** B.
If it is not found, signal **syntax-error**.
If it is found, call the corresponding evaluation sequence, passing **token g** as the value of G.
Return the **token** it returns.

6.3.8 Evaluate-Indexed-Reference

Pattern $A[K]$

Evaluation Sequence:

If A is not a **value**, signal **value-error**.
If **index-origin** is **nil**, signal **implicit-error**.
If the **number-of-items** in the **index-list** K differs from the **rank** of A, signal **rank-error**.
If the **rank** of A is **greater-than one**,
Search the **form-table** for $Z \leftarrow A[I]$.
Call the corresponding evaluation sequence, passing K as the value of I.
Return the **token** it returns.
Otherwise,
If **first-item** in K is an **elided-index-marker**, return A.
Otherwise,
Set X to **first-item** in the **index-list** K.
If any **item** of the **ravel-list** of X is not a **near-integer**, signal **domain-error**.
Generate $X1$, a **numeric array** with the **shape-list** of X such that each **item** of the **ravel-list** of $X1$ is (**one minus index-origin**) **plus** the **integer-nearest-to** X.
If any **item** of the **ravel-list** of $X1$ is not in the **index-set** of A, signal **index-error**.
Return Z, an array with the **shape-list** of $X1$, such that for each **integer** I in the **index-set** of Z, **item** I of the **ravel-list** of Z is **item** J of the **ravel-list** of A, where J is **item** I of the **ravel-list** of $X1$. The **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of A.

Note: *Since an* **index-list** *will never have zero* **items**, *indexing will always signal a* **rank-error** *when argument* A *is a* **scalar**.

6.3.9 Evaluate-Assignment

Pattern *V* ← *B*

Evaluation Sequence:

If *B* is not a **value**, signal **value-error**.
If *V* is a **shared-variable-name**,
  If the **current-class** of *V* is **shared-variable**,
    Search the **form-table** for *Z* ←*SHV* ←*B*.
    Call the corresponding evaluation sequence, passing **token** *V* as the value of *SHV*.
    Return the **token** it returns.
  Otherwise, signal **syntax-error**.
If *V* is a **system-variable-name**,
  Search the **form-table** for *Z* ← **q** ←*B*, where **q** is the **content** of *V*.
  If it is not found, signal **syntax-error**.
  Otherwise,
    Call the corresponding evaluation sequence
    Return the **token** it returns.
If *V* is a **variable-name**,
  If the **current-class** of *V* is **nil** or **variable**,
    Set the **current-referent** of *V* to a **token** whose **class** is **variable** and whose **content** is the **content** of *B*.
    Return a **token** whose **class** is **committed-value** and whose **content** is *B*.
  Otherwise, signal **syntax-error**.

Note: *The phrase* ABC←⍎'→3' *yields* **value-error**. *The phrase* V←⎕SVR 'V' *where* V *was a* **shared-variable** *yields* **syntax-error**.

6.3.10 Evaluate-Indexed-Assignment

Pattern *V*[*K*] ←*B*

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If *B* is not a **value**, signal **value-error**.
If *V* is a **shared-variable-name**,
  If the **current-class** of *V* is **shared-variable**,
    Search the **form-table** for *Z* ←*SHV*[*I*] ←*B*.
    Call the corresponding evaluation sequence, passing **token** *V* as the value of *SHV*, and *K* as the value of *I*.
    Return the **token** it returns.
  Otherwise, signal **syntax-error**.
If *V* is a **system-variable-name**,
  Search the **form-table** for *Z* ← **q**[*I*] ←*B*, where **q** is the **content** of *V*.

If it is not found, signal **syntax-error**.
Otherwise,
Call the corresponding evaluation sequence, passing K as the value of I.
Return the **token** it returns.
If V is a **variable-name**,
If the **current-class** of V is **nil**, signal **value-error**.
If the **current-class** of V is a **variable**,
Search the **form-table** for $Z \leftarrow V[I] \leftarrow B$.
Call the corresponding evaluation sequence, passing K as the value of I.
Return the **token** it returns.
Otherwise, signal **syntax-error**.

6.3.11 Evaluate-Variable

Pattern V

Evaluation Sequence:

If V is a **shared-variable-name**,
If the **current-class** of V is **shared-variable**,
Search the **form-table** for $Z \leftarrow SHV$.
Call the corresponding evaluation sequence, passing **token** V as the value of SHV.
Return the **token** it returns.
Otherwise, signal **syntax-error**.
If V is a **system-variable-name**,
If the **current-class** of V is **nil**, signal **value-error**.
Search the **form-table** for $Z \leftarrow$**q**, where **q** is the **content** of V.
Call the corresponding evaluation sequence.
Return the **token** it returns.
If V is a **variable-name**,
If the **current-class** of V is **nil**, signal **value-error**.
If the **current-class** of V is **variable**, return the **current-content** of V.
Otherwise, signal **syntax-error**.

6.3.12 Build-Index-List

Pattern]

Pattern **;** I

Pattern **;** B I

Pattern [I

Pattern [B I

Evaluation Sequence:

For pattern `]`
Return *J*, a **partial-index-list** with **content** the **index-list** of length **zero**.
For pattern **;** *I*
Return *J*, a **partial-index-list** with **content** *Z*, an **index-list** such that the **first-item** in *Z* is an **elided-index-marker** and the **rest-of** *Z* is *I*.
For pattern **;** *B I*
Return *J*, a **partial-index-list** with **content** *Z*, an **index-list** such that the **first-item** in *Z* is *B* and the **rest-of** *Z* is *I*.
For pattern `[`*I*
Return *J*, a **complete-index-list** with **content** *Z*, an **index-list** such that the **first-item** in *Z* is an **elided-index-marker** and the **rest-of** *Z* is *I*.
For pattern `[`*B I*
Return *J*, a **complete-index-list** with **content** *Z*, an **index-list** such that the **first-item** in *Z* is *B* and the **rest-of** *Z* is *I*.

6.3.13 Process-End-of-Statement

Pattern *L R*

Pattern *L A R*

Pattern *L* →*R*

Pattern *L* →*A R*

Evaluation Sequence:

For pattern *L R*
Return a **token** whose **class** is **nil**.
For pattern *L A R*
Return *A*.
For pattern *L* →*R* Process-End-of-Statement
Return a **token** whose **class** is **escape**.
Process-End-of-Statement
For pattern *L* →*A R*
If the **rank** of *A* is **greater-than one**, signal **rank-error**.
If *A* is **empty**, return a **token** whose **class** is **nil**.
Otherwise, set *A1* to the **first-scalar** in *A*.
If *A1* is not a **near-integer**, signal **domain-error**.
Return a **token** whose **class** is **branch** and whose **content** is the **numeric-scalar** with value the **integer-nearest-to** *A1*.

6.4 The Form Table

The **form-table** is the **list** of all **lists** of **syntactic-units** for which evaluation sequences exist.

The following matching rules apply in the **form-table**.

A , *B* , *Z* match **constant**.
I , *K* match **complete-index-list**.
f, g match **primitive-function** or **defined-function**, but not **defined-operator**.
A given ideogram, such as ⊛, matches a **primitive-function token** that contains it.
A given **distinguished-identifier**, such as ⎕IO, matches any **system-variable-name token** or **system-function-name token** that contains it.

The behaviour of operations in the **form-table** that do not create new **contexts** is **atomic**.

This behaviour is observable only for those operations that have **side-effects**. For example, if any of the elements of an argument array is not in the domain of **roll**, the value of the system parameter **random-link** following execution will be as it was when **roll** was called.

Form	Operation Name	Page
Z ← + B	**Conjugate**	76
Z ← − B	**Negative**	76
Z ← × B	**Direction**	77
Z ← ÷ B	**Reciprocal**	77
Z ← ⌊ B	**Floor**	78
Z ← ⌈ B	**Ceiling**	78
Z ← ⋆ B	**Exponential**	79
Z ← ⍟ B	**Natural Logarithm**	79
Z ← ∣ B	**Magnitude**	80
Z ← ! B	**Factorial**	81
Z ← ○ B	**Pi times**	82
Z ← ∼ B	**Not**	83
Z ← A + B	**Plus**	84
Z ← A − B	**Minus**	84
Z ← A × B	**Times**	85
Z ← A ÷ B	**Divide**	85
Z ← A ⌈ B	**Maximum**	86
Z ← A ⌊ B	**Minimum**	86
Z ← A ⋆ B	**Power**	87
Z ← A ⍟ B	**Logarithm**	88
Z ← A ∣ B	**Residue**	89
Z ← A ! B	**Binomial**	90
Z ← A ○ B	**Circular Functions**	91
Z ← A ∧ B	**And/LCM**	93
Z ← A ∨ B	**Or/GCD**	94
Z ← A ⍲ B	**Nand**	94
Z ← A ⍱ B	**Nor**	95
Z ← A = B	**Equal**	96
Z ← A < B	**Less than**	97
Z ← A ≤ B	**Less than or equal to**	98
Z ← A ≠ B	**Not equal**	99
Z ← A ≥ B	**Greater than or equal to**	100
Z ← A > B	**Greater than**	101
Z ← , B	**Ravel**	102
Z ← ⍴ B	**Shape**	103
Z ← ⍳ B	**Index Generator**	104
Z ← ⍪ B	**Table**	105
Z ← A ⍴ B	**Reshape**	107
Z ← A , B	**Join**	109
Z ← A ⍪ B	**Join**	109

Table 4: The Form Table

Form	Operation Name	Page
Z ← **f**/ *B*	**Reduction**	110
Z ← **f**/[*K*] *B*	**Reduction**	110
Z ← **f**⌿ *B*	**Reduction**	110
Z ← **f**⌿[*K*] *B*	**Reduction**	110
Z ← **f**\ *B*	**Scan**	113
Z ← **f**\[*K*] *B*	**Scan**	113
Z ← **f**⍀ *B*	**Scan**	113
Z ← **f**⍀[*K*] *B*	**Scan**	113
Z ← *N* **f** / *B*	**N-wise Reduction**	115
Z ← *N* **f** /[*K*] *B*	**N-wise Reduction**	115
Z ← *N* **f** ⌿ *B*	**N-wise Reduction**	115
Z ← *N* **f** ⌿[*K*] *B*	**N-wise Reduction**	115
Z ← **f** ⍨ *B*	**Duplicate**	118
Z ← *A* **f** ⍨ *B*	**Commute**	118
Z ← *A* ∘ . **f** *B*	**Outer Product**	120
Z ← *A* **f** . **g** *B*	**Inner Product**	121
Z ← **f** ⍤ **y** *B*	**Rank**	124
Z ← *A* **f** ⍤ **y** *B*	**Rank**	125
Z ← ? *B*	**Roll**	127
Z ← ⍋ *B*	**Grade Up**	129
Z ← ⍒ *B*	**Grade Down**	131
Z ← ⌽ *B*	**Reverse**	132
Z ← ⊖ *B*	**Reverse**	132
Z ← ⌽[*K*] *B*	**Reverse**	132
Z ← ⊖[*K*] *B*	**Reverse**	132
Z ← ⍉ *B*	**Monadic Transpose**	133
Z ← ⌹ *B*	**Matrix Inverse**	134
Z ← ⍎ *B*	**Execute**	135
Z ← ∪ *B*	**Unique**	136
Z ← *A* ,[*K*] *B*	**Join Along an Axis**	137
Z ← *A* ⍳ *B*	**Index of**	140
Z ← *A* ∊ *B*	**Member of**	141
Z ← *A* ? *B*	**Deal**	142
Z ← *A* / *B*	**Replicate**	143
Z ← *A* ⌿ *B*	**Replicate**	143
Z ← *A* /[*K*] *B*	**Replicate**	143
Z ← *A* ⌿[*K*] *B*	**Replicate**	143
Z ← *A* \ *B*	**Expand**	145
Z ← *A* ⍀ *B*	**Expand**	145
Z ← *A* \[*K*] *B*	**Expand**	145
Z ← *A* ⍀[*K*] *B*	**Expand**	145

Table 4: (Continued)

Form	Operation Name	Page
Z ← A ⌽ B	**Rotate**	147
Z ← A ⊖ B	**Rotate**	147
Z ← A ⌽[K] B	**Rotate**	147
Z ← A ⊖[K] B	**Rotate**	147
Z ← A ⊥ B	**Base Value**	149
Z ← A ⊤ B	**Representation**	151
Z ← A ⍉ B	**Dyadic Transpose**	153
Z ← A ↑ B	**Take**	155
Z ← A ↓ B	**Drop**	156
Z ← A ⌹ B	**Matrix Divide**	157
Z ← A[I]	**Indexed Reference**	158
Z ← V[I] ← B	**Indexed Assignment**	159
Z ← A ~ B	**Without**	161
Z ← A ⊣ B	**Left**	161
Z ← A ⊢ B	**Right**	162
Z ← A ⍒ B	**Character Grade Down**	163
Z ← A ⍋ B	**Character Grade Up**	164
Z ← ⎕TS	**Time Stamp**	171
Z ← ⎕AV	**Atomic Vector**	172
Z ← ⎕LC	**Line Counter**	172
Z ← ⎕EM	**Event Message**	173
Z ← ⎕ET	**Event Type**	174
Z ← ⎕DL B	**Delay**	174
Z ← ⎕NC B	**Name Class**	175
Z ← ⎕EX B	**Expunge**	176
Z ← ⎕NL B	**Name List**	177
Z ← ⎕STOP B	**Query Stop**	178
Z ← ⎕TRACE B	**Query Trace**	179
⎕ES B	**Monadic Event Simulation**	180
Z ← A ⎕NL B	**Name List**	180
Z ← A ⎕STOP B	**Set Stop**	181
Z ← A ⎕TRACE B	**Set Trace**	182
Z ← A ⎕EA B	**Execute Alternate**	183
A ⎕ES B	**Dyadic Event Simulation**	184
Z ← A ⎕TF B	**Transfer Form**	185
Z ← ⎕CT ← B	**Comparison Tolerance**	187
Z ← ⎕CT	**Comparison Tolerance**	187
Z ← ⎕RL ← B	**Random Link**	188
Z ← ⎕RL	**Random Link**	188
Z ← ⎕PP ← B	**Print Precision**	189
Z ← ⎕PP	**Print Precision**	189

Table 4: (Continued)

Form	Operation Name	Page
Z ← ⎕IO ← B	**Index Origin**	190
Z ← ⎕IO	**Index Origin**	190
Z ← ⎕LX ← B	**Latent Expression**	191
Z ← ⎕LX	**Latent Expression**	191
Z ← ⎕LX[I] ← B	**Latent Expression**	191
Z ← DFN	**Call-Defined-Function**	200
Z ← DFN B	**Call-Defined-Function**	200
Z ← A DFN B	**Call-Defined-Function**	200
Z ← **f** DFN B	**Call-Defined-Function**	200
Z ← A **f** DFN B	**Call-Defined-Function**	200
Z ← **f** DFN **g** B	**Call-Defined-Function**	200
Z ← A **f** DFN **g** B	**Call-Defined-Function**	200
Z ← ⎕FX B	**Function Fix**	203
Z ← ⎕CR B	**Character Representation**	204
Z ← SHV	**Shared Variable Reference**	217
Z ← SHV ← B	**Shared Variable Assignment**	218
Z ← SHV[I] ← B	**Shared Variable Indexed Assignment**	219
Z ← ⎕SVC B	**Shared Variable Access Control Inquiry**	219
Z ← ⎕SVQ B	**Shared Variable Query**	221
Z ← ⎕SVO B	**Shared Variable Degree of Coupling**	222
Z ← A ⎕SVO B	**Shared Variable Offer**	223
Z ← ⎕SVR B	**Shared Variable Retraction**	224
Z ← A ⎕SVC B	**Shared Variable Access Control Set**	225
Z ← ⎕SVS B	**Shared Variable State Inquiry**	226
⎕SVE ← B	**Shared Variable Event**	227
Z ← ⎕SVE	**Shared Variable Event**	227
Z ← ⍕ B	**Monadic Format**	233
Z ← A ⍕ B	**Dyadic Format**	237
Z ← ⎕	**Quad Input**	244
Z ← ⍞	**Quote Quad Input**	245
Z ← ⎕ ← B	**Quad Output**	245
Z ← ⍞ ← B	**Quote Quad Output**	246

Table 4: (Concluded)

7 Scalar Functions

Note: *The primitive functions described in this chapter are called* **scalar-functions***. All* **scalar-functions** *have uniform behaviour with respect to the structure of their argument arrays. The shape of the result of a* **scalar-function** *is determined solely by the shapes of its arguments.*

This section defines **scalar-functions** *individually for scalar arguments. Their common behaviour is described in this informal description by the expository device of an implicit operator, called the* **scalar-extension-operator**.

If the argument to a monadic scalar function is not a scalar, a monadic scalar extension operator can be thought of as being invoked to produce a derived function which, in turn, applies the monadic scalar function to every element of the argument array, producing a result array of the same shape as the argument. The order in which the elements of the argument array are presented to the monadic scalar function is not specified by this International Standard. Monadic scalar functions never signal **rank-error** *or* **length-error**.

If either of the arguments of a dyadic scalar function is not a scalar, a dyadic scalar extension operator can be thought of as being invoked to produce a derived function, which provides pairs of scalars to the scalar function as follows:

The dyadic scalar extension operator first tests whether the two argument arrays have the same shape. Arguments to a dyadic scalar function must have the same shape. If they do not, and the argument of lesser rank is a scalar or one-element vector, the argument of lesser rank is reshaped to the shape of the argument of greater rank.

If the arguments cannot be made to have the same shape, the dyadic scalar extension operator signals a **rank-error** *if the arguments are of different ranks and a* **length-error** *otherwise.*

When the dyadic scalar extension operator succeeds, it produces a derived function which generates a scalar for each position in its result **array** *by applying the subject* **scalar-function** *to pairs of scalars selected from corresponding positions in the argument arrays. The order in which the elements of the result* **array** *are produced is not specified by this International Standard.*

Because the derived function produced by either scalar extension operator never calls its **scalar-function** *argument for empty arrays,* **domain-error** *can never be signalled for empty array arguments or for arrays reshaped by scalar extension to empty.*

The type of all empty results produced by the functions derived from monadic and dyadic scalar extension is a property of the function to scalar extension. Since all scalar functions specified in this International Standard produce **numeric** *results, the type of all empty results produced by scalar extension is specified as* **numeric***.*

For example, `''∧5` *and* `−''` *return empty* **numeric** *results rather than signalling an error.*

7.1 Monadic Scalar Functions

Note: *The definitions in this section cover only scalar arguments. The* **phrase-evaluator evaluate-monadic-function** *handles non-scalar cases. All* **scalar-functions** *yield scalar results when applied to scalar arguments.*

Note that, in this International Standard, **roll** *is not a* **scalar-function***.*

7.1.1 Conjugate

```
Z ← + B
```

Informal Description: Z is the conjugate of B. Geometrically, it is the reflection of the number about the real axis.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return B with its **imaginary-part** negated.

Example:

```
      + 3 ¯4 0 0.5 3J4 ¯3J4 3J¯4
3 ¯4 0 0.5 3J¯4 ¯3J¯4 3J4
```

7.1.2 Negative

```
Z ← - B
```

Informal Description: Z is the **negation** of B.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return **zero minus** B.

Example:

```
      - 7 0 ¯7
¯7 0 7
```

7.1.3 Direction

```
Z ← × B
```

Informal Description: *Z* is **zero** if *B* is **zero**, and otherwise is the **number** with **magnitude one** that has the same **direction** as *B*. Geometrically, it is the number determined by the radial projection of *B* onto the **unit-circle**. For real numbers, this is identical with the **sign** or **signum** function, and takes on only the values **one**, **zero**, and **negative one**.

Evaluation Sequence:

If *B* is not a **number**, signal **domain-error**.
If *B* is **zero**, return **zero**; otherwise, return *B* **divided-by** the **magnitude** of *B*.

Examples:

```
      ×1 ¯.5 .33 0 ¯1E¯20
1 ¯1 1 0 ¯1
      ×3J4 3J¯4 ¯3J4 ¯3J¯4
0.6J0.8 0.6J¯0.8 ¯0.6J0.8 ¯0.6J¯0.8
```

7.1.4 Reciprocal

```
Z ← ÷ B
```

Informal Description: *Z* is `1÷B`.

Evaluation Sequence:

If *B* is not a **number**, signal **domain-error**.
If *B* is **zero**, signal **domain-error**.
Return **one divided-by** *B*.

Examples:

```
      ÷¯.25 .5 1 2 ¯4
¯4 2 1 0.5 ¯0.25

      ÷0
```
domain-error

7.1.5 Floor

```
Z ← ⌊ B
```

Informal Description: Z is the **complex-integer** U associated with the **unit-square** containing B, unless the sum of the **fractional-parts** of the **real-part** and **imaginary-part** of B is **greater-than-or-equal-to one**, in which case it is the nearer of U **plus one** or U **plus imaginary-one**, In case of a tie, it is U **plus one**. For **real-numbers**, Z is the greatest **integer** tolerantly less than or equal to B. Uses **comparison-tolerance**

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If B is not a **number**, signal **domain-error**.
Return the **tolerant-floor** of B **within comparison-tolerance**.

Examples:

– In the following, **comparison-tolerance** is `1E¯10`.

```
      ⌊¯3.1416 3.1416 .99999999999 5E20 ¯0.5E¯10
¯4 3 1 5E20 0
      ⌊0.3J0.6 0.6J0.8 0.8J0.6 0.6J0.3 0.8J0.2 0.5J0.5
0 0J1 1 0 1 1
```

Note: *The following article describes the design of the floor function for complex arguments:*

McDonnell, E. E., “Complex Floor”, **APL Congress 73**, *North Holland Publishing Co., 1973*

7.1.6 Ceiling

```
Z ← ⌈ B
```

Informal Description: Z is the **negation** of the **floor** of the **negation** of B. For B a **real-number**, Z is the least **integer** tolerantly greater than or equal to B. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If B is not a **number**, signal **domain-error**.
Return `-⌊-B`.

Example:

– In the following, **comparison-tolerance** is `1E¯10`.

```
      ⌈¯3.1416 3.1416 5.00000000001
¯3 4 5
```


7.1.7 Exponential

```
Z ← ⋆ B
```

Informal Description: Z is e raised to the power B, where e is the base of the natural logarithms.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return the **exponential** of B.

Examples:

```
      ⋆ ¯1E50 ¯2 ¯1 0 1 2
0 0.135335 0.367879 1 2.71828 7.38906
      ⋆ .693147
2
```

7.1.8 Natural Logarithm

```
Z ← ⍟ B
```

Informal Description: Z is the natural logarithm of B.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
If B is **zero**, signal **domain-error**.
Return the **natural-logarithm** of B.

Examples:

```
      ⍟1828459045 2 1E¯50 1E50
1 0.693147 ¯115.129 115.129

      ⍟⋆1
1
```


7.1.9 Magnitude

```
Z ← | B
```

Informal Description: Z is the magnitude of B.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return the **magnitude** of B.

Example:

```
      |1 ¯0.5 0.33 ¯0.25 0 1E¯20
1 0.5 0.33 0.25 0 1E¯20
```

7.1.10 Factorial

```
Z ← ! B
```

Informal Description: Z is the **gamma-function** of B+1. If B is a **nonnegative-integer**, this is factorial B.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
If B is a **negative-integer**, signal **domain-error**.
Set $B1$ to B **plus one**.
Return **gamma-function** of $B1$.

Examples:

```
     ! 0 1 2 3 4 5 6 7 8 9
1 1 2 6 24 120 720 5040 40320 362880

     !¯.5
1.77245
     5 1 ρ! - 1.502 1.503 1.504 1.505 1.506
¯3.54471
¯3.54466
¯3.54464
¯3.54466
¯3.5447
```

Note: *The* **gamma-function** *is defined in, for example, the* **National Bureau of Standards Handbook of Mathematical Functions**, *U.S. Government Printing Office, Washington D.C., 1964.*

See also Hart, J. F., **Computer Approximations**, *Robert C. Krieger Publishing Company, Huntington, NY, 1978.*

7.1.11 Pi times

```
Z ← ○ B
```

Informal Description: Z is π times B.

Evaluation Sequence:

If B is not a **number**, signal **domain-error**.
Return **pi-times** B.

Example:

```
      ○1 10 100
3.14159 31.4159 314.159
```

7.1.12 Not

```
Z ← ~ B
```

Informal Description: Z is the Boolean complement of B.

Evaluation Sequence:

If B is not **near-Boolean**, signal **domain-error**.
If the **integer-nearest-to** B is **one**, return **zero**.
Otherwise, return **one**.

Example:

– For the following, the **implementation-parameter integer-tolerance** is `1E¯10`.

```
      ~0 1 1E¯11 .999999999999
1 0 1 0
```

7.2 Dyadic Scalar Functions

Note: *The definitions in this section cover only scalar arguments. The* **phrase-evaluator evaluate-dyadic-function** *handles non-scalar cases. All* **scalar-functions** *yield scalar results when applied to scalar arguments.*

The outer product operator, which has not yet been formally introduced at this point in the document, is used in the examples in this section as a convenient way of generating tables. The use of outer product in this section is limited to vector arguments. The same results could be obtained from each example, although not so compactly, by supplying the elements of the left argument one at a time, starting from the leftmost, as left arguments to the scalar function.

For example,

Example:

```
      0 1∘.=0 1 2
1 0 0
0 1 0
```

is equivalent to

Example:

```
      0 = 0 1 2
1 0 0
      1 = 0 1 2
0 1 0
```


7.2.1 Plus

```
Z ← A + B
```

Informal Description: *Z* is *A* plus *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
Return *A* **plus** *B*.

Example:

```
      ¯2 ¯1 0 1 ∘.+ ¯2 ¯1 0 1
¯4  ¯3  ¯2  ¯1
¯3  ¯2  ¯1   0
¯2  ¯1   0   1
¯1   0   1   2
```

7.2.2 Minus

```
Z ← A - B
```

Informal Description: *Z* is *A* minus *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
Return *A* **minus** *B*.

Example:

```
      ¯2 ¯1 0 1 ∘.- ¯2 ¯1 0 1
 0  ¯1  ¯2  ¯3
 1   0  ¯1  ¯2
 2   1   0  ¯1
 3   2   1   0
```

7.2.3 Times

```
Z ← A × B
```

Informal Description: *Z* is *A* times *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
Return *A* **times** *B*.

Example:

```
      ¯2 ¯1 0 1 ∘.×¯2 ¯1 0 1
 4   2   0  ¯2
 2   1   0  ¯1
 0   0   0   0
¯2  ¯1   0   1
```

7.2.4 Divide

```
Z ← A ÷ B
```

Informal Description: *Z* is *A* divided by *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
If *B* is **zero** and *A* is not **zero**, signal **domain-error**.
If *B* is **zero** and *A* is **zero**, return **one**.
Otherwise, return *A* **divided-by** *B*.

Example:

```
      0 1 2 3 4 ∘.÷1 2 3 4
0  0    0         0
1  0.5  0.333333  0.25
2  1    0.666666  0.5
3  1.5  1         0.75
4  2    1.33333   1
```

7.2.5 Maximum

```
Z ← A ⌈ B
```

Informal Description: *Z* is the larger of *A* and *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **near-real number**, signal **domain-error**.
Set *A*1 to the **real-number** nearest to *A*.
Set *B*1 to the **real-number** nearest to *B*.
If *A*1 is **greater-than** *B*1, return *A*1.
Otherwise, return *B*1.

Example:

```
      ¯2 ¯1 0 1 ∘.⌈¯2 ¯1 0 1
¯2  ¯1   0   1
¯1  ¯1   0   1
 0   0   0   1
 1   1   1   1
```

7.2.6 Minimum

```
Z ← A ⌊ B
```

Informal Description: *Z* is the smaller of *A* and *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **near-real number**, signal **domain-error**.
Set *A*1 to the **real-number** nearest to *A*.
Set *B*1 to the **real-number** nearest to *B*.
If *A*1 is **greater-than** *B*1, return *B*1.
Otherwise, return *A*1.

Example:

```
      ¯2 ¯1 0 1 ∘.⌊¯2 ¯1 0 1
¯2  ¯2  ¯2  ¯2
¯2  ¯1  ¯1  ¯1
¯2  ¯1   0   0
¯2  ¯1   0   1
```

7.2.7 Power

$Z \leftarrow A \star B$

Informal Description: Z is A raised to the Bth power.

Evaluation Sequence:

If either of A or B is not a **number**, signal **domain-error**.
If A is **zero** and B is **zero**, return **one**.
If A is **zero** and the **real-part** of B is a **positive-number**, return **zero**, otherwise signal **domain-error**.
Return A **to-the-power** B.

Examples:

– In the following, **print-precision** is `12`.

```
      2⋆32
4294967296
      4⋆0.5
2
      ¯8⋆÷3
1J1.73205080757
```

Additional Requirement:

The foregoing example shows that, when the optional complex arithmetic facility is implemented, the **implementation-algorithm** should yield the principal value of the nth (odd n) root of a negative number, not the real negative root. If it is not implemented, a domain error should be signalled.

7.2.8 Logarithm

```
Z ← A ⍟ B
```

Informal Description: Z is the logarithm of B to the base A.

Evaluation Sequence:

If either of A or B are not **numbers** signal **domain-error**.
If A and B are **equal**, return **one**.
If A is **one**, signal **domain-error**.
Set $A1$ to the **natural-logarithm** of A.
Set $B1$ to the **natural-logarithm** of B.
Return $B1$ **divided-by** $A1$.

Example:

```
      10 2 10 0.1 ⍟2 65536 1E15 1E15
0.30103 16 15 ¯15
```

7.2.9 Residue

```
Z ← A | B
```

Informal Description: *Z* is *B* modulo *A*. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of *A* or *B* is not a **number**, signal **domain-error**.
If *A* is **zero**, return *B*.
If **comparison-tolerance** is not **zero**, and *B* **divided-by** *A* is **integral-within comparison-tolerance**, return **zero**.
Otherwise set *Z* to *B* **modulo** *A*.
If *Z* is *A*, return **zero**.
Otherwise, return *Z*.

Examples:

– In the following, **print-precision** is `16` and **comparison-tolerance** is `1E¯10`.

```
      7 ¯7 ∘.|31 28 ¯30
 3 0  5
¯4 0 ¯2
      0.2 |1.4 1.5 1.6
0 0.1 0
      1 |1E30 1E¯30 ¯1E¯30 .99999999999
0 0 0 0
```

– In the following, **comparison-tolerance** is **zero**.

```
      1 |1E30 1E¯30 ¯1E¯30 .99999999999
0 1E¯30 0 0.99999999999
```

Additional Requirement:

The range of residue is those numbers that are the product of a **fraction** and *A*, except when *A* is **zero**, in which case the range is the single **number** *B*.

Note: *The* **implementation-algorithm** *P* **modulo** *Q provides an exact modulo operation for* **real-numbers** *P and Q. It evaluates* `R←P-(×P)×|Q×⌊|P÷Q` *exactly, and returns R if* `(×R)=×Q`*, or* `R+Q` *otherwise.*

The definition of "mod" in the IEEE standard for Binary Floating-Point Arithmetic (754) provides an example of this exact evaluation.

Implementations should avoid signalling **limit-error** *in* **residue**. *If the operation B* **divided-by** *A causes* **exponent-overflow**, *return* **zero**. *If it causes* **exponent-underflow**, *and if A and B have the same signs, return B. If they have different signs, return* **zero**.

7.2.10 Binomial

`Z ← A ! B`

Informal Description: `Z` is `(`**gamma**`(1+B))÷((`**gamma**`(1+A))×`**gamma**`(1+B-A))`

If `A` and `B` are **nonnegative-integers**, `Z` is the number of combinations of `B` things taken `A` at a time.

Evaluation Sequence:

If either of `A` or `B` is not a **number**, signal **domain-error**.
Determine if each of `A`, `B`, and `B-A` is a **negative-integer**.
Select the appropriate case from the following table, where a **one** indicates that the corresponding value is a **negative-integer** and a **zero** indicates that it is not.

Case			Rule
A	B	B-A	
0	0	0	Return `(!B)÷(!A)×!B-A`.
0	0	1	Return **zero**.
0	1	0	Signal **domain-error**.
0	1	1	Return `(¯1*A)×A!A-B+1`.
1	0	0	Return **zero**.
1	0	1	(Case cannot arise.)
1	1	0	Return `(¯1*B-A)×(|B+1)!(|A+1)`.
1	1	1	Return **zero**.

Example:

```
      ¯4 ¯3 ¯2 ¯1 0 1 2 3 4 ∘.! ¯4 ¯3 ¯2 ¯1 0 1 2 3 4
   1   ¯3    3   ¯1    0    0    0    0    0
   0    1   ¯2    1    0    0    0    0    0
   0    0    1   ¯1    0    0    0    0    0
   0    0    0    1    0    0    0    0    0
   1    1    1    1    1    1    1    1    1
  ¯4   ¯3   ¯2   ¯1    0    1    2    3    4
  10    6    3    1    0    0    1    3    6
 ¯20  ¯10   ¯4   ¯1    0    0    0    1    4
  35   15    5    1    0    0    0    0    1
```

Note: *The APL expressions in the* **rule** *column indicate the result required, not the algorithm to be used. For example,* `64!65` *should be* `65` *even if* `!65` *signals* **limit-error**.

7.2.11 Circular Functions

`Z ← A ○ B`

Informal Description: *Z* is the result of applying a function designated by *A* to *B*.

Evaluation Sequence:

If *A* is not a **near-integer**, signal **domain-error**.
If *B* is not a **number**, signal **domain-error**.
Set `A1` to the **integer-nearest-to** *A*.
If `A1` is not in the **closed-interval-between** `¯12` and `12`, signal **domain-error**.
If `A1` is `¯12`, return `⋆0J1×B`.
If `A1` is `¯11`, return `0J1×B`.
If `A1` is `¯10`, return `+B`.
If `A1` is `¯9`, return *B*.
If `A1` is `¯8`, return `-(¯1-B⋆2)⋆.5`.
If `A1` is `¯7`,
  If *B* is **negative-one** or **one**, signal **domain-error**.
  Return the **inverse-hyperbolic-tangent** of *B*.
If `A1` is `¯6`,
  Return *Z*, the principal value of the **inverse-hyperbolic-cosine** of *B*, where *Z* is a **nonnegative-number**.
If `A1` is `¯5`, return the **inverse-hyperbolic-sine** of *B*.
If `A1` is `¯4`,
  If *B* is `¯1` return **zero**
  Otherwise return `(B+1)×((B-1)÷B+1)⋆0.5`.
If `A1` is `¯3`, return *Z*, the principal value in radians of the **inverse-tangent** of *B*, where *Z* is in the **open-interval-between** $-\pi/2$ and $\pi/2$.
If `A1` is `¯2`,
  Return *Z*, the principal value in radians of the **inverse-cosine** of *B*, where *Z* is either **zero** or a **number** in the **open-interval-between zero** and π.
`A1` is `¯1`,
  Return *Z*, the principal value in radians of the **inverse-sine** of *B*, where *Z* is either $\pi/2$, or a **number** in the **open-interval-between** $-\pi/2$ and $\pi/2$.
If `A1` is `0`,
  If *B* is not in the **closed-interval-between negative-one** and **one**, signal **domain-error**.
  Return `(1-B⋆2)⋆0.5`.
If `A1` is `1`, return the **sine** of *B* radians.
If `A1` is `2`, return the **cosine** of *B* radians.
If `A1` is `3`,
  If *B* is an odd multiple of $\pi/2$, signal **domain-error**.
  Return the **tangent** of *B* radians.
If `A1` is `4`, return `(1+B⋆2)⋆0.5`.
If `A1` is `5`, return the **hyperbolic-sine** of *B*.
If `A1` is `6`, return the **hyperbolic-cosine** of *B*.

If $A1$ is `7`, return the **hyperbolic-tangent** of B.
If $A1$ is `8`, return `(¯1-B*2)*.5`.
If $A1$ is `9`, return the **real-part** of B.
If $A1$ is `10`, return `|B`.
If $A1$ is `11`, return the **imaginary-part** of B.
If $A1$ is `12`, return the **arc** of B.

Examples:

```
      2 ○¯1 ○.6
.8
      2 ○0
1
      3 ○○÷4
1
      6 ○0
1
```

Note: *The APL expressions used for* `0○X`*,* `¯4○X`*, and* `4○X` *above indicate the result desired, not the algorithm to be used.*

Note: *The following article describes the reasons for the choices made in defining the circular functions for complex arguments,*

Penfield, Paul, "Principal Values and Branch Cuts in Complex APL", **APL81 Conference Proceedings**, *ACM, San Francisco, 1981*

Note: *Values of* A *greater than 7 in magnitude are required only for an implementation which includes the optional* **complex-arithmetic-facility**.

7.2.12 And/LCM

```
Z ← A ∧ B
```

Informal Description: *Z* is the least common multiple of *A* and *B*. For Boolean arguments it is the Boolean product of *A* and *B*. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either *A* or *B* is not a **number**, signal **domain-error**.
If both *A* and *B* are **near-boolean**,
  Set *A*1 to the **integer-nearest-to** *A*.
  Set *B*1 to the **integer-nearest-to** *B*.
  If either *A*1 or *B*1 is **zero**, return **zero**.
  Otherwise, return **one**.
Otherwise, set *A*1 to the **greatest-common-divisor** of *A* and *B*, using the **implementation-algorithm greatest-common-divisor**.
If *A*1 is **zero**, return **zero**.
Otherwise, return *A* **times** (*B* **divided-by** *A*1).

Examples:

```
     0 1 ∘.∧0 1
0 0
0 1
     30 ∧ 36
180
     3 ∧ 3.6
18
     ¯29J53 ∧ ¯1J107
¯853J¯329
```

7.2.13 Or/GCD

```
Z ← A ∨ B
```

Informal Description: *Z* is the greatest common divisor of *A* and *B*. For Boolean arguments, *Z* is the Boolean sum of *A* and *B*. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either *A* or *B* is not a **number**, signal **domain-error**.
If both *A* and *B* are **near-boolean**,
  Set *A*1 to the **nearest-integer** to *A*.
  Set *B*1 to the **nearest-integer** to *B*.
  If either *A*1 or *B*1 is **one**, return **one**.
  Otherwise, return **zero**.
Otherwise, return the **greatest-common-divisor** of *A* and *B*, using the **implementation-defined-algorithm greatest-common-divisor**.

Examples:

```
      0 1 ∘.∨0 1
0 1
1 1
      30 ∨ 36
6
      3 ∨ 3.6
0.6
      ¯29J53 ∨ ¯1J107
7J1
```

7.2.14 Nand

```
Z ← A ⍲ B
```

Informal Description: *Z* is the Boolean complement of the Boolean product of *A* and *B*.

Evaluation Sequence:

If either *A* or *B* is not **near-Boolean**, signal **domain-error**.
Otherwise, return ~*A*∧*B*, with **comparison-tolerance** set to **zero**.

Example:

```
      0 1 ∘.⍲0 1
1 1
1 0
```


7.2.15 Nor

```
Z ← A ⍱ B
```

Informal Description: Z is the Boolean complement of the Boolean sum of A and B.

Evaluation Sequence:

If either A or B is not **near-Boolean**, signal **domain-error**.
Otherwise, return ~A∨B, with **comparison-tolerance** set to **zero**.

Example:

```
      0 1 ∘.⍱0 1
1 0
0 0
```


7.2.16 Equal

```
Z ← A = B
```

Informal Description: Z is **one** if A and B are considered equal and **zero** otherwise. A and B are equal if they are the same **character**, or if they are both **numeric** and A is tolerantly equal to B within **comparison-tolerance**. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If the **type** of A is not the same as the **type** of B, return **zero**.
If both A and B are **characters**,
  If A is the same **character** as B, return **one**.
  Otherwise, return **zero**.
If both A and B are **numbers**,
  If A is **tolerantly-equal to** B **within comparison-tolerance**, return **one**.
  Otherwise, return **zero**.

Example:

```
      1 2 3 ∘.= 1 2 3
1 0 0
0 1 0
0 0 1
```

– In the following, **comparison-tolerance** is `1E¯13`.

```
      4 = 4 + 5E¯13 2E¯13 ¯2E¯13 ¯5E¯13
0 1 1 0
      0 = ¯1E¯20 1E¯20 0
0 0 1
      3 = 'A3'
0 0
```

Note: *Comparisons of* **numbers** *whose signs differ are not affected by* **comparison-tolerance***.*

For any value of **comparison-tolerance** *and any two real numbers A and B, exactly one of the expressions $A<B$, $A=B$, and $A>B$ is* **one***.*

Equal *should not signal a* **limit-error***. For example, the result of* **positive-number-limit** = **negative-number-limit** *is* **zero***. The following is a sample technique for handling exponent-overflow and exponent-underflow when scaling* **comparison-tolerance***.*

Set C to the larger of the magnitudes of A and B.
Set D to **comparison-tolerance times** *C.*
If **exponent-underflow** *occurs,*
  Set $A1$ to A **divided-by** *C.*
  Set $B1$ to B **divided-by** *C.*
  Set $C1$ to the magnitude of $A1$ **minus** *$B1$.*

If $C1$ *is* **greater-than comparison-tolerance***, return* **zero**.
Otherwise, return **one**.
Set E *to the magnitude of* A **minus** B.
If **exponent-overflow** *occurs, return* **zero**.
If **exponent-underflow** *occurs,*
Set $A1$ *to* A **divided-by** C.
Set $B1$ *to* B **divided-by** C.
Set $C1$ *to the magnitude of* $A1$ **minus** $B1$.
If $C1$ *is* **greater-than comparison-tolerance***, return* **zero**.
Otherwise, return **one**.
If E *is not* **greater-than** D*, return* **one**.
Otherwise, return **zero**.

7.2.17 Less than

```
Z ← A < B
```

Informal Description: Z is **one** if A is tolerantly less-than B, and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of A or B is not a **near-real number**, signal **domain-error**.
Set $A1$ to the **real-number** nearest to A.
Set $B1$ to the **real-number** nearest to B.
If $A1$=$B1$, evaluated with the current value of **comparison-tolerance**, is **one**, return **zero**.
If $A1$ is **less-than** $B1$, return **one**.
Otherwise, return **zero**.

Examples:

```
      1 2 3 ∘.< 1 2 3
0 1 1
0 0 1
0 0 0
      0 1 ∘.< 0 1
0 1
0 0
```


7.2.18 Less than or equal to

```
Z ← A ≤ B
```

Informal Description: *Z* is **one** if *A* is less than or tolerantly equal to *B*, and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of *A* or *B* is not a **near-real number**, signal **domain-error**.
Set *A*1 to the **real-number** nearest to *A*.
Set *B*1 to the **real-number** nearest to *B*.
If *A*1=*B*1, evaluated with the current value of **comparison-tolerance**, is **one**, return **one**.
If *A*1 is **less-than** *B*1, return **one**.
Otherwise, return **zero**.

Examples:

```
      1 2 3 ∘.≤1 2 3
1 1 1
0 1 1
0 0 1
      0 1 ∘.≤0 1
1 1
0 1
```


7.2.19 Not equal

```
Z ← A ≠ B
```

Informal Description: Z is **one** if A does not equal B, and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
Return ~A=B, evaluated with the current value of **comparison-tolerance**.

Examples:

```
      'A' ≠41
1
      1 2 3 ∘.≠1 2 3
0 1 1
1 0 1
1 1 0
      0 1 ∘.≠0 1
0 1
1 0
```


7.2.20 Greater than or equal to

```
Z ← A ≥ B
```

Informal Description: *Z* is **one** if *A* is greater than or tolerantly equal to *B*, and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of *A* or *B* is not a **near-real number**, signal **domain-error**.
Set *A*1 to the **real-number** nearest to *A*.
Set *B*1 to the **real-number** nearest to *B*.
If *A*1=*B*1, evaluated with the current value of **comparison-tolerance**, is **one**, return **one**.
If *A*1 is **greater-than** *B*1, return **one**.
Otherwise, return **zero**.

Examples:

```
      1 2 3 ∘.≥1 2 3
1 0 0
1 1 0
1 1 1
      0 1 ∘.≥0 1
1 0
1 1
```

7.2.21 Greater than

```
Z ← A > B
```

Informal Description: *Z* is **one** if *A* is tolerantly greater than *B*, and **zero** otherwise. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of *A* or *B* is not a **near-real number**, signal **domain-error**.
Set *A*1 to the **real-number** nearest to *A*.
Set *B*1 to the **real-number** nearest to *B*.
If *A*1=*B*1, evaluated with the current value of **comparison-tolerance**, is **one**, return **zero**.
If *A*1 is **greater-than** *B*1, return **one**.
Otherwise, return **zero**.

Examples:

```
      1 2 3 ∘.> 1 2 3
0 0 0
1 0 0
1 1 0
      0 1 ∘.> 0 1
0 0
1 0
```


8 Structural Primitive Functions

8.1 Introduction

Note: *The functions in this chapter are used in the evaluation sequences of many non-scalar operations. They are defined here to avoid forward references.*

8.2 Monadic Structural Primitive Functions

8.2.1 Ravel

```
Z ← , B
```

Informal Description: *Z* is a vector containing the elements of *B* in row-major order.

Evaluation Sequence:

Return *Z*, a **vector** such that the **ravel-list** of *Z* is the same as the **ravel-list** of *B*, the **type** of *Z* is the same as the the **type** of *B*, and the **shape-list** of *Z* is a **list** of length **one** containing the **count** of *B* as its only **item**.

Examples:

```
      ,N22
11 12 21 22
      ,N222
111 112 121 122 211 212 221 222
      ,N2221
1111 1121 1211 1221 2111 2121 2211 2221
```

Note: *Ravel always produces a vector result. The expressions* **ravel-list** *of Z,* **type** *of Z, and* **shape-list** *of Z refer to attributes of an* **array** *object.*

8.2.2 Shape

```
Z ← ρ B
```

Informal Description: Z is a **numeric** vector containing the shape of the array B.

Evaluation Sequence:

Return Z, an **array** such that the **type** of Z is **numeric**, the **ravel-list** of Z is the **shape-list** of B, and the **shape-list** of Z is a **list** whose only **item** contains the **number-of-items** in the **shape-list** of B.

Examples:

```
          ρN

          ρ,N
1
          ρρN
0
          ρN3
3
          ρρN3
1
          ρN34
3 4
```

Note: *Shape always produces a vector result. The expression* **shape-list** *of* Z *refers to an attribute of an* **array** *object.*

8.2.3 Index Generator

```
Z ← ⍳ B
```

Informal Description: *Z* is a **numeric** vector of *B* consecutive ascending **integers**, the first of which is **index-origin**. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If the **count** of *B* is not **one**, signal **length-error**.
If *B* is not a **near-integer**, signal **domain-error**.
Set *B1* to the **integer-nearest-to** *B*.
If *B1* is not a **nonnegative-integer**, signal **domain-error**.
If *B1* is **zero**, return an **empty numeric vector**.
Generate a **numeric vector** *Z1* of length *B1* such that the **ravel-list** of *Z1* consists of the **integers** in the **closed-interval-between one** and *B1* in ascending order. Generate *Z*, a **numeric array** with the **shape-list** of *Z1* such that each **item** of the **ravel-list** of *Z* is (**index-origin minus one**) **plus** the corresponding element of *Z1*.
Return *Z*.

Examples:

– In the following example, **index-origin** is **zero**.

```
      ⍳4
0 1 2 3
```

– In the following example, **index-origin** is **one**.

```
      ⍳4
1 2 3 4
```


8.2.4 Table

```
Z ← ⍪ B
```

Informal Description: Z is a **matrix** containing the elements of B. Z is formed by ravelling the subarrays of B along its first axis. For example, if B is a scalar, Z has shape `1 1`; if B is a five-element vector, Z has shape `5 1`; if B has shape `5 4 3 2`, Z has shape `5 24`.

Evaluation Sequence:

If B is a **scalar** set C to `1 1`.
Else set C to a two-element **list** as follows:
  Set the **first-item** of C to the **first-item** of the **shape-list** of B.
  Set the **last-item** of C to the **product-of** the **rest-of** the **shape-list** of B.
Set Z to the C **reshape** of B.
Return Z.

Examples:

```
      ⍪0
0
      ρ⍪0
1 1
      ⍪N4
1
2
3
4
      ⍪N22
11 12
21 22
      ⍪N222
111 112 121 122
211 212 221 222
      ⍪N2221
1111 1121 1211 1221
2111 2121 2211 2221
```

8.2.5 Depth

```
Z ← ≡ B
```

Informal Description: *Z* is a numeric scalar describing the level of nesting of *B*. For **simple-scalars**, this is 0. For arrays, this is one greater than the depth of the element with the greatest depth.

Note: *An enclosed array is an array.*

Evaluation Sequence:

If *B* is a **simple-scalar**, return **zero**.
Return 1+⌈/,≡¨*B*.

Examples:

```
      ≡5
0
      ≡1 2 3
1
      ≡N234
1
      ≡'ABC',1 2 3
1
      ≡⊂1 2 3
2
      ≡,⊂1 2 3
2
      ≡'HERO',⊂2 3,⊂2 3ρ⊂5 8
4
```


8.2.6 Enlist

```
Z ← ∊ B
```

Informal Description: If B is a **simple-array**, then enlist has the same effect as ravel. Otherwise, enlist has the effect of recursively raveling each element of B (in **ravel-list** order) and joining them together. Thus, Z will always be a **simple-array** of rank one, containing all items of B.

Note: *Enlist is sometimes called "super ravel".*

Evaluation Sequence:

If B is empty, return `0⍴∊1↑,B`.
If B is a **simple-array**, return `,B`.
Set `B1` to the first item of the **ravel-list** of B.
Return `(∊B1),(∊1↓,B)`.

Examples:

```
      ∊1 2 3,⊂4 5 6
1 2 3 4 5 6
      ≡1 2 3,⊂4 5 6
2
      ≡∊1 2 3,⊂4 5 6
1
```

8.3 Dyadic Structural Primitive Functions

8.3.1 Reshape

```
Z ← A ⍴ B
```

Informal Description: Z is an array of shape `,A` whose elements are taken sequentially from `,B` repeated cyclically as required.

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
Set `A1` to the **integer-array-nearest-to** `,A`.
If any **item** of the **ravel-list** of `A1` is not a **nonnegative-counting-number**, signal **domain-error**.
Let `RA` stand for the **product-of** the **ravel-list** of `A1`.
Let `CB` stand for the **count** of B.

If *RA* is not **zero** and *CB* is **zero**, signal **length-error**.

Return an **array** *Z* such that the **type** of *Z* is the **sufficient-type** of the **ravel-list** of *Z* under the **type** of *B*, the **shape-list** of *Z* is the same as the **ravel-list** of *A*1, and the **ravel-list** of *Z* is a **list** with *RA* **items** such that for all *I* in the **index-set** of *Z*, **item** *I* of the **ravel-list** of *Z* is **item** 1+*CB*|*I*−1 of the **ravel-list** of *B*.

Examples:

```
      2 4⍴N213
111 112 113 211
212 213 111 112
      ⍴0⍴'ABCD'
0
      B ←1E7 1E7 1E7 0 1E7 1E7 1E7 ⍴ 42
      ⍴B
1E7 1E7 1E7 0 1E7 1E7 1E7
```

Note: *For any X that is not* **empty***,* ''⍴*X and* (⍳0)⍴*X produce the same result: a* **scalar** *whose value is that of the* **first-scalar** *in X.*

8.3.2 Join

`Z ← A , B`

`Z ← A ⍪ B`

Informal Description: If A and B are scalars or vectors, Z is the vector of length `(⍴,A)+⍴,B` whose first `⍴,A` elements are `,A` and whose last `⍴,B` elements are `,B`.

If either A or B has rank greater than one,
For form `Z ← A , B`
Z is `A,[(⍴⍴A)⌈⍴⍴B]B`,

For form `Z ← A ⍪ B`
Z is `A,[1]B`,

as defined under **Join Along an Axis**.

Evaluation Sequence:

If A is a **scalar** and B is a **scalar**, return `(,A),,B`.
If A is a **scalar** and B is a **vector**, return `(,A),B`.
If A is a **vector** and B is a **scalar**, return `A,,B`.
If A is a **vector** and B is a **vector**,
If A is **empty** and B is **empty** and the **type** of A differs from the **type** of B, return an **empty-list** whose **type** is determined by the **implementation-algorithm typical-element-for-mixed**.
If B is **empty**, return A.
If A is **empty**, return B.
Otherwise, return a **vector** Z, such that the **shape-list** of Z is `(⍴A)+⍴B`, the **ravel-list** of Z is a **list** whose first `⍴A` **items** are the **ravel-list** of A and whose last `⍴B` **items** are the **ravel-list** of B, and the **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type mixed**.
Otherwise, when either A or B has rank greater than **one**, return `A,[1]B` for form `A⍪B` and `A,[(⍴⍴A)⌈⍴⍴B]B` for form `A,B`.

Example:

```
      '',0
0
```

Note: *This subsection intentionally contains a forward reference to* **Join Along an Axis**. **Join** *and* **Join Along an Axis** *are defined separately because the description of* **Join Along an Axis** *requires APL operations that depend for their definitions in turn upon* **Join**.

9 Operators

9.1 Introduction

Note: *The forms in this chapter are referred to as* **operators***.*

9.2 Monadic Operators

9.2.1 Reduction

Z ← *f*/ *B*

Z ← *f*/[*K*] *B*

Z ← *f*⌿ *B*

Z ← *f*⌿ [*K*] *B*

Informal Description: *Z* is the value produced by placing the dyadic function **f** between adjacent items along a designated axis of *B* and evaluating the resulting expression. The axis designated determines how the subarrays are chosen. Uses **index-origin**.

There are two conforming definitions for Reduction; the **Implementation Parameter Reduction-Style** specifies which definition a particular implementation uses.

The **Enclose-Reduction-Style** style, informally known as the APL2 style, uses a definition based on enclose which, for all **f**, preserves the identity:
ρρ*Z* *is* 0⌈¯1+ρρ*B*.

The **Insert-Reduction-Style** style, informally known as the Sharp/J style, does not universally preserve the above identity. Instead **f** is inserted between each successive **cell** along the **frame** axis (the axis of reduction). The rank of **f** controls the subsequent evaluation.

Note: *Since reduction is used in the evaluation sequences of other operators, the choice of the* **Implementation Parameter Reduction-Style** *has a pervasive effect on the implementation, and its importance should not be underestimated.*

Note: *The functions listed in* **Table 5** *are* **scalar** *functions, that is their left and right function ranks are* **zero**.

Evaluation Sequence:

If **Implementation Parameter Reduction-Style** is **Enclose-Reduction-Style**:
  If **f** is not a **dyadic-function**, signal **syntax-error**.
  For form **f**/B
    If B is a **scalar**, return B.
    Otherwise, return **f**/[¯1↑⍳⍴⍴B] B .
  For form **f**⌿B
    If B is a **scalar**, return B.
    Otherwise, return **f**/[⍳1] B .
  For forms **f**/[K] B and **f**⌿[K] B
    If K is not a **valid-axis** for B, signal **axis-error**.
    Otherwise, set K1 to the **integer-nearest-to** K.
    If B is a **vector**,
      If the **length** of B is **zero**, take the action designated in **Table 5** for **f**.
      If the **length** of B is **one**, return a **scalar** Z such that the **type** of Z is the **type** of B and the **ravel-list** of Z is the **ravel-list** of B.
      If the **length** of B is **greater-than one**,
        Set B1 to the **first-scalar** in B.
        Set B2 to the **remainder-of** B.
        Return ⊂B1 **f** ⊃**f**/B2.
    If the **rank** of B is **greater-than one**, return an **array** Z such that the **shape-list** of Z is the **shape-list** of B with **item** K1 omitted, and the **ravel-list** of Z has the property that if Z1 is an **item** of Z and B3 is the corresponding **vector-item along-axis** K1 of B, then Z1 is **f**/B3.

Otherwise **Implementation Parameter Reduction-Style** is **Insert-Reduction-Style**:
  If **f** is not a **dyadic-function**, signal **syntax-error**.
  For form **f**/B
    If B is a **scalar**, return B.
    Otherwise, return **f**/[¯1↑⍳⍴B] B .
  For form **f**⌿B
    If B is a **scalar**, return B.
    Otherwise, return **f**/[⍳1] B .
  For forms **f**/[K] B and **f**⌿[K] B
    If K is not a **valid-axis** for B, signal **axis-error**.
    Otherwise, set K1 to the **integer-nearest-to** K.
    Set B1 to (⍋K1,(⍳⍴⍴B)~K1)⍉B.
    Set C1 to 1↓⍴B1.
    If the number of C1-cells in B1 is **zero**, take the action designated in **Table 5** for **f** to choose a result item Z0. Return an **array** Z such that the **shape-list** of Z is C1, the **type** of Z is the **type** of Z0, and each item of the **ravel-list** of Z is the

ravel-list of *Z*0.

If the number of *C*1-cells in *B*1 is **one**, return an **array** *Z* such that the **shape-list** of *Z* is C1, the **type** of *Z* is the **type** of *B*1, and the **ravel-list** of *Z* is the **ravel-list** of *B*1.

If the number of *C*1-cells in *B*1 is **greater-than one**,

Set *B*2 to an array such that the **shape-list** of *B*2 is C1, the **type** of *B*2 is the **type** of *B*1, and the **ravel-list** of *B*2 is the **ravel-list** of the first *C*1-cell in *B*1.

Set *B*3 to an array such that the **shape-list** of *B*3 is `((¯1+1↑ρB1),C1)`, the **type** of *B*3 is the **type** of *B*1, and the **ravel-list** of *B*3 is the **ravel-list** of all the the *C*1-cells of *B*1 except the first.

Return *B*2 **f f**⌿*B*3.

Examples:

```
      +/1 2 3
6
      ×/1 2
2
      =/'A'
A
      =/'AA'
1
      =/'AAA'
0
      (3 8ρ0) ≡,⌿2 3 4ρ0 ⍝Insert-Reduction-Style
1
      (3 4ρ⊂0 0) ≡,⌿2 3 4ρ0 ⍝Enclose-Reduction-Style
1
```

Additional Requirement:

If *Z* is **empty**, the **type** of *Z* is determined by the argument function **f**.

When applied along an empty **axis** in an array, reduction produces an array whose shape is that of the argument array with the designated **axis** deleted. The elements of the array, if any, are determined by the argument function and **Table 5**. In some cases, an error is signalled.

Example:

```
      +/2 0 ρ5.1
0 0
      ρ+/2 0ρ5.1
2
```


Dyadic Function		**Action**
Plus	$+$	Return **zero**.
Minus	$-$	Return **zero**.
Times	$\times$	Return **one**.
Divide	$\div$	Return **one**.
Residue	$\vert$	Return **zero**.
Minimum	⌊	Return **positive-number-limit**.
Maximum	⌈	Return **negative-number-limit**.
Power	⋆	Return **one**.
Binomial	!	Return **one**.
And	$\wedge$	Return **one**.
Or	$\vee$	Return **zero**.
Less	$<$	Return **zero**.
Not greater	$\leq$	Return **one**.
Equal	$=$	Return **one**.
Not less	$\geq$	Return **one**.
Greater	$>$	Return **zero**.
Not equal	$\neq$	Return **zero**.
all others		Signal **domain-error**.

Table 5: Actions for the Reduction of an Empty Vector.

9.2.2 Scan

Z ← **f** *B*

Z ← **f** \\[*K*] *B*

Z ← **f** ⍀*B*

Z ← **f** ⍀[*K*] *B*

Informal Description: *Z* is an array having the same shape as *B* and containing the results produced by **f** reduction over all prefixes of a designated axis of *B*. Uses **index-origin**.

Evaluation Sequence:

If **f** is not a **dyadic-function**, signal **syntax-error**.
For form **f** *B*
  If *B* is a **scalar**, return *B*.
  Otherwise, return **f** \\[⍴⍴*B*] *B* evaluated with **index-origin** set to **one**.
For form **f** ⍀*B*
  If *B* is a **scalar**, return *B*.
  Otherwise, return **f** \\[1] *B* evaluated with **index-origin** set to **one**.

For forms **f** `\[K]B` and **f** `⍀[K]B`

If *K* is not a **valid-axis** for *B*, signal **axis-error**.

Otherwise, set `K1` to the **integer-nearest-to** *K*.

If *B* is a **vector**,

If the **count** of *B* is **less-than two**, return *B*.

Otherwise return *Z*, a **vector** such that the **shape-list** of *Z* is the **shape-list** of *B*, and the **ravel-list** of *Z* is such that **item** *I* of the **ravel-list** of *Z* is **f**`/B[⍳I]` for all *I* in the **index-set** of the **ravel-list** of *B*. The **type** of *Z* is the **sufficient-type** of the **ravel-list** of *Z* under the **type mixed**.

If the **rank** of *B* is **greater-than one**, each **vector-item along-axis** `K1` of *Z* is **f** `\B1`, where `B1` is the corresponding **vector-item along-axis** `K1` of *B*.

Examples:

```
      +\1 1 1
1 2 3
      ^\1 1 1 0 0 0 1 1 1
1 1 1 0 0 0 0 0 0
      -\'A'
A
      =\'AB'
A 0
```

Additional Requirement:

If the operator **reduction** signals an error when called by **scan**, **scan** returns the resultant error **token**.

The evaluation sequence above describes a quadratic algorithm for **scan**. If the function **f** is associative, **scan** may be implemented with a linear algorithm.

Note: *Various error checks have been performed on K by the phrase evaluators, and* **index-origin** *is* **one**, *before this evaluation sequence is called.*

9.2.3 N-wise Reduction

$Z \leftarrow N\ f/\ B$

$Z \leftarrow N\ f/[K]\ B$

$Z \leftarrow N\ f⌿\ B$

$Z \leftarrow N\ f⌿[K]\ B$

Informal Description: N-wise reduction is the dyadic invocation of a function derived by **reduction**. Z is the value produced by placing the primitive scalar dyadic function **f** between subarrays of B and evaluating the resulting expression. If N is negative the subarrays are reversed before application of **f**. Each subarray has length N along a common dimension. The axis designated determines how the subarrays are chosen. Uses **index-origin**.

Evaluation Sequence:

If the **rank** of N is **greater-than one**, signal **rank-error**.
If the length of the **ravel-list** of N is **greater-than one**, signal **length-error**.
If N is not a **near-integer**, signal **domain-error**.
  Set $N1$ to the **integer-nearest-to** N.
  Set $M1$ to the **magnitude** of $N1$.
If **f** is not a **dyadic-function**, signal **syntax-error**.
For form N **f**$/B$
  If B is a **scalar**,
    If $M1$ is **greater-than two**, signal **domain-error**.
    If $M1$ is **zero**, set R to **two** and take the action designated in **Table 6** for R and **f**.
    Otherwise, return $(2-M1)\rho B$.
  Otherwise, return N **f**$/[\rho\rho B]\ B$ evaluated with **index-origin** set to **one**.
For form N **f**$⌿B$
  If B is a **scalar**,
    If $M1$ is **greater-than two**, signal **domain-error**.
    If $M1$ is **zero**, set R to **two** and take the action designated in **Table 6** for R and **f**.
    Otherwise, return $(2-M1)\rho B$.
  Otherwise, return N **f**$/[1]\ B$ evaluated with **index-origin** set to **one**.
For forms N **f**$/[K]\ B$ and N **f**$⌿[K]\ B$
  If K is not a **valid-axis** for B, signal **axis-error**.
  Otherwise, set $K1$ to the **integer-nearest-to** K.
  If B is a **vector**,
    If $M1$ is **greater-than one plus** the **length** of B, signal **domain-error**.
    If $M1$ is **zero**, set R to **one plus** the **length** of B and take the action designated in **Table 6** for R and **f**.
    Set $B1$ to $M1$ **take** B.
    If $N1$ is **less-than zero**,
      set $B2$ to $\phi B1$

otherwise set $B2$ to $B1$.
If $M1$ **equals** the **length** of B, return **f** / $B2$.
Otherwise,
Set $B3$ to **f** / $B2$.
Set $B4$ to **one drop** B.
Return $B3$, N **f** / $B4$.

If the **rank** of B is **greater-than one**, return an **array** Z such that the **shape-list** of Z is the **shape-list** of B with **item** $K1$ replaced by **one plus** the value of **item** $K1$ **minus** $M1$, and the **ravel-list** of Z has the property that if $Z1$ is the **vector-item along-axis** $K1$ of Z and $B3$ is the corresponding **vector-item along-axis** $K1$ of B, then $Z1$ is N **f** / $B3$.

Examples:

```
      0+/1 2 3
0 0 0 0
      1+/1 2 3
1 2 3
      2+/1 2 3
3 5
      3+/1 2 3
6
      4+/1 2 3

      2-/1 4 9 16 25
¯3 ¯5 ¯7 ¯9
      ¯2-/1 4 9 16 25
3 5 7 9
      ¯3-/1 2 3 4 5 6 7
2 3 4 5 6
```


Dyadic Function		**Action**
Plus	+	Return R reshape **zero**.
Minus	−	Return R reshape **zero**.
Times	×	Return R reshape **one**.
Divide	÷	Return R reshape **one**.
Residue	∣	Return R reshape **zero**.
Minimum	⌊	Return R reshape **positive-number-limit**.
Maximum	⌈	Return R reshape **negative-number-limit**.
Power	⋆	Return R reshape **one**.
Logarithm	⍟	Signal **domain-error**.
Circular	○	Signal **domain-error**.
Binomial	!	Return R reshape **one**.
And	∧	Return R reshape **one**.
Or	∨	Return R reshape **zero**.
Nand	⍲	Signal **domain-error**.
Nor	⍱	Signal **domain-error**.
Less	<	Return R reshape **zero**.
Not greater	≤	Return R reshape **one**.
Equal	=	Return R reshape **one**.
Not less	≥	Return R reshape **one**.
Greater	>	Return R reshape **zero**.
Not equal	≠	Return R reshape **zero**.
all others		Signal **domain-error**.

Table 6: Actions for the N-wise Reduction of an Empty Vector.

9.2.4 Duplicate

Z ← **f** ⍨ *B*

Informal Description: *Z* is *B* **f** *B*.

Evaluation Sequence:

Return *B* **f** *B*.

Example:

```
        ∘.≤⍨N3
1 1 1
0 1 1
0 0 1
```

Note: *If* **f** *is not an* **ambivalent-function**, *a* **valence-error** *will be signalled.*

9.2.5 Commute

Z ← *A* **f** ⍨ *B*

Informal Description: *Z* is *B* **f** *A*.

Evaluation Sequence:

Return *B* **f** *A*.

Examples:

```
        3-⍨4
1
        +/2*⍨2 2⍴4 7 1 8
65 65
```

9.2.6 Each

$Z \leftarrow \mathbf{f}^{\ddot{}}\ B$

$Z \leftarrow A\ \mathbf{f}^{\ddot{}}\ B$

Informal Description: The operand function **f** is applied independently to corresponding **items** of the arguments, or (monadic case) independently to the **items** of the argument. The corresponding results are assembled in an array of the same shape as the argument(s).

Evaluation Sequence:

For form *A***f**¨*B*:
  If the **rank** of *A* differs from the **rank** of *B*,
    If *A* is a **scalar** or **one-element-vector** and *B* is not a **scalar**, set *A* to (ρ*B*)ρ*A*.
    Otherwise,
      If *B* is a **scalar** or **one-element-vector**, set *B* to (ρ*A*)ρ*B*.
      Otherwise, Signal **rank-error**.
  If the **shape-list** of *A* differs from the **shape-list** of *B*, Signal **length-error**.
For both forms:
  If *B* is not empty, Signal **domain-error**.
  Create *Z*, an array having the same **shape-list** as *B*.
  For each *I* in the **index-set** of the **ravel-list** of *Z*:
    For form **f**¨*B*,
      Set *X* to item *I* of the **ravel-list** of *B*.
      Evaluate-Monadic-Function with **f***X* giving **token T**.
    For form *A***f**¨*B*,
      Set *X* to item *I* of the **ravel-list** of *A*.
      Set *Y* to item *I* of the **ravel-list** of *B*.
      For each pair *X* and *Y* of corresponding items from *A* and *B*, **Evaluate-Dyadic-Function** with *X***f***Y* giving **token T**.
    If **T** is an error, return **T**.
    Set *Q* to the content of **T**.
    Set item *I* of the **ravel-list** of *Z* as follows:
      If *Q* is a **simple-scalar**, the **first-item** of the **ravel-list** of *Q*.
      Otherwise, *Q*.
  If all items of the **ravel-list** of *Z* are **nil**, return **nil**.
  If all items of the **ravel-list** of *Z* are **values**, return *Z*.
  Otherwise, signal **value-error**.

Example:

```
ρ¨⊂'AB',⊂'CDE'
  2 3
```

9.3 Dyadic Operators

9.3.1 Outer Product

Z ← A ∘.**f** B

Informal Description: Z is an array of shape (⍴A),⍴B. The elements of Z are the result of applying the **dyadic-function f** to every possible combination of scalar arguments where the left argument is an element of A and the right an element of B.

Z is such that if I is an **index-list** that selects a single element of Z, the first ⍴A **items** of I are the **index-list** that would select from A the element used as the left argument to **f** and the last ⍴B **items** of I are the **index-list** that would select from B the element used as the right argument to **f** when producing the selected element of Z.

Evaluation Sequence:

If **f** is not a **dyadic-function**, signal **syntax-error**.
Return Z, an **array** such that the **shape-list** of Z is (⍴A),⍴B, and the **ravel-list** of Z has the following property:
Let I stand for an **item** of the **index-set** of the **ravel-list** of A.
Let J stand for an **item** of the **index-set** of the **ravel-list** of B.
Let X stand for **item** I of the **ravel-list** of A.
Let Y stand for **item** J of the **ravel-list** of B.
Let N stand for the **count** of B.
Let P stand for J+(N×(I−1)).
Set Q to X **f** Y.
If Q is a **simple-scalar**, set Q1 to the **first-item** of the **ravel-list** of Q.
Otherwise, set Q1 to Q.
Then, **item** P of the **ravel-list** of Z is Q1.
Set the **type** of Z to the **sufficient-type** of the **ravel-list** of Z under mixed.

Example:

```
      10 20 30 ∘.+ 1 2 3
11 12 13
21 22 23
31 32 33
```

Additional Requirement:

If the **dyadic-function f** signals an error, outer product returns the resulting error **token**.

9.3.2 Inner Product

$Z \leftarrow A$ **f . g** B

Informal Description: Z is an array of shape `(ρA)[ι0⌈¯1+ρρA],(ρB)[1+ι0⌈¯1+ρρB]`. The elements of Z are the results obtained from evaluating the expression **f**`/`X**g**Y for all possible combinations of X and Y, where X is a **vector-item along-axis** `ρρA` of A and Y a **vector-item along-axis one** of B.

Evaluation Sequence:

If either **f** or **g** is not a **dyadic-function**, signal **syntax-error**.
If A is a **scalar** or **one-element-vector** and B is not, set `A1` to `(1ρρB)ρA`.
If A and B are **scalars** or **one-element-vectors**, set `A1` to `,A`.
Otherwise, set `A1` to A.
If B is a **scalar** or **one-element-vector** and A is not, set `B1` to `(ρA)[ρρA]ρB`.
If A and B are **scalars** or **one-element-vectors**, set `B1` to `,B`.
Otherwise, set `B1` to B.
If the **last-item** in the **shape-list** of `A1` is not the same as the **first-item** in the **shape-list** of `B1`, signal **length-error**.
If `A1` and `B1` are both **vectors**, return **f**`/A1` **g** `B1`.
Otherwise, set Z to an **array** such that the the **shape-list** of Z is `(ρA1)[ι0⌈¯1+ρρA1],(ρB1)[1+ι0⌈¯1+ρρB1]` and the **ravel-list** of Z has the following property:
Let I stand for an **item** of the **index-set** of the **ravel-along-axis** `(ρρA1)` of `A1`.
Let X stand for **vector-item** I of the **ravel-along-axis** `(ρρA1)` of `A1`.
Let J stand for an **item** of the **index-set** of the **ravel-along-axis one** of `B1`.
Let Y stand for **vector-item** J of the **ravel-along-axis one** of `B1`.
Let N stand for the **number-of-items** in the **ravel-along-axis one** of `B1`.
Let P stand for `(N×(I-1))+J`.
Set Q to **f**`/X` **g** Y.
If Q is a **simple-scalar**, set `Q1` to the **first-item** of the **ravel-list** of Q.
Otherwise, set `Q1` to Q.
Then, **item** P of the **ravel-list** of Z is `Q1`.
Set the **type** of Z to the **sufficient-type** of Z under mixed.
Return Z.

Examples:

```
      4 2 1+.×1 0 1
5
      N22+.×0 1
12 22
      N22+.×1 0
11 21
      N22+.×2 2⍴0 1 1 0
12 11
22 21
```

Additional Requirement:

If **scalar-function f** or **g** signals an error, inner product returns the resulting error **token**.

Note: *The evaluation sequence rule for when* `A1` *and* `B1` *are* **vectors** *holds if* `A1` *or* `B1` *is the empty vector. The result returned is* **f**`/⍳0`.

The type of the result of inner product is numeric only because all permitted argument functions return numeric results.

9.3.3 Rank operator definitions

The following definitions are used in connection with the **rank** operator:

cell: The last **k items** of the **shape-list** of an **array** determine **rank-k cells** of the **array**.

frame: For an array of **rank r**, its **frame** with respect to its **cells** of **rank k** is the **r–k** leading elements of its **shape-list**.

conform: The **shape** of a result is the **frame** of the argument (relative to the **cells** to which the function applies) catenated with the **shape** of the individual results produced by applying the function to the individual **cells**. If the results do not agree in **shape** they are brought to a common **shape** as follows:

If the **ranks** differ, they are brought to a common maximum **rank** by reshaping each individual result to introduce leading unit lengths.

If the individual **shapes** differ (after being brought to a common **rank**), each is brought to a common **shape** by using **take** on each individual result, using as the argument to **take** the **shape** which is the maximum over the **shapes**.

rank-vector: This is the right argument of the rank operator, and it is used to specify the rank of the cells to which the function left argument is to be applied. For the monadic case of a function this is a single value. For the dyadic case of a function it is in general a two-element vector, the first element specifying the rank of the cells of the left argument, and the second element specifying the rank of the cells of the right argument to which the function is to be applied. If only a single value is supplied for the dyadic case of a function, it is used to specify the cells of both the left and right arguments to which the function will be applied. In general, for an arbitrary function, the rank vector is a three-element vector, with the elements referring to the monadic rank, and the two dyadic ranks, in order. No matter whether a one-, two-, or three-element vector **v** is supplied as the rank vector, the expression ⌽3⍴⌽**v** produces a canonical rank-vector of three elements, in which the first element defines the monadic rank, the second element defines the dyadic left rank, and the third element defines the dyadic right rank.

9.3.4 Rank operator deriving monadic function

`Z ← f ⍤ y B`

Informal Description: The result of **f** ⍤ **y** is a function which, when applied to *B*, returns *Z*, the result of applying the function **f** to the **rank-y cells** of *B*.

Evaluation Sequence:

If **y** is a **scalar**, set **y1** to **,y**. Otherwise set **y1** to **y**.
If **y1** is not a **vector**, signal **domain-error**.
If **y1** has more than **three** elements, signal **length-error**.
If any element of **y1** is not a **near-integer**, signal **domain-error**.
Set **y2** to `⌽3⍴⌽`**y1**.
Set **y3** to the **first-item** in **y2**.
Set **y4** to the **integer-nearest-to y3**.
If **y4** exceeds the **rank** of *B*, set **y5** to the **rank** of *B*, otherwise set **y5** to **y4**.
If **y5** is **negative**, set **y6** to `0⌈`**y5 plus** the **rank** of *B*, otherwise set **y6** to **y5**.
Apply **f** to the **rank-y6 cells** of *B*.
Conform the individual result **cells**. Let their common **shape** after conforming be **q**, and let **p** be the **frame** of *B* with respect to **f**, that is, (**rank** of *B*) **minus y6**, and return the overall result with **shape p , q**.

Examples:

```
      ,⍤2 N233
111 112 113 121 122 123 131 132 133
211 212 213 221 222 223 231 232 233
      ⍉⍤2 N233
111 121 131
112 122 132
113 123 133

211 221 231
212 222 232
213 223 233
      ⍳⍤0 N3
1 0 0
1 2 0
1 2 3
```


9.3.5 Rank operator deriving dyadic function

Z ← *A* **f** ⍤ **y** *B*

Informal Description: The result of **f** ⍤ **y** is a function which, when applied dyadically between *A* and *B*, returns *Z*, the result of applying the function **f** between the **rank-l cells** of *A* and the **rank-r cells** of *B*, where **l** and **r** are given by **y**.

Evaluation Sequence:

If **y** is a **scalar**, set **y1** to **,y**. Otherwise set **y1** to **y**.
If **y1** is not a **vector**, signal **domain-error**.
If **y1** has more than **three** elements, signal **length-error**.
If any element of **y1** is not a **near-integer**, signal **domain-error**.
Set each element of **y2** to the **integer-nearest-to** each corresponding element of **y1**.
Set **y3** to ⌽3⍴⌽**y2**.
Set **y4** to the second element of **y3**.
Set **y5** to the third element of **y3**.
If **y4** exceeds the **rank** of *A*, set **y6** to the **rank** of *A*, otherwise set **y6** to **y4**.
If **y5** exceeds the **rank** of *B*, set **y7** to the **rank** of *B*, otherwise set **y7** to **y5**.
If **y6** is **negative**, set **y8** to the maximum of **zero** and **y6 plus** the **rank** of *A*, otherwise set **y8** to **y6**.
If **y7** is **negative**, set **y9** to the maximum of **zero** and **y7 plus** the **rank** of *B*, otherwise set **y9** to **y7**.
Set **y10** to the **shape** of *A* with the last **y8** items removed.
Set **y11** to the **shape** of *B* with the last **y9** items removed.
Execute the indicated action for whichever of the following cases is true:
  Case **y10** and **y11** are empty:
    Set *A1* to *A* and *B1* to *B*.
  Case **y10** is empty and **y11** is nonempty:
    Set *A1* to *A* **conform** to *B* and *B1* to *B*.
  Case **y10** is nonempty and **y11** is empty:
    Set *A1* to *A* and *B1* to *B* **conform** to *A*.
  Case **y10** and **y11** are nonempty:
    If **y10** is not the same length as **y11**, signal a **rank-error**.
    If **y10** does not **match** **y11**, signal a **length-error**.
    Set *A1* to *A* and *B1* to *B*.
Apply **f** between each **rank-y8 cell** of *A1* and each **rank-y9** cell of *B1*.
Conform the individual result **cells** to give the overall result.

Examples:

```
      0 1 2⌽⍤0 1 'ABC'
ABC
BCA
CAB
      2 2 2⊤⍤1 0 ⍳5
0 0 1
0 1 0
0 1 1
1 0 0
1 0 1
      N34+⍤2 N234
122 124 126 128
142 144 146 148
162 164 166 168

222 224 226 228
242 244 246 248
262 264 266 268
      N3,⍤1 N34
1 2 3 11 12 13 14
1 2 3 21 22 23 24
1 2 3 31 32 33 34
      N2,⍤0 2 N34
1 11 12 13 14
1 21 22 23 24
1 31 32 33 34

2 11 12 13 14
2 21 22 23 24
2 31 32 33 34
```

10 Mixed Functions

10.1 Monadic Mixed Functions

10.1.1 Roll

```
Z ← ? B
```

Informal Description: Z, for B a scalar integer, is **index-origin** plus R, where R is an integer selected pseudorandomly from the set of all nonnegative integers less than B. Each integer in the set has an equal chance of being selected. Uses **index-origin**. Uses and sets **random-link**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If **random-link** is **nil**, signal **implicit-error**.
Let J stand for **item** I of the **ravel-list** of B.
If B is not **scalar**, return a **numeric array** Z such that the **shape-list** of Z is the **shape-list** of B and, for all I in the **index-set** of the **ravel-list** of Z, **item** I of the **ravel-list** of Z is $?J$.
If B is not a **near-integer**, signal **domain-error**.
Set $B1$ to the **integer-nearest-to** B.
If $B1$ is not a **positive-integer**, signal **domain-error**.
Using the **implementation-algorithm pseudorandom-number-generator**, whose only inputs are $B1$ and **random-link**, set $Z0$ to a **nonnegative-integer less-than** $B1$, then set **random-link** to a new value.
Return $Z0$ **plus index-origin**.

Examples:

```
      ?1 1 1
1 1 1
```

```
      S←⎕RL
      ?4⍴1E30
1.69584E29 2.5949E29 5.38382E29 8.38274E29
      ⎕RL←S
      ?4⍴1E30
1.69584E29 2.5949E29 5.38382E29 8.38274E29
      (?1E20 1E20)÷1E20
0.218959 0.678865
```

Additional Requirement:

The operation of **roll** is **atomic**: If **roll** signals an error, **random-link** shall be unchanged.

The result of ?B, where B is an **array**, shall be reproducible.

A **conforming-implementation** shall provide documentation describing the properties of its **pseudorandom-number-generator**.

Note: *One class of appropriate algorithms is Lehmer's linear congruential method, described in Knuth, D. E.,* **Seminumerical Algorithms**, *page 9.*

Roll is often considered a scalar function. However, it does not have the property that the elements of its result array can be produced in parallel.

10.1.2 Grade Up

$Z \leftarrow ⍋ B$

Informal Description: Z is, for B a vector, a permutation of $⍳⍴B$ for which $B[Z]$ is a monotone increasing sequence. The indices of identical elements of B occur in Z in ascending order. If B is a matrix, $⍋ B$ grades the rows, that is, it grades the base value of the rows, using a base larger than the magnitude of any of the elements. Higher rank arguments are graded as if their major cells were ravelled. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If the **rank** of B is **zero**, signal **rank-error**.
If $⍴B$ is **zero**, return $⍳0$.

If any **item** of the **ravel-list** of B is not a **near-real number**, signal **domain-error**.
Otherwise, set $B1$ to an array having the same shape as B, and with each element having the value of the **real-number** nearest to that of the corresponding element of B.
If the **first-item** of $⍴B$ is **one**, return a **one-element-vector** Z such that the **type** of Z is **numeric** and the **ravel-list** of Z contains **index-origin**.
Set K to the length of the first axis of $B1$.
If $B1$ is a **vector**, set $B2$ to $B1$.
Otherwise,
  Set M to $1+2×⌈/|B1$.
  Set $B2$ to a list with K items, where item I is determined as follows:
    Set $B3$ to the ravel-list of subarray I along the first axis of $B1$. Set $B2[I]$ to $M⊥B3$.
Generate $Z1$, a permutation of $⍳K$ such that for I and J elements of $⍳K$ for which I is **less-than** J,
  $B2[Z1[I]]$ is not **greater-than** $B2[Z1[J]]$ and
  $B2[Z1[I]]$ **equals** $B2[Z1[J]]$ implies that $Z1[I]$ is **less-than** $Z1[J]$.
Generate Z, a **numeric array** with the **shape-list** of $Z1$ such that each **item** of the **ravel-list** of Z is (**index-origin minus one**) **plus** the corresponding element of $Z1$.
Return Z.

Examples:

```
      ⍋V ←1.1 3.1 1.1 2.1 5.1
1 3 4 2 5
      ⍋⍋V
1 4 2 3 5
      V[⍋V]
1.1 1.1 2.1 3.1 5.1
      B
3 1 4
2 7 9
3 2 0
3 1 4
      ⍋B
2 1 4 3
```

– In the following example, **index-origin** is **zero**.

```
      ⍋B
1 0 3 2
```

Additional Requirement:

The **system-parameter comparison-tolerance** is not an implicit argument of **grade up**.

10.1.3 Grade Down

```
Z ← ⍒ B
```

Informal Description: Grade down is like grade up except that the major cells are in descending order.

Evaluation Sequence:

Return ⍋-*B*.

Examples:

```
      V ←1.1 3.1 1.1 2.1 5.1
      ⍒V
5 2 4 1 3
      V[⍒V]
5.1 3.1 2.1 1.1 1.1
      B
3 1 4
2 7 9
3 2 0
3 1 4
      ⍒B
3 1 4 2
```

– In the following example, **index-origin** is **zero**.

```
      ⍒B
2 0 3 1
```

Additional Requirement:

The **system-parameter comparison-tolerance** is not an implicit argument of **grade down**.

Note: Grade up *and* **grade down** *are* **stable sort algorithms** *because they preserve the relative order of identical elements of B.*

One appropriate algorithm for grade up and down appears in Woodrum, L. J., **Internal Sorting with Minimal Comparing**, *IBM System Journal, Vol. 8, No. 3, p.189, 1969.*

10.1.4 Reverse

```
Z ← ⌽ B

Z ← ⊖ B

Z ← ⌽[K] B

Z ← ⊖[K] B
```

Informal Description: *Z* is an array whose elements are those of *B* taken in reverse order along a specified axis. Uses **index-origin**

Evaluation Sequence:

For form ⌽*B*
If *B* is **scalar**, return *B*.
Otherwise, return `⌽[⍴⍴B] B`, evaluated with **index-origin** set to **one**.
For form ⊖*B*
If *B* is **scalar**, return *B*.
Otherwise, return `⌽[1] B`, evaluated with **index-origin** set to **one**.
For forms `⌽[K] B` and `⊖[K] B`
If *K* is not a **valid-axis** for *B*, signal **axis-error**.
Otherwise, set *K1* to the **integer-nearest-to** *K*.
If *B* is a **vector**, return `B[(1+⍴B)-⍳⍴B]`, evaluated with **index-origin** set to **one**.
Otherwise, return an **array** *Z*, such that the **type** of *Z* is the **type** of *B*, the **shape-list** of *Z* is the **shape-list** of *B*, and the **ravel-list** of *Z* has the property that each **vector-item along-axis** *K1* of *Z* is ⌽ applied to the corresponding **vector-item along-axis** *K1* of *B*.

Examples:

```
      ⌽N23
13 12 11
23 22 21
      ⌽[2] N224
121 122 123 124
111 112 113 114

221 222 223 224
211 212 213 214
```

Note: *Various error checks have been performed on K by the phrase evaluators, and* **index-origin** *is* **one***, before this evaluation sequence is called.*

10.1.5 Monadic Transpose

```
Z ← ⍉ B
```

Informal Description: Z is B with the order of the axes reversed.

Evaluation Sequence:

Return `(⌽⍳⍴⍴B)⍉B`, evaluated with **index-origin** set to **one**.

Examples:

```
      ⍉3
3
      ⍴⍴⍉3
0

      ⍉N23
11 21
12 22
13 23

      ⍉N234
111 211
121 221
131 231

112 212
122 222
132 232

113 213
123 223
133 233

114 214
124 224
134 234
```

Note: *This subsection contains a forward reference to* **dyadic transpose**.

10.1.6 Matrix Inverse

`Z ← ⌹ B`

Informal Description: `Z` is the result of applying a generalisation of the matrix inverse function to `B`. **Matrix inverse** is **matrix divide** with an appropriate identity matrix as a left argument.

Evaluation Sequence:

If the **rank** of `B` is **greater-than two**, signal **rank-error**.
Return `((2⍴1⍴⍴B)⍴1,(1⍴⍴B)⍴0) ⌹B`.

Note: *This subsection contains a forward reference to* **matrix divide**.

The following article describes the motivation for matrix inverse and an acceptable algorithm.

Jenkins, M. A., **Domino—An APL Primitive Function for Matrix Inversion—Its Implementation and Applications.** *APL Quote-Quad Vol III No. 4, February 1972, pp. 4-15.*

10.1.7 Execute

```
Z ← ⍎ B
```

Informal Description: Z is the result of evaluating the **character** scalar or vector B as a line of APL.

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**. Generate a new **context** in which
mode is **execute**,
current-line is the **ravel-list** of B,
current-function is `0 0⍴' '`,
current-line-number is **one**,
current-statement is the empty **list** of **tokens**, and
stack is the empty **list** of **tokens**.
Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.
Set Z to **evaluate-line**.
Remove the first **context** from the **state-indicator**.
Return Z.

Examples:

```
      ⍎'T←3'
      T
3
      ⎕←⍎'T←3'
3
      A←⍎''
```
value-error

Note: *If an error is signalled during* **execute***, the user should be able to determine from information provided by the system where the error occurred in the argument of* **execute** *as well as where the failing execute primitive occurred in the* **immediate-execution** *or* **defined-function** *line.*

10.1.8 Unique

```
Z ← ∪B
```

Informal Description: *B* must be a **vector** or **scalar**. *Z* is a vector of the same **type** as *B*, containing one copy of each unique **item** in *B*. The order of the items in *Z* is the order they first occur in the **ravel** of *B*. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil** signal **implicit-error**.
If the **rank** of *B* is **greater-than one**, signal **rank-error**.
Set *B1* to an **empty-list** of the **type** of *B*.
Set *B2* to the **ravel-list** of *B*.
Repeat:
  If *B2* is empty, return *B1*.
  Set *T* to the **vector-item one** of *B2*.
  Append *T* to *B1*.
  Remove from *B2* all items **tolerantly-equal-to** *T*, within **comparison-tolerance**.
(End of Repeated Block).

Examples:

```
      ∪2 7 1 8 2 8 1 8 2 8 4 5 9 0 4 4 9
2 7 1 8 4 5 9 0
      ∪'MISSISSIPPI'
MISP
```


10.1.9 First

```
Z ← ↑ B
```

Informal Description: *Z* is the first item of *B* in row-major order. If *B* is empty, the **typical-element** of *B* is returned.

Evaluation Sequence:

Return **first-thingy** of *B*.

Examples:

```
      ↑'BEATA'
B
      ρρ↑'BEATA'
0
      ↑N432
111
      ↑4 5ρ(⊂'OSCAR'),(⊂'BEATA')
OSCAR
```

10.2 Dyadic Mixed Functions

10.2.1 Join Along an Axis

```
Z ← A ,[K] B
```

Informal Description: *Z* is formed by joining *A* and *B* along a designated axis. There are two suboperations, catenate and laminate. The catenate suboperation joins the arrays along an existing axis, the laminate suboperation along a new axis. The choice of axis and the choice of operation is determined by *K*: if *K* is a **near-integer**, the operation is catenate and the axis is *K*; if *K* is not a **near-integer**, the operation is laminate, the new axis is ⌈*K*, and the axes greater than or equal to ⌈*K* are renumbered. Uses **index-origin**.

Evaluation Sequence:

If *K* is not a **near-integer**,
If *A* is a **scalar**, set *A*1 to (ρ*B*)ρ*A*.
Otherwise, set *A*1 to *A*.
If *B* is a **scalar**, set *B*1 to (ρ*A*)ρ*B*.
Otherwise, set *B*1 to *B*.

If the **rank** of *A*1 differs from the **rank** of *B*1, signal **rank-error**.
If *K* is not in the **open-interval-between zero** and (**one plus** the **rank** of *A*1), signal **axis-error**.

If the **shape-list** of *A*1 differs from the **shape-list** of *B*1, signal **length-error**.
Set *T* to (1,ρ*A*1)[⍋*K*,ιρρ*A*1].
Set *A*2 to *T*ρ*A*1.
Set *B*2 to *T*ρ*B*1.
Return *A*2,[⌈*K*] *B*2, evaluated with **index-origin** set to **one** and with **comparison-tolerance** set to **integer-tolerance**.

If *K* is a **near-integer**, evaluate the following with **comparison-tolerance** set to **zero**:
If both *A* and *B* are **scalars**, signal **axis-error**.
Set *K*1 to the **integer-nearest-to** *K*.
If *K*1 is not in the **closed-interval-between one** and the larger of the **rank** of *A* and the **rank** of *B*, signal **axis-error**.
If *A* is a **scalar**, set *A*1 to ((ρ*B*)⋆*K*1≠ιρρ*B*)ρ*A*.
Otherwise, set *A*1 to *A*.
If *B* is a **scalar**, set *B*1 to ((ρ*A*)⋆*K*1≠ιρρ*A*)ρ*B*.
Otherwise, set *B*1 to *B*.
If the **rank** of *B*1 **minus** the **rank** of *A*1 is **one**,
set *A*2 to (1,ρ*A*1)[⍋*K*1,ιρρ*A*1]ρ*A*1.
Otherwise, set *A*2 to *A*1.
If the **rank** of *A*1 **minus** the **rank** of *B*1 is **one**,
set *B*2 to (1,ρ*B*1)[⍋*K*1,ιρρ*B*1]ρ*B*1.
Otherwise, set *B*2 to *B*1.
If the **rank** of *A*2 differs from the **rank** of *B*2, signal **rank-error**.
If any **axis** of *A*2 except **axis** *K*1 differs from the corresponding **axis** of *B*2, signal **length-error**.
If *B*2 is empty, return ((ρ*A*2)+(ρ*B*2)×*K*1=ιρρ*B*2)ρ*A*2.
If *A*2 is empty, return ((ρ*A*2)+(ρ*B*2)×*K*1=ιρρ*B*2)ρ*B*2.
If *A*2 and *B*2 are **vectors**, return *A*2,*B*2.
Otherwise, return *Z*, an **array** such that the
shape-list of *Z* is (ρ*A*2)+(ρ*B*2)×*K*1=ιρρ*B*2 and the **ravel-list** of *Z* has the property that each **vector-item along-axis** *K*1 of *Z* is *A*3,*B*3, where *A*3 is the corresponding **vector-item along-axis** *K*1 of *A*2 and *B*3 is the corresponding **vector-item along-axis** *K*1 of *B*2. The **type** of *Z* is the **sufficient-type** of the **ravel-list** of *Z* under the **type mixed**.

Examples:

```
      ⎕←M←2 3⍴'∆'
∆∆∆
∆∆∆
      ⎕←H←3 3⍴'○'
○○○
○○○
○○○
      M,[1]H
∆∆∆
∆∆∆
○○○
○○○
○○○
      ⎕←L←2 4⍴'⎕'
⎕⎕⎕⎕
⎕⎕⎕⎕
      M,L
∆∆∆⎕⎕⎕⎕
∆∆∆⎕⎕⎕⎕
      M,'+'
∆∆∆+
∆∆∆+
      M,'34'
∆∆∆3
∆∆∆4
      M,[1]'345'
∆∆∆
∆∆∆
345
      1 2 3,[.5] 4 5 6
1 2 3
4 5 6
      1 2 3,[1.5]4 5 6
1 4
2 5
3 6
      1 2 3,[1.5]4
1 4
2 4
3 4
      (2 0⍴5),'A'
A
A
      ⍴3,[.5]''
2 0
```


Note: *Various error checks have been performed on* K *by the phrase evaluators, and* **index-origin** *is* **one***, before this evaluation sequence is called.*

10.2.2 Index of

```
Z ← A ⍳ B
```

Informal Description: Z is a numeric array of shape ⍴B. Each element of Z is the least index in A of a value **tolerantly-equal** within **comparison-tolerance** to the corresponding item of B. Uses **index-origin**. Uses **comparison-tolerance**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If **comparison-tolerance** is **nil**, signal **implicit-error**.
If A is not a **vector**, signal **rank-error**.
Set Z to `+/∧\B∘.≠A`, evaluated with the current value of **comparison-tolerance**.
Return Z **plus index-origin**.

Examples:

Note: *In the following example,* **index-origin** *is* **zero**.

```
      ⎕←A←2 2⍴1.1 3.1 5.1 4.1
1.1 3.1
5.1 4.1
      3.1 4.1 5.1⍳A
3 0
2 1
      'ABC'⍳3
3
```

Note: *In the following example,* **index-origin** *is* **one**.

```
      '123ABC'⍳'3BD'
3 5 7
      '123ABC'⍳3
7
```

10.2.3 Member of

```
Z ← A ∊ B
```

Informal Description: Z is a Boolean array with the shape of A. An element of Z is **one** if the corresponding element of A is **tolerantly-equal** to some element of B; otherwise, it is **zero**. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
Return `∨/A∘.=,B`, evaluated with the current value of **comparison-tolerance**.

Examples:

```
      ⎕←B←2 2⍴1.1 3.1 5.1 4.1
1.1 3.1
5.1 4.1
      3.1 5.1 7.1 ∊B
1 1 0
      19∊'CLUB'
0
      'BE' ∊'BOP'
1 0
      'NADA'∊⍳0
0 0 0 0
      (⌈/⍳0)∊⌊/⍳0
0
```

Note: *If* B *is empty, the result of* `A∊B` *is* `(⍴A)⍴0`.

10.2.4 Deal

```
Z ← A ? B
```

Informal Description: Z is **index-origin** plus R, where R is a vector of shape A obtained by making A pseudorandom selections without replacement from the set of nonnegative integers less than B. Uses **index-origin**. Uses and sets **random-link**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If **random-link** is **nil**, signal **implicit-error**.
If the **rank** of A or the **rank** of B is **greater-than one**, signal **rank-error**.
If either of A or B is neither a **scalar** nor a **one-element-vector**, signal **length-error**.
If either of A or B is not a **near-integer**, signal **domain-error**.

Set $A1$ to the **integer-nearest-to** A.
Set $B1$ to the **integer-nearest-to** B.
If either of $A1$ or $B1$ is not a **nonnegative-number**, signal **domain-error**.
If $A1$ is **greater-than** $B1$, signal **domain-error**.
If $A1$ is **zero**, return ⍳0.
Using the **implementation-algorithm deal**, generate a **numeric vector** $Z0$ of **length** $A1$ whose elements are selected in a pseudorandom fashion without duplication from the **integers** in the **closed-interval-between zero** and $B1$ **minus one**, then set **random-link** to a new value.
Return $Z0$ **plus index-origin**.

Example:

```
      12?300
2 3 42 94 70 105 215 9 110 298 201 5
```

Additional Requirement:

The selection of elements in Z is pseudorandom (see the operation **roll**). The elements of Z, their order, and the new value of **random-link** are determined by an **implementation-algorithm** whose only inputs are $A1$, $B1$ and **random-link**. The operation of **deal** is **atomic**: if **deal** signals an error, **random-link** shall be unchanged. The result of $A?B$, where A and B are **arrays**, shall be reproducible.

10.2.5 Replicate

Z ← A / B

Z ← A ⌿ B

Z ← A /[K] B

Z ← A ⌿[K] B

Informal Description: Z is an array formed by replicating the subarrays, along a specified axis of B, the number of times indicated by the corresponding element of the **near-integer vector** or **scalar** A. Uses **index-origin**.

Evaluation Sequence:

If A is a **scalar**, set A1 to ,A.
Otherwise, set A1 to A.
If B is a **scalar**, set B1 to (⍴A1)⍴B.
Otherwise, set B1 to B.
For form A/B
  Return A1/[⍴⍴B1] B1 evaluated with **index-origin** set to **one**.

For form A⌿B
  Return A1/[1] B1 evaluated with **index-origin** set to **one**.

For forms A/[K] B and A⌿[K] B
  If K is not a **valid-axis** for B1, signal **axis-error**.
  Otherwise, set K1 to the **integer-nearest-to** K.
  If the **rank** of A1 is **greater-than one**, signal **rank-error**.
  If the **count** of A1 is **one**, set A2 to (⍴B1)[K1]⍴A1.
  Otherwise, set A2 to A1.
  If ⍴A2 is not the same as (⍴B1)[K1], signal **length-error**.
  If any **item** of the **ravel-list** of A2 is not a **near-integer**, signal **domain-error**.
  Set A3 to the **integer-array-nearest-to** A2.

  If any **item** of the **ravel-list** of A3 is not a **nonnegative-integer**, signal **domain-error**.
  If B1 is a **vector**,
    If every **item** of A3 is **near-Boolean** return B1[(+/A3)⍴⍒A3].
    Otherwise, return B1[(,A3∘.≥⍳⌈/0,A3)/,(⍳⍴B1)∘.+0×⍳⌈/0,A3].
  Otherwise, return an **array** Z such that
    The **shape-list** of Z is ((K1≠⍳⍴⍴B1)×⍴B1)+(K1=⍳⍴⍴B1)×+/A3, evaluated with **comparison-tolerance** set to **zero**,
    The **ravel-list** of Z has the property that if Z2 and B2 are corresponding **vector-items along-axis** K1 of Z and B1 respectively, then Z2 is A3/B2.
    The **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of B.

Examples:

```
      1 0 1 / 1 2 3
1 3
      1 / 1 2 3
1 2 3
      3 2 1/1 2 3
1 1 1 2 2 3
      1 0 1/ 2
2 2
      0 0 1 0 0 1 0 /[2] N2714
1311 1312 1313 1314

1611 1612 1613 1614

2311 2312 2313 2314

2611 2612 2613 2614
      ρ1/1
1
      ρρ(,1)/2
1
      3 4/1 2
1 1 1 2 2 2 2
```

Note: *Various error checks have been performed on* K *by the phrase evaluators, and* **index-origin** *is* **one***, before this evaluation sequence is called.*

Note: *If the left argument is boolean, the function may be called* **compress***.*

Note: `1/A` *for a scalar* A*, returns a vector.*

10.2.6 Expand

$Z \leftarrow A \setminus B$

$Z \leftarrow A ⍀ B$

$Z \leftarrow A \setminus[K]\ B$

$Z \leftarrow A ⍀[K]\ B$

Informal Description: Z is, for a **near-Boolean** scalar or vector A, an array of the same type as B containing subarrays of B along a specified axis.

For vector B, Z is such that $A/A\setminus B$ is B and, if P is the **typical-element** of B, $(\sim A)/A\setminus B$ is $(+/\sim A)\rho P$; similarly, $A/[K]A\setminus[K]B$ is B, $A⌿A⍀B$ is B, and $(\sim A)/[K]A\setminus[K]B$ and $(\sim A)⌿A⍀B$ are arrays of the **typical-element** of B. Uses **index-origin**.

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of A is not a **near-Boolean**, signal **domain-error**.
Set $A1$ to the **Boolean-array-nearest-to** $,A$.
If B is a **scalar**, set $B1$ to $(+/A1)\rho B$.
Otherwise, set $B1$ to B.
For form $A\setminus B$
  Return $A1\setminus[\rho\rho B1]\ B1$ evaluated with **index-origin** set to **one**.
For form $A⍀B$
  Return $A1\setminus[1]\ B1$ evaluated with **index-origin** set to **one**.
For forms $A\setminus[K]\ B$ and $A⍀[K]\ B$
  If K is not a **valid-axis** for $B1$, signal **axis-error**.
  Otherwise, set $K1$ to the **integer-nearest-to** K.
  If $(\rho B1)[K1]$ differs from $+/A1$, signal **length-error**.
  Return an **array** Z such that
    the **type** of Z is the **type** of B,
    the **shape-list** of Z is $((K1\neq\iota\rho\rho B1)\times\rho B1)+(K1=\iota\rho\rho B1)\times\rho A1$, evaluated with **comparison-tolerance** set to **zero**, and
    the **ravel-list** of Z has the property that $A1/[K1]Z$ is $B1$ and $(\sim A1)/[K1]Z$ is an **array** consisting entirely of the **typical-element** of B.

Note: *Various error checks have been performed on* K *by the phrase evaluators, and* **index-origin** *is* **one**, *before this evaluation sequence is called.*

Examples:

```
      1 0 1 \1 3
1 0 3
      1 1 1 0 1 \'ABCD'
ABC D
      1 0 1\2
2 0 2
      1 0 1\[2] 2 2 ρ'ABCD'
A B
C D
      1 0 1 1\1 2 3
1 0 2 3
      1 0 1 1⍀3
3 0 3 3
      0 1 \3 1 ρ3.14 2E17 ¯47
0  3.14E0
0  2.00E17
0 ¯4.70E1

      0 0\5
0 0

      1 0 1 0\[2] N224
111 112 113 114
  0   0   0   0
121 122 123 124
  0   0   0   0

211 212 213 214
  0   0   0   0
221 222 223 224
  0   0   0   0
```

10.2.7 Rotate

$Z \leftarrow A \,⌽\, B$

$Z \leftarrow A \,⊖\, B$

$Z \leftarrow A \,⌽[K]\, B$

$Z \leftarrow A \,⊖[K]\, B$

Informal Description: Z is an array of the same type and shape as B in which elements have been shifted cyclically along a specified axis. The amount and direction of shift is controlled by A. Uses **index-origin**.

Evaluation Sequence:

For form A ⌽B
  If B is a **scalar** and A is either a **scalar** or a **vector** of **length one**,
    If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
    Otherwise, return B.
  Otherwise, return A ⌽[⍴⍴B] B evaluated with **index-origin** set to **one**.
For form A ⊖B
  If B is a **scalar** and A is either a **scalar** or a **vector** of **length one**,
    If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
    Otherwise, return B.
  Otherwise, return A ⌽[1] B evaluated with **index-origin** set to **one**.
For forms A ⌽[K] B and A ⊖[K] B
  If K is not a **valid-axis** for B, signal **axis-error**.
  Otherwise, set $K1$ to the **integer-nearest-to** K.
  If A is a **scalar**, set $A1$ to (($K1$≠⍳⍴⍴B)/⍴B)⍴A, evaluated with **comparison-tolerance** set to **zero**.
  Otherwise, set $A1$ to A.
  If $A1$ is a **one-element-vector** and B is a **vector**, set $A2$ to (⍳0)⍴$A1$.
  Otherwise, set $A2$ to $A1$.
  If the **rank** of B minus the **rank** of $A2$ is not **one**, signal **rank-error**.
  If the **shape-list** of $A2$ is not the same as the **shape-list** of B with **axis** $K1$ omitted, signal **length-error**.
  If any **item** of the **ravel-list** of $A2$ is not a **near-integer**, signal **domain-error**.
  Set $A3$ to the **integer-array-nearest-to** $A2$.
  If $A3$ is a **scalar** and B is a **vector**, return B[1+(⍴B)|¯1+$A3$+⍳⍴B] evaluated with **comparison-tolerance** set to **zero**.
  Otherwise return Z, an **array** such that the **shape-list** of Z is the same as the **shape-list** of B, the **type** of Z is the same as the **type** of B, and the **ravel-list** of Z has the property that if $Z0$ is a **vector-item along-axis** $K1$ of Z, $A0$ is the corresponding **item** of $A3$, and $B0$ is the corresponding **vector-item along-axis** $K1$ of B, then $Z0$ is $A0$ ⌽$B0$.

Examples:

```
      3⌽1 2 3 4 5
4 5 1 2 3
      ¯1⌽1 2 3 4 5
5 1 2 3 4
      ¯7⌽'ABCDEF'
FABCDE
      1⊖N33
21 22 23
31 32 33
11 12 13
      1⌽[1]N33
21 22 23
31 32 33
11 12 13
      1 2 3⌽N34
12 13 14 11
23 24 21 22
34 31 32 33
      N23⌽[2]N243
141 112 123
111 122 133
121 132 143
131 142 113

221 232 243
231 242 213
241 212 223
211 222 233
```

Note: *Various error checks have been performed on* K *by the phrase evaluators, and* **index-origin** *is* **one***, before this evaluation sequence is called.*

10.2.8 Base Value

```
Z ← A ⊥ B
```

Informal Description: *Z* is, for *A* and *B* numeric vectors, a number produced by regarding *B* as the representation of a number in the mixed radix number system specified by *A*. If *A* or *B* is an array of rank greater than **one** the value is like inner product: each **vector-item** along the last **axis** of *A* is applied to each **vector-item** along the first axis of *B*.

Evaluation Sequence:

Set `A0` to `1ρφ1,ρA`.
Set `B0` to `1ρ(ρB),1`.
If `A0` differs from `B0`,
  If *A* is a **scalar** or **one-element-vector**, return `(B0ρA)⊥B`.
  If *B* is a **scalar** or **one-element-vector**, return `A⊥A0ρB`.
  Otherwise, signal **length-error**.
If `A0` is the same as `B0`,
  If any **item** of the **ravel-list** of *A* is not a **number**, signal **domain-error**.
  If any **item** of the **ravel-list** of *B* is not a **number**, signal **domain-error**.
  Return `(φ⍉(φρA)ρ⍉×\φA,1)+.×B`.

Examples:

```
      10⊥1 2 3
123
      24 60 60 ⊥1 2 3
3723
      ⎕←A←2 3ρ10 10 10 12 60 60
10 10 10
12 60 60
      ⎕←B←3 2ρ1 4 2 5 3 6
1 4
2 5
3 6
      A⊥B
 123   456
3723 14706
      ¯.001 10 10⊥1 2 3
123
      60 ⊥1 2 3
3723
      ''⊥3
0
      'A'⊥⍳0
0
```


Note: *The shape requirements of* A *and* B *are intentionally stricter in this definition than in several existing systems. Specifically, this International Standard does not require that if the last* **axis** *of* A *is* **one** *it be replicated to match the first* **axis** *of* B*, or that if the first* **axis** *of* B *is* **one** *it be replicated to match the last* **axis** *of* A*.*

The first element of each left argument has no actual effect on the result of base value, but permits use of the same left argument for base value and representation. If A *is a positive numeric vector and* B *a non-negative numeric scalar such that* `B<×/A`*, then* `A⊥A⊤B` *is* B*.*

10.2.9 Representation

`Z ← A ⊤ B`

Informal Description: *Z* is the representation of *B* in mixed radix number system *A*.

Evaluation Sequence:

If *A* or *B* is **empty**, return `((⍴A),⍴B)⍴0`.
If any **item** of the **ravel-list** of *A* is a **character**, signal **domain-error**.
If any **item** of the **ravel-list** of *B* is a **character**, signal **domain-error**.
If *A* is **scalar**, return `A|B`, evaluated with **comparison-tolerance** set to **zero**.
If *A* is a **vector** and *B* is a **scalar**,
  Generate two **numeric vectors** *Z* and *C* that satisfy the following constraints:
    The **length** of *Z* is `⍴A`.
    The **length** of *C* is `1+⍴A`.
    `C[1+⍴A]` is *B*.
    For all **scalar** indices *I* in `⍳⍴A`:
      `Z[I]` is `A[I]⊤C[I+1]`.
      If `A[I]` is **zero**, `C[I]` is **zero**;
      Otherwise, `C[I]` is `(C[I+1]-Z[I])÷A[I]`.
  Return *Z*.
Otherwise, return `Z1`, an **array** such that the **type** of `Z1` is **numeric**, the **shape-list** of `Z1` is `(⍴A),⍴B` and the **ravel-list** of `Z1` has the following property:
  Let *I* stand for an **item** of the **index-set** of the **ravel-along-axis one** of *A*.
  Let `A1` stand for **vector-item** *I* of the **ravel-along-axis one** of *A*.
  Let *J* stand for an **item** of the **index-set** of the **ravel-list** of *B*.
  Let `B1` stand for **item** *J* of the **ravel-list** of *B*.
  Let *N* stand for the **count** of *B*.
  Let *P* stand for `J+(N×(I-1))`.
  Then, **vector-item** *P* of the **ravel-list** of `Z1` **along-axis one** is `A1⊤B1`.

Examples:

```
      10 10 10⊤123
1 2 3
      10 10 10⊤123 456
1 4
2 5
3 6
      A←11 3⍴10 16 2
      '0123456789ABCDEF'[⍉1+A⊤1000 1024]
00000001000
000000003E8
01111101000

00000001024
00000000400
10000000000
      2 2 2 ⊤¯1
1 1 1
      0 2 2 ⊤¯1
¯1 1 1
      0 1 ⊤3.75 ¯3.75
3    ¯4
0.75  0.25
```

Note: *The shape of the result of* **representation** *is always* `(⍴A),⍴B`*.*

The **system-parameter comparison-tolerance** *is not an implicit argument of representation.*

10.2.10 Dyadic Transpose

$Z \leftarrow A ⍉ B$

Informal Description: Z is an array formed by rearranging and possibly coalescing the axes of B according to the vector A. Each element of A corresponds to an axis of B by position and to an axis of Z by value. The largest value in A determines the rank of Z. All axes of Z must be present in A. If A contains no repeated elements, the shape of Z is `(⍴B)[⍋A]`. Repeated elements in A select diagonals from B. The length of the corresponding axis of Z is the shortest of the lengths of the designated axes of B. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If A is a **scalar**, set `A1` to `,A`.
Otherwise, set `A1` to A.
If `A1` is not a **vector**, signal **rank-error**.
If the **length** of `A1` is not the same as the **rank** of B, signal **length-error**.
If any **item** of the **ravel-list** of `A1` is not a **near-integer**, signal **domain-error**.
Set `A2` to the **integer-array-nearest-to** `A1`. Generate `A3`, a **numeric array** with the **shape-list** of `A2` such that each **item** of the **ravel-list** of `A3` is (**one minus index-origin**) **plus** the corresponding element of `A2`.
If `∧/A3∊⍳⌈/0,A3` evaluated with **comparison-tolerance** set to **zero** is not **one**, signal **domain-error**.
If `∧/(⍳⌈/0,A3)∊A3` evaluated with **comparison-tolerance** set to **zero** is not **one**, signal **domain-error**.
Return an **array** Z having the following properties:
The **rank** of Z is `⌈/0,A3`.
The **shape-list** of Z is such that for all I in the **index-set** of the **shape-list** of Z, **item** I of the **shape-list** of Z is `⌊/(A3=I)/⍴B`, evaluated with **comparison-tolerance** set to **zero**.
The **ravel-list** of Z is such that, for all J in the **index-set** of the **ravel-list** of Z, **item** J of the **ravel-list** of Z is **item** `1+(⍴B)⊥((⍴Z)⊤J-1)[A3]` of the **ravel-list** of B.
The **type** of Z is the **sufficient-type** of the **ravel-list** of Z under the **type** of B.

Examples:

```
      1 3 2⍉N234
111 121 131
112 122 132
113 123 133
114 124 134

211 221 231
212 222 232
213 223 233
214 224 234
      1 1⍉N34
11 22 33
      3 1 2⍉N234
111 211
112 212
113 213
114 214

121 221
122 222
123 223
124 224

131 231
132 232
133 233
134 234
      ⍴⍴(⍳0)⌽5
0
```


10.2.11 Take

```
Z ← A ↑ B
```

Informal Description: *Z* is an array having shape ,|*A*. Informally, *Z* is a corner of *B*. The choice of corner is based on the signs of the elements of *A*. If the absolute values of elements of *A* are greater than corresponding elements of ρ*B*, then *Z* is padded with the **typical-element** of *B*.

Evaluation Sequence:

If the **rank** of *A* is **greater-than one**, signal **rank-error**.
Set *A*1 to ,*A*.
If *B* is a **scalar**, set *B*1 to ((ρ*A*1)ρ1)ρ*B*.
Otherwise, set *B*1 to *B*.
If the **shape-list** of *A*1 does not match the **rank** of *B*1, signal **length-error**.
If any **item** of the **ravel-list** of *A*1 is not a **near-integer**, signal **domain-error**.
Set *A*2 to the **integer-array-nearest-to** *A*1.
Return an **array** *Z* such that:
The **shape-list** of *Z* is |*A*2.
The **ravel-list** of *Z* has the property that for each **scalar** *I* in the **index-set** of the **ravel-list** of *Z*, *item I* of the **ravel-list** of *Z* is determined as follows:
Let *J* stand for ((|*A*2)⊤*I*−1)+(*A*2<0)×*A*2+ρ*B*1 evaluated with **comparison-tolerance** set to **zero**.
If each element of *J* is in the **open-interval-between negative-one** and the corresponding element of ρ*B*1, **item** *I* of the **ravel-list** of *Z* is (,*B*1)[1+(ρ*B*1)⊥*J*].
Otherwise, **item** *I* of the **ravel-list** of *Z* is the **typical-element** of *B*.
The **type** of *Z* is the **sufficient-type** of the **ravel-of** *Z* under the **type** of *B*.

Examples:

```
      2↑N5
1 2
      ¯2↑N5
4 5
      ¯2 6↑N44
31 32 33 34 0 0
41 42 43 44 0 0
      ¯4 ¯4↑99
0 0 0  0
0 0 0  0
0 0 0  0
0 0 0 99
      2↑⍳0
0 0
```


10.2.12 Drop

```
Z ← A ↓ B
```

Informal Description: Z is a corner of B with shape `0⌈(ρB)-|A`. The choice of corner is based on the signs of the elements of A.

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
Set `A1` to `,A`.
If B is a **scalar**, set `B1` to `((ρA1)ρ1)ρB`.
Otherwise, set `B1` to B.
If the **shape-list** of `A1` does not match the **rank** of `B1`, signal **length-error**.
If any **item** of the **ravel-list** of `A1` is not a **near-integer**, signal **domain-error**.
Set `A2` to the **integer-array-nearest-to** `A1`.
Return `(((A2<0)×0⌈A2+ρB1)+(A2≥0)×0⌊A2-ρB1) ↑B1` evaluated with **comparison-tolerance** set to **zero**.

Examples:

```
      1↓N5
2 3 4 5
      2 ¯1↓N44
31 32 33
41 42 43
      ρ1↓5
0
      ρ0↓5
1
      ρ1 2 3↓4
0 0 0
      ''↓5
5
      ρρ''↓5
0
```


10.2.13 Matrix Divide

`Z ← A ⌹ B`

Informal Description: *Z* is the least squares solution to a linear system specified by *A* and *B*.

Evaluation Sequence:

If the **rank** of *A* or the **rank** of *B* is **greater-than two**, signal **rank-error**.
Set `A1` to `(2↑(ρA),1 1)ρA`.
Set `B1` to `(2↑(ρB),1 1)ρB`.
If `1↑ρA1` is not the same as `1↑ρB1`, signal **length-error**.
If `1↑ρB1` is **less-than** `¯1↑ρB1`, signal **length-error**.
If any **item** of the **ravel-list** of *A* is not a **number**, signal **domain-error**.
If any **item** of the **ravel-list** of *B* is not a **number**, signal **domain-error**.
Use the **implementation-algorithm matrix-divide** to generate an **array** *Z*, such that the **type** of *Z* is **numeric**, the **shape-list** of *Z* is `(¯1↑ρB1),¯1↑ρA1`, and the **ravel-list** of *Z* has the property that for each **scalar** *I* in `ι¯1↑ρA1`, `Z[;I]` is such that it minimises `+/(A1[;I]-B1+.×Z[;I])*2`.
If *Z* cannot be uniquely determined within a reasonable round-off criterion by the above constraints, signal **domain-error**.
Return `((1↓ρB),1↓ρA)ρZ`.

Additional Requirement:

The round-off criterion discussed above is not specified by this International Standard.

Note: *The following article describes an acceptable algorithm for matrix division:*

Jenkins, M. A., **Domino—An APL Primitive Function for Matrix Inversion—Its Implementation and Applications.** *APL Quote-Quad Vol III No. 4, February 1972, pp 4-15.*

10.2.14 Indexed Reference

$Z \leftarrow A[I]$

Informal Description: Z is an array consisting of elements of the array A selected and structured by the position and values of the arrays in the **index-list** I. Uses **index-origin**.

Evaluation Sequence:

If the **number-of-items** in the **index-list** I does not match the **rank** of A, signal **rank-error**.
In the following,
  Let IX stand for (array) **item** X of I.
  Let JX stand for (array) **item** X of J.
  Let LX stand for (array) **item** X of L.
  Let LY stand for (array) **item** X+1 of L.
Generate J, an **index-list** having the same **count** as I such that for every **item** X of the **index-set** of I,
  If IX is an **elided-index-marker**, set JX to ¯1+⍳(⍴A)[X].
  Otherwise,
    If any **item** of the **ravel-list** of IX is not a **near-integer**, signal **domain-error**.
    Set JX to the **integer-array-nearest-to** IX.
    Set JX to JX **minus index-origin**.
    If any element of JX is not an element of ¯1+⍳(⍴A)[X], signal **index-error**.
  Set JX to JX ××/ X↓⍴A.
Generate L, an **index-list** with 1+⍴⍴A (array) **items** such that
  Item 1+⍴⍴A of L is **one**.
  For all X in ⍳⍴⍴A, LX is JX ∘.+ LY.
Let K stand for **first-item** in the **list** of **arrays** L.

Return (,A)[K].

Examples:

```
      1 2 3[2]
2
      N222[2 1;;2]
212 222
112 122
```

Note: *Since an* **index-list** *has at least one* **item**, *indexing will always signal* **rank-error** *when argument* A *is a scalar.*

The vector indexing on which this subsection is based is defined in the subsection **evaluate-indexed-reference**.

10.2.15 Indexed Assignment

`Z ← V[I] ← B`

Informal Description: *Z* is *B*. As a side effect, indexed assignment sets elements of the array *V* selected by the position and values of the arrays in the **index-list** *I* to corresponding elements of *B*. Note that, at this stage, *V* is known to be a **variable-name**, not a **value**. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
Set *A* to the **current-content** of *V*.
If the **number-of-items** in the **index-list** *I* does not match the **rank** of *A*, signal **rank-error**.
In the following,
  Let *IX* stand for (array) **item** *X* of *I*.
  Let *JX* stand for (array) **item** *X* of *J*.
  Let *LX* stand for (array) **item** *X* of *L*.
  Let *LY* stand for (array) **item** *X*+1 of *L*.
Generate *J*, an **index-list** having the same **count** as *I* such that for every **item** *X* of the **index-set** of *I*,
  If *IX* is an **elided-index-marker**, set *JX* to `¯1+⍳(⍴A)[X]`.
  Otherwise,
    If any **item** of the **ravel-list** of *IX* is not a **near-integer**, signal **domain-error**.
    Set *JX* to the **integer-array-nearest-to** *IX*.
    Set *JX* to *JX* **minus index-origin**.
    If any element of *JX* is not an element of `¯1+⍳(⍴A)[X]`, signal **index-error**.
  Set *JX* to `JX ××/ X↓⍴A`.
Generate *L*, an **index-list** with `1+⍴⍴A` (array) **items** such that
  Item `1+⍴⍴A` of *L* is **one**.
  For all *X* in `⍳⍴⍴A`, *LX* is `JX ∘.+ LY`.
Let *K* stand for **first-item** in the **list** of **arrays** *L*.
Set *K*1 to `((1≠⍴K)/⍴K)⍴K` evaluated with **comparison-tolerance** set to **zero**.
Set *B*1 to `((1≠⍴B)/⍴B)⍴B` evaluated with **comparison-tolerance** set to **zero**.
If *B*1 is a **scalar**, set *B*2 to `(⍴K1)⍴B1`.
Otherwise, set *B*2 to *B*1.
If the **rank** of *K*1 is not the same as the **rank** of *B*2, signal **rank-error**.
If any **item** of the **shape-list** of *K*1 is not the same as the corresponding **item** of the **shape-list** of *B*2, signal **length-error**.
Set *M* to **one**.
Repeat:
  If *M* is not **greater-than** the **count** of *K*1,
    Let *K*2 stand for **item** *M* of the **ravel-list** of *K*1.
    Set **item** *K*2 of the **ravel-list** of *A* to **item** *M* of the **ravel-list** of *B*.
    Set *M* to *M* **plus one**.
  Otherwise,

Set the **type** of *A* to the **sufficient-type** of the **ravel-list** of *A* under the **type mixed**.
Set the **current-referent** of *V* to a **token** whose **class** is **variable** and whose **content** is *A*.
Return a **token** whose **class** is **committed-value** and whose **content** is *B*.
(End of repeated block)

Examples:

```
      X 2 3
      ⎕←X[3 2]←4 5
4 5
      X
1 5 4
      ⎕←Y←N222
111 112
121 122

211 212
221 222
      ⎕←Y[2 1;;1]←N12121
11111
11121


12111
12121
      Y
12111 112
12121 122

11111 212
11121 222
```

Additional Requirements:

The evaluation sequence requires that after the statement `A[1 1 1]←1 2 3` has been evaluated the value of `A[1]` be `3`.

Indexed assignment exhibits **atomic** behaviour. If **indexed assignment** signals an error, the value of *V* shall be unchanged.

Note: *Since an* **index-list** *has at least one* **item**, *indexing will always signal* **rank-error** *when argument A is a scalar.*

10.2.16 Without

$Z \leftarrow A \sim B$

Informal Description: Z is a **vector** whose elements are the elements of A without those that are also elements of B. Uses **comparison-tolerance**.

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**. If the **rank** of A or B is **greater-than one**, signal **rank-error**. Set $A1$ to $,A$; set $B1$ to $,B$. Return $(\sim A1 \in B1)/A1$, evaluated with the current value of **comparison-tolerance**.

Examples:

```
      1 2 3 4 5~3 4 2
1 5
      1 1 2 3 3 4 5~3 4 2
1 1 5
      'NAPOLEON'~'NEON'
APL
```

10.2.17 Left

$Z \leftarrow A \dashv B$

Informal Description: Z is A.

Evaluation Sequence:

Return A.

Examples:

```
      0⊣1
0
      1⊣0
1
      N2 ⊣'FRANCE'
1 2
```


10.2.18 Right

```
Z ← A ⊢ B
```

Informal Description: *Z* is *B*.

Evaluation Sequence:

Return *B*.

Examples:

```
      0⊢1
1
      1⊢0
0
      N2 ⊢'FRANCE'
FRANCE
```

10.2.19 Character Grade Definitions

A **precedes** *B* **in** *C* **order**: An operation that, for **character-vectors** *A* and *B* and character **array** *C* of **rank greater-than zero**, returns a **Boolean** result determined by the following evaluation sequence:

Evaluation Sequence:

Set *AX* to the **rank** of *C*.
Repeat
Set `C1` to the **array-of-vectors along-axis** *AX* of *C*.
Generate *AC*, a **numeric vector** with the same **length** as *A*, such that for all *I* in the **index-set** of *A*, `AC[I]` is the minimum of `V⍳A[I]` over all items *V* in `C1`.
Generate *BC*, a **numeric vector** with the same **length** as *B*, such that for all *I* in the **index-set** of *B*, `BC[I]` is the minimum of `V⍳B[I]` over all items *V* in `C1`.
If `((AC<BC)⍳1) < (AC>BC)⍳1`, return **one**.
Set *AX* to `AX-1`.
If *AX* **equals zero**, return **zero**.
(end of repeated sequence).

A **identifies-with** *B* **in** *C* **order** : An operation that returns **zero** if *A* **precedes** *B* **in** *C* **order**or *B* **precedes** *A* **in** *C* **order** and otherwise returns **one**.

10.2.20 Character Grade Down

$Z \leftarrow A$ ⍒ B

Informal Description: Z is a permutation of the index set of the first axis of B which specifies a stable sort of the character subarrays crossing that axis into a non-increasing sequence according to an ordering relation given in the collating array A. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
Set IO to the **numeric-scalar** with value **index-origin minus one**.
If any **item** of the **ravel-list** of A is not a **character**, signal **domain-error**.
If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.
If A is a **scalar** signal **rank-error**.
If A is empty, return IO+⍳1↑⍴B.
Set B1 to ((1↑⍴B),×/1↓⍴B)⍴B.
If 1↑⍴B is **zero**, return ⍳0.
If 1↑⍴B is **one**, return a **one-element-vector** Z such that the **ravel-list** of Z contains **index-origin**.
Otherwise, generate Z, a permutation of ⍳1↑⍴B such that for every pair (I and J) of elements of ⍳1↑⍴B where I is **less-than** J either:
B1[Z[J];] **precedes** B1[Z[I];] **in** A**-order** or
B1[Z[I];] **identifies-with** B1[Z[J];] **in** A**-order** and Z[I] is **less-than** Z[J].
Return Z+IO.

10.2.21 Character Grade Up

$Z \leftarrow A \Delta\!\!\mid B$

Informal Description: Z is a permutation of the index set of the first axis of B which specifies a stable sort of the character subarrays crossing that axis into a non-decreasing sequence according to an ordering relation given in the collating array A. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
Set `IO` to the **numeric-scalar** with value **index-origin minus one**.
If any **item** of the **ravel-list** of A is not a **character**, signal **domain-error**.
If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.
If A is a **scalar** signal **rank-error**.
If A is empty, return `IO+⍳1↑⍴B`.
Set `B1` to `((1↑⍴B),×/1↓⍴B)⍴B`.
If `1↑⍴B` is **zero**, return `⍳0`.
If `1↑⍴B` is **one**, return a **one-element-vector** Z such that the **ravel-list** of Z contains **index-origin**.
Otherwise, generate Z, a permutation of `⍳1↑⍴B` such that for every pair (I and J) of elements of `⍳1↑⍴B` where I is **less-than** J either:
`B1[Z[I];]` **precedes** `B1[Z[J];]` **in** A**-order** or
`B1[Z[I];]` **identifies-with** `B1[Z[J];]` **in** A**-order** and `Z[I]` is **less-than** `Z[J]`.
Return `Z+IO`.

Note: *In comparing rows of* `B1`*, differences at low index positions are more significant than those at higher index positions. Thus, if* `'a'` *comes before* `'b'` *in A-order, the rows* `'aa'`, `'ab'`, `'ba'`*, and* `'bb'` *are in ascending A-order.*

Every axis of the collating array A defines an ordering of the character set. A character 's lowest index along an axis defines its position in the order described by that axis (low indices precede high indices). Characters not found in A are equal and occur after all characters in A.

The order given along A's last axis is the most significant. The result of character grade groups together rows of B which are equal under this (major) ordering. Within each such group, the result arranges B's rows according to the hierarchy of (minor) orderings along axes of A ending with the first (least significant) axis. Character grade is stable; its results are equal under all the A-orderings in the order that they occur in B.

Hierarchical sorting allows distinctions such as spelling, case, font, and length to carry their proper lexicographic weight. For example, the rows of W (below) are taken in order from Webster's New Collegiate Dictionary.

Examples:

```
      (2 2ρ'ABBA') ⍋ 'AB'[?5 2ρ2] ⍝ A AND B ARE EQUIVALENT
 1 2 3 4 5
      A←2 14ρ' abcdegiklmnrt ABCDEGIKLMNRT'
 W            W[(,A)⍋W;]   W[(,⍉A)⍋W;]  W[A⍋W;]
 Ab           aba          aba          Ab
 AB           abaca        abaca        AB
 aba          abecedarian  abecedarian  aba
 ABA          black        Ab           ABA
 abaca        black belt   Abelian      abaca
 abecedarian  blackball    AB           abecedarian
 Abelian      blacking     ABA          Abelian
 black        Ab           black        black
 blackball    Abelian      black belt   black belt
 black belt   AB           blackball    Black Mass
 blacking     ABA          blacking     blackball
 Black Mass   Black Mass   Black Mass   blacking
```

Note: *Character grade is equivalent to the function* SRT4 *defined in Smith, Howard J., "Sorting – A New/Old Problem",* **APL Quote-Quad 9**, *1, June 1979, pp 123-127*

10.2.22 Pick

```
Z ← A ⊃ B
```

Informal Description: Z is an item selected from some level of B. A is a scalar or vector. The items of A are selectors applying to one or more successive levels of B, such that the first (or only) item of A selects an item from B, the next (if any) selects an item from this item, and so on. The length of each item in A must agree with the rank of the object to which it is to be applied. Uses **index-origin**.

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If the **rank** of A is **greater-than one**, signal **rank-error**.
If A is empty, return B.
If the **count** of `a` is **one**:
  Set `A1` to the **first-thingy** of A.
  If `A1` is not a **simple array**, signal **domain-error**.
  If the **rank** of `A1` is **greater-than one**, signal **rank-error**
  Set `A2` to the **ravel-list** of `A1`.
  If any item in `A2` is not a **near-integer**, signal **domain-error**.
  Set `A3` to a list with each item the **integer-nearest-to** the corresponding element of `A2`.
  If the **number-of-items** in `A3` is not the same as the `rank` of B, signal **rank-error**
  Set `A4` to a list, with each item equal to the corresponding item in `A3`, minus **index-origin**.
  For every member X of the **index-set** of `A4`:
    If item X of `A4` is **less-than zero**, or **greater-than-or-equal-to** item X of the **shape-list** of B, signal **index-error**.
  Return item `1+(⍴B)⊥A4` of the **ravel-list** of B.
Otherwise, return `(1↓A)⊃(1↑A)⊃B`.

Examples:

```
      3⊃'OSCAR'
C
      (,3)⊃10 11 12 13
12
      (⊂3 2)⊃N44
32
      1 5⊃(⊂'OSCAR'),(⊂'BEATA')
R
```


10.2.23 Identical

`Z ← A ≡ B`

Informal Description: Returns `1` if *A* and *B* match with respect to structure and content, and `0` otherwise.

Note: *Identical is sometimes called "match".*

Evaluation Sequence:

If *A* and *B* are both **simple-scalars**, return *A*=*B*.
If the **shape-list** of *A* is different from the **shape-list** of *B*, return **zero**.
If *A* is empty, return `(1↑,A)≡1↑,B`.
Set *A*`1` to the first item of the **ravel-list** of *A*, and *B*`1` to the first item of the **ravel-list** of *B*.
Set *Z*`1` to *A*`1`≡*B*`1`.
If *Z*`1` is **zero**, return **zero**.
Set *A*`2` to the rest of the **ravel-list** of *A*, and *B*`2` to the rest of the **ravel-list** of *B*.
If *A*`2` is empty, return **one**.
Return *A*`2`≡*B*`2`.

Examples:

```
      'SOLAR'≡'SOLAR'
1
      (2 3 ρι6)≡ι6
0
      (1,0ρ' ')≡1,0ρ0
1
```


10.2.24 Disclose

```
Z ← ⊃ B
```

Informal Description: For *B*, an array of shape *A*, all of whose items have the same **rank** *K*, *Z* is an array of shape `A,M` where *M* is the maximum over the shapes of the items in *B*, and each **rank-*K* cell** of *Z* is the result of `M↑` of the corresponding item in *B*.

Evaluation Sequence:

Set *M* to **max-shape-of** *B*.
Set `M1` to `(ρB),M`.
Set *K* to `ρM`.
Set *S* to the **index-set** of the **ravel-list** of *B*.
Return an **array** *Z* such that `ρZ` is `M1`, and such that for every member *I* of *S*, the *I*'th **rank-*K* cell** is `m↑C`, where *C* is **item** *I* of the **ravel-list** of *B*.

Examples:

```
      T ← (⊂'OSCAR'),(⊂'FRED')
      ρT
2
      ⊃T
OSCAR
FRED
      ρ⊃T
2 5
```

10.2.25 Disclose with Axis

```
Z ←⊃[K] B
```

Informal Description: This is the same as Disclose, except that the function specifies which axes in the result *Z*, are new. Uses **index-origin**.

Evaluation Sequence:

Set `Z1` to `⊃B`.

If **index-origin** is **nil**, signal **implicit-error**.
If *A* is not a **simple vector**, signal **axis-error**.
If any **thingy-in** *A* is not a **valid-axis-of** `Z1`, signal **axis-error**.
Set `A1` to the **integer-array-nearest-to** *A*.
If any item of the **ravel-list** of `A1` is not distinct, signal **axis-error**.
If the **count-of** `A1` is not the **rank** of the **first-thingy-of** *B*, signal **axis-error**.
Return `(((⍳ρρZ1)~A),A)⍉Z1`.

10.2.26 Enclose

```
Z ← ⊂ B
```

Informal Description: *Z* is a scalar array whose only item is B.

Note: *If B is a* **simple-scalar**, *Z is B.*

Evaluation Sequence:

If *B* is a **simple-scalar**, return *B*.
Return an array whose **shape-list** is empty, and whose **ravel-list** has a single **item**, the array *B*, and whose **type** is **mixed**.

Examples:

```
      ⊂'OSCAR'
 OSCAR
      ⍴⍴⊂'OSCAR'
0
      2=⊂2
1
```

10.2.27 Enclose with Axis

```
Z ← ⊂ [K] B
```

Informal Description: *Z* is *B* with the indicated axes eliminated and replace by a corresponding new level of nesting. K specifies the axis to be enclosed, and their order in the resulting items. Uses **index-origin**.

Evaluation Sequence:

If, for some *N*, `K = (-N)↑⍳⍴⍴B`, return an array whose shape is `(-N)↓⍴B`, and whose **ravel-list** contains the **rank-*N* cells** of *B*.
Otherwise, return `⊂[(-⍴K)↑⍳⍴⍴B](⍋((⍳⍴⍴B)~K),K)⍉B`.

11 System Functions

11.1 Introduction

Note: *System functions are primitive functions whose names are* **distinguished-identifiers** *rather than* **primitives**. *System function names are not permitted in the* **locals-list** *of a* **defined-function header-line**.

The system functions related to **shared-variables** *and* **defined-functions** *are specified in those chapters.*

11.2 Definitions

Note: *Some system functions take an argument which is treated as an* **identifier-array**.

– **Identifier-Array**: A **character array** of **rank two** each of whose **rows** matches the **character-diagram identifier-row**.

– **Event-Report**: A representation of the most recent error.

– **Event-Message**: A **character array** giving, in an **implementation-defined** form, the **event-report** and any other information the implementation deems helpful in locating the error's source.

– **Empty-Event-Message**: An empty **array**, of an **implementation-defined** shape, indicating that the most recent **event-type** corresponds to no error.

11.3 Diagram

Identifier-Row

11.4 Niladic System Functions

Note: *A* **niladic system function** *is an implementation-provided facility associated with a* **distinguished-identifier**. *The primary difference between a* **niladic system function** *and a* **system variable** *is that a* **niladic system function** *causes an error to be signalled if a value is assigned to it, or its name is included in the list of locals of a defined function header.*

11.4.1 Time Stamp

```
Z ← ⎕TS
```

Informal Description: *Z* is a seven-element numeric vector representing the current date and time relative to an epoch and a time zone.

Evaluation Sequence:

Return the seven-element **numeric vector** result of the **implementation-algorithm time-stamp**.

Example:

```
      ⎕TS
1789 7 14 11 14 45 586.4
```

Additional Requirements:

The first through sixth elements of the result returned by **time-stamp** are integral and represent respectively the current year, month, day, hour, minute, and second relative to the underlying epoch and time zone. The hour corresponds to the number of integral hours that have elapsed within the current day.

The seventh and last element is a quantity, not necessarily integral, representing the milliseconds that have elapsed within the current second.

The accuracy, resolution, epoch, and time zone for **time-stamp** are not specified by this International Standard.

11.4.2 Atomic Vector

$Z \leftarrow$ ⎕AV

Informal Description: Z is an **implementation-defined character** vector containing every element of the **required-character-set** exactly once.

Evaluation Sequence:

Return **atomic-vector**.

Note: *The elements of the* **atomic-vector** *are often ordered so that, in* **index-origin zero**, ((⌈2⍟⍴⎕AV)⍴2) ⊤⎕AV⍳'X' *gives the bit representation of* X*. This behaviour is not required.*

11.4.3 Line Counter

$Z \leftarrow$ ⎕LC

Informal Description: Z is a vector of statement numbers of active defined functions ordered so that the most recently called function has the lowest index.

Evaluation Sequence:

Set Y to a **numeric vector** whose **length** is the same as the **number-of-items** in the **state-indicator**, such that for every I in ⍳⍴Y, Y[I] is the **current-line-number** of **item** I of the **state-indicator**, if the **mode** of **item** I of the **state-indicator** is **defined-function**, and **zero** otherwise.
Set M to a **Boolean vector** whose **length** is the same as the **number-of-items** in the **state-indicator**, such that for every I in ⍳⍴M, M[I] is **one** if the **mode** of **item** I of the **state-indicator** is **defined-function**, and **zero** otherwise.
Return M/Y.

Note: *The subsections that affect the value of* ⎕LC *are discussed in the subsection* **defined-function-control**.

11.4.4 Event Message

Z ← ⎕*EM*

Informal Description: When an error occurs, three lines of information are usually displayed: the first line indicates what happened, the second line shows the line in which the error was found, and the third line contains one or more carets pointing to the place(s) in the second line where the error was detected. **Event-message** is commonly a character matrix corresponding to this three-line message. It is set according to the contents of **event-type**. If **event-type** is `0 0` then **event-message** is an empty three-row matrix. Otherwise, **event-message** is set to an appropriate message. In some cases its first row is blank (when **event-message** is set via ⎕*ES*) because there is no message for the given value of **event-type**. (See the description of **event-type** for proposed values and messages.)

event-message is local to a **context**.

Evaluation Sequence:

Return **event-message**.

Note: *While the informal description speaks of a three line message of a particular format, this is an attempt to describe common usage, rather than prescribed behaviour. The format of* **event-message** *is implementation defined.*

11.4.5 Event Type

Z ← ⎕*ET*

Informal Description: The default value for **event-type** is 0 0 (no error). When an event happens which halts execution **event-type** is set to a pair of integers specifying the class and sub-class of the event. **event-type** is local to a **context**.

Evaluation Sequence:

Return **event-type**.

Note: *Classes and subclasses of* **event-type***:*
0 0 – *no error*
0 1 – *unclassified event*

There may be additional **implementation-defined** *types.*

11.5 Monadic System Functions

Note: *Refer to the* **Defined Functions** *chapter for definitions of* ⎕*FX and* ⎕*CR.*

11.5.1 Delay

Z ← ⎕*DL B*

Informal Description: *Z* is the time, in seconds, for completion of this operation. As a side effect, ⎕*DL* causes a delay in execution of at least *B* seconds.

Evaluation Sequence:

Set *T*0 to **current-time** seconds.
If *B* is not a **scalar**, signal **rank-error**.
If *B* is not a **near-real-number**, signal **domain-error**.
Set *T*1 to *T*0 **plus** *B*.
Wait until **current-time** is not **less-than** *T*1.
Return *Z*, a **numeric scalar** such that the **first-item** in the **ravel-list** of *Z* is **current-time minus** *T*0.

Note: *B may be fractional or negative.*

11.5.2 Name Class

```
Z ← ⎕NC B
```

Informal Description: *Z* is a vector of name classes giving the usage of the identifier in the corresponding row of *B*. 0 means an identifier is available without current referent; 1, a label; 2, a variable or a shared variable; 3, a defined function; 4, a defined operator; 5, a system variable; 6, a system function.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
If *B* is **empty**, return ⍳0.
Set *B*1 to (¯2↑1 1,⍴*B*)⍴*B*.
If any row of *B*1 does not match **identifier-row**, signal **domain-error**.
Generate *Z*, a **numeric vector** such that the **length** of *Z* is the same as the **number-of-rows** in *B*1, and the **ravel-list** of *Z* has the property that for every *I* in ⍳1↑⍴*B*, the following holds:
  Let *ZI* stand for **item** *I* of the **ravel-list** of *Z*.
  Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in the **identifier-row**.
  If the **current-class** of *N* is **nil**, *ZI* is **zero**.
  If the **current-class** of *N* is **label**, *ZI* is **one**.
  If the **current-class** of *N* is **variable** or **shared-variable**, *ZI* is **two**.
  If the **current-class** of *N* is **defined-function** or **niladic-defined-function**, *ZI* is **three**.
  If the **current-class** of *N* is **defined-operator**, *ZI* is **four**.
  If the **current-class** of *N* is **system-variable**, *ZI* is **five**.
  If the **current-class** of *N* is **system-function**, *ZI* is **six**.
  Otherwise, signal **domain-error**.
Return *Z*.

Example:

```
      ∇Z←TEST;R0;R21;R22;R31;R32
[1] R1:R21←1 ⎕SVO 'R22'
[2] Z←⎕FX 1 3⍴'R31'
[3] Z←⎕FX 1 5⍴'R32 X'
[4] Z←⎕NC 6 3⍴'R0 R1 R21R22R31R32'
   ∇
      TEST
0 1 2 2 3 3
```


11.5.3 Expunge

Z ← ⎕*EX B*

Informal Description: *Z* is a Boolean vector such that an element of *Z* is **one** if the identifier in the corresponding row of *B* is available for use when the operation completes. Expunge changes the **current-referent** of **symbols** whose **current-class** is **variable**, **shared-variable**, **defined-function**, **defined-operator**, or **niladic-defined-function** to **nil**, making them available for redefinition. The **symbols** to be changed are named by the rows of *B*, an **identifier-array**.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
Set *B*1 to (¯2↑1 1,ρ*B*)ρ*B* .
For each **row** *J* of *B*1,
  If **row** *J* of *B*1 does not match **identifier-row**, signal **domain-error**.
  Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
  If the **current-class** of *N* is **shared-variable**, **retract** *N*.
  If the **current-class** of *N* is **variable**, set the **current-referent** of *N* to a **token** whose **class** is **nil**.
  If the **current-class** of *N* is **defined-function** or **niladic-defined-function**, and if the **current-referent** of *N* is **locally-erasable**, set the **current-referent** of *N* to a **token** whose **class** is **nil**.
Return ~×⎕*NC B*.

Additional Requirement:

The operation of Expunge is **atomic**: if Expunge signals an error, **current-referents** shall be unchanged.

Note: *Because of the definition of* **locally-erasable**, *a* **defined-function** *or* **defined-operator** *cannot be expunged by a* **conforming-program** *while it is* **pendent** *or* **waiting**. *This does not prevent a* **conforming-implementation** *from permitting such behaviour; it simply leaves the consequences to a* **conforming-program** *undefined.*

11.5.4 Name List

`Z ← ⎕NL B`

Informal Description: *Z* is an **identifier-array**. An identifier occurs in *Z* if its **name-class** is in *B*. Elements of *B* can be: `1`, for labels; `2`, for variables or shared variables; `3`, for defined functions; `4`, for defined operators; `5`, for system variables; `6`, for system functions.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of *B* is not a **near-integer**, signal **domain-error**.
Set *B*`1` to the **integer-array-nearest-to** *B*.
If *B*`1` contains any **numbers** other than **one**, **two**, **three**, **four**, **five**, or **six**, signal **domain-error**.
Generate *Z*, an **identifier-array** consisting of the **names** of **symbols** whose **current-class** is **label**, **variable**, **shared-variable**, **niladic-defined-function**, **defined-function**, **defined-operator**, **system-variable** or **system-function**. The **symbol names** are left-justified in the rows of *Z*.
Set *Z* to `((⎕NC Z)∊B1)⌿Z`.
Remove any all-blank trailing columns from *Z*.
Return *Z*.

Note: *The* **rank** *of Z is always* **two**. *If there are no* **symbols** *with a* **name-class** *designated in B, Z is* `0 0⍴''`.

11.5.5 Query Stop

Z ← ⎕*STOP B*

Informal Description: *Z* is an integer vector of line numbers in the function or operator whose name is *B* that have stop control set.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If *B* does not match **identifier-row**, signal **domain-error**.
Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
If the **current-class** of *N* is not **defined-function**, **defined-operator** or **niladic-defined-function**, signal **domain-error**.
Return the **stop-vector** of the **current-content** of *N*.

Additional Requirement:

Query Stop is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Query Stop is not origin sensitive.*

If no lines have stop control set, the result is ⍳0.

11.5.6 Query Trace

Z ← ⎕*TRACE B*

Informal Description: *Z* is an integer vector of line numbers in the function or operator whose name is *B* that are being traced.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If *B* does not match **identifier-row**, signal **domain-error**.
Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
If the **current-class** of *N* is not **defined-function**, **defined-operator** or **niladic-defined-function**, signal **domain-error**.
Return the **trace-vector** of the **current-content** of *N*.

Additional Requirement:

Query Trace is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Query Trace is not origin sensitive.*

If no lines are being traced, the result is ⍳0.

11.5.7 Monadic Event Simulation

⎕*ES B*

Informal Description: An exception is created in the caller's **context**. *B* is either a pair of integers specifying the class and sub-class of the event, or a character vector.

If *B* is a pair of integers, **event-type** is set to *B* and **event-message** is set to an appropriate message. If the pair of integers is undefined, the message part of **event-message** is blank, but an exception is signalled and **event-type** is set. If *B* is `0 0` (no error), **event-type** is set to `0 0` and **event-message** is set to **empty-event-message**, but no exception is signalled.

If *B* is empty no action is taken. This allows for conditional signalling of an event.

Evaluation Sequence:

If the rank of *B* is **greater-than one**, signal a **rank-error**.
If *B* is **empty**, return **nil**.
If *B* is **numeric**,
  If the **rank** of *B* is not **one**, signal **rank-error**.
  If the **count** of **items** in *B* is not **two**, signal **length-error**.
  If any **item** in *B* is not a **near-integer**, signal **domain-error**.
  Set *B*`1` to the **integer-nearest-to** *B*.
  Set **event-type** to *B*`1`.
  Set **event-message** to **empty-event-message**.
  If *B*`1` is not `0 0`,
    Set **event-message** as appropriate.
    Signal event corresponding to **event-type**.
If *B* is **character**,
  Set **event-type** to `0 1`.
  Set message portion of **event-message** to *B*.
  Signal event corresponding to **event-type**.

11.6 Dyadic System Functions

11.6.1 Name List

Z ← *A* ⎕*NL B*

Informal Description: *Z* is an **identifier-array** of all names having a **name-class** in *B* that begin with a letter in *A*.

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of A does not match **letter**, signal **domain-error**.
Set `Z1` to `⎕NL B`.
If `0∊⍴Z1`, return `Z1`.
Return `(Z1[;1]∊A)⌿Z1`.

11.6.2 Set Stop

`Z ← A ⎕STOP B`

Informal Description: Z is a numeric vector representing the stop controls in effect for the function or operator whose name is in B before `⎕STOP` began evaluation. As a side effect, `⎕STOP` sets stop controls for the function or operator whose name is in B at lines specified by A.

Evaluation Sequence:

If the **rank** of B is **greater-than one**, signal **rank-error**.
If B does not match **identifier-row**, signal **domain-error**.
Let N stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
If the **current-class** of N is not **defined-function**, **defined-operator** or **niladic-defined-function**, signal **domain-error**.
If the **rank** of A is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
Set `A1` to the **integer-array-nearest-to** A.
If any **item** of the **ravel-list** of `A1` is not a **positive-integer**, signal **domain-error**.
Set Z to `⎕STOP B`.
Let M stand for the **last-line-number** of the **current-content** of N.
Set the **stop-vector** of the **current-content** of N to `((⍳M)∊A1)/⍳M`, evaluated with **index-origin one**.
Return Z.

Additional Requirement:

Set Stop is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Stop controls are an attribute of* **defined-function** *objects.* `⎕STOP` *affects the* **current-referent** *of* B *only.*

11.6.3 Set Trace

Z ← *A* ⎕*TRACE B*

Informal Description: *Z* is a numeric vector representing the lines being traced in the function or operator whose name is in *B* before ⎕*TRACE* began evaluation. As a side effect, ⎕*TRACE* begins tracing the lines specified by *A* in the function or operator whose name is in *B*.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If *B* does not match **identifier-row**, signal **domain-error**.
Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
If the **current-class** of *N* is not **defined-function**, **defined-operator**, or **niladic-defined-function**, signal **domain-error**.
If the **rank** of *A* is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of *A* is not a **near-integer**, signal **domain-error**.
Set *A*1 to the **integer-array-nearest-to** *A*.
If any **item** of the **ravel-list** of *A*1 is not a **positive-integer**, signal **domain-error**.
Set *Z* to ⎕*TRACE B*.
Let *M* stand for the **last-line-number** of the **current-content** of *N*.
Set the **trace-vector** of the **current-content** of *N* to ((⍳*M*)∊*A*1)/⍳*M*, evaluated with **index-origin one**.
Return *Z*.

Additional Requirement:

Set Trace is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Trace controls are an attribute of* **defined-function** *objects.* ⎕*TRACE affects the* **current-referent** *of B only.*

11.6.4 Execute Alternate

Z ← A ⎕EA B

Informal Description: A and B are both character vectors representing executable expressions. If execution of B is successful the result is Z. If there is an error or an interrupt in the execution of B, execution is abandoned and A is executed. The execution of A is subject to normal error handling.

It is possible to get a **value-error** if the successful execution of B does not return a result and an explicit result is needed. A is not executed in this case.

Evaluation Sequence:

If the **rank** of either A or B is **greater-than one** signal **rank-error**.
If any **item** of the **ravel-list** of A is not a **character**, signal **domain-error**.
If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.
Generate a new **context** in which
  mode is **execute**,
  current-line is the **ravel-list** of B,
  current-function is 0 0⍴' ',
  current-line-number is **one**,
  current-statement is the empty **list** of **tokens**,
  stack is the empty **list** of **tokens**.
Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.
Set Z to **evaluate-line**.
Remove the first **context** from the **state-indicator**.
If Z is not a member of **error** return Z,
Else return ⍎A

Note: *See* **Conforming Programs** *in* **Compliance** *regarding hazards to conforming programs if this facility is used.*

11.6.5 Dyadic Event Simulation

A ⎕*ES B*

Informal Description: An exception is created in the caller's **context**. *A* is a **character scalar** or **vector** and *B* is either a pair of integers specifying the class and sub-class of the event, or an empty **vector**.

Event-type is set to *B* and the message portion of **event-message** is set to *A*. If *B* is `0 0` (no error), **event-type** is set to `0 0` and **event-message** is set to **empty-event-message**, and no event is signalled.

If *B* is empty no action is taken. This allows for conditional signalling of an event (for example *A* ⎕*ES* **cond** /*B*).

Evaluation Sequence:

If the **rank** of *A* is **greater-than one**, signal **rank-error**.
If any **item** of *A* is not a **character**, signal **domain-error**.
If the **rank** of *B* is not **one**, signal **rank-error**.
If *B* is empty, return **nil**.
If the **count** of *B* is not **two**, signal **length-error**.
If any **item** in *B* is not a **near-integer**, signal **domain-error**.
Set *B*1 to the **integer-nearest-to** *B*.
Set **event-type** to *B*1.
Set **event-message** to **empty-event-message**.
If *B*1 is not `0 0`,
  Set the message portion of **event-message** to *A*.
  Set the rest of **event-message** as appropriate.
  Signal event corresponding to **event-type**.

11.6.6 Transfer Form

Z ← *A* ⎕*TF* *B*

Informal Description: *B* is a character vector giving the name of the object for which the transfer form is desired. If there is no transfer form for the item named by *B* (for example, *B* names a system function, a label, or has no value) the result is an empty character vector.

A is either **one** or **two**. If it is **one** *B* may name only defined functions, defined operators, and simple homogeneous arrays. If it is **two**, *B* may also name **mixed** arrays.

For a given *A*, ⎕*TF* is its own inverse. For example, if *ABC* names a transferable object, *A* ⎕*TF* *A* ⎕*TF* '*ABC*' will reestablish the object named *ABC* and return the character list '*ABC*'.

If *A* is **two**, the result is an executable character vector giving the name and contents of the object named by *B*. (Numeric conversions are carried out to **full-print-precision**.)

Evaluation Sequence:

If the **rank** of *A* is **greater-than one** signal **rank-error**.
If the **count** of *A* is not **one** signal **length-error**.
If any **item** of the **ravel-list** of *A* is not a **near-integer**, signal **domain-error**.
Set *A*1 to the **integer-nearest-to** *A*.
If *A*1 has the value **one**, set *Z* to **produce-canonical-representation-vector** of the object named by *B*.
If *A*1 has the value **two**,
  If the **rank** of *B* is **greater-than one** signal **rank-error**.
  If the **type** of *B* is not **character** signal **domain-error**.
  Set *NC* to the **name-class** of *B*.
  If *NC* is **two**:
    Set *B*1 to ⍎*B*
    Set *Z* to an executable string that will recreate the object named by *B*
  If *NC* is **three** or **four**:
    Set *B*1 to the **character-representation** of the object named by *B*
    If the **count** of the **shape** of *B* is not **zero**
      Set *Z* to an executable string that will recreate the object named by *B*
    Else set *Z* to an empty character vector.
  If *NC* is **negative-one**:
    If ⍎*B* is successful, set *Z* to the name of the object created
    Otherwise, set *Z* to an empty character vector
  Otherwise signal **domain-error**
Otherwise, signal **domain-error**.

12 System Variables

12.1 Definitions

– **System-Variable-Symbol**: A **symbol**, having as its name a **distinguished-identifier**, and as its **referent-list** a **list** of **tokens** representing the value assigned to an associated **system-parameter** in each **context** of the **state-indicator**.

Note: *System parameters are values used implicitly and set as side effects by primitive operations.*

– **Internal-Value-Set**: An **implementation-defined** set of values that the **tokens** in the **referent-list** of a **system-variable-symbol** can take on.

Note: *When a* **system-variable-symbol** *is* **localised** *the initial* **class** *of its associated* **system-parameter** *is* **nil**. *If a primitive operation is invoked that requires a system parameter whose class is currently* **nil**, *the primitive operation signals* **implicit-error**. *Therefore, a* **conforming-program** *that* **localises system-variable-symbols** *should assign them values from their* **internal-value-set** *before calling primitive operations that require them.*

The assignment operation for a system variable rejects attempts to set its associated system parameter to a value outside its **internal-value-set**.

12.2 Evaluation Sequences

12.2.1 Comparison Tolerance

Z ← ⎕*CT* ← *B*

Z ← ⎕*CT*

Informal Description: *Z* is the current value of **comparison-tolerance**.

The distance within which numbers are to be considered equal by certain primitives is a function of **comparison-tolerance**.

Evaluation Sequence:

For form ⎕*CT*
  Return **comparison-tolerance**.
For form ⎕*CT* ← *B*
  If the **rank** of *B* is **greater-than one**, signal **rank-error**.
  If the **count** of *B* is not **one**, signal **length-error**.
  Let *B*1 be the **first-scalar** in *B*.
  If *B*1 is not a **nonnegative-number**, signal **domain-error**.
  If *B*1 is not in the **internal-value-set** of **comparison-tolerance**, signal **limit-error**.
  Set **comparison-tolerance** to *B*1.
  Return a **token** whose **class** is **committed-value** and whose **content** is *B*.

Additional Requirement:

The **internal-value-set** for **comparison-tolerance** consists of **nonnegative-numbers** not greater than the **implementation-parameter comparison-tolerance-limit**.

The initial value of **comparison-tolerance** in a **clear-workspace** is that member of the **internal-value-set** for **comparison-tolerance** given by the **implementation-parameter initial-comparison-tolerance**.

Note: *The following subsections reference* **comparison-tolerance***:* **ceiling**, **equal**, **floor**, **greater than or equal to**, **greater than**, **index of**, **less than or equal to**, **less than**, **member of**, **not equal**, **residue**, **and**, **or** *and* **unique**.

12.2.2 Random Link

$Z \leftarrow \square RL \leftarrow B$

$Z \leftarrow \square RL$

Informal Description: Z is the current value of **random-link**; **random-link** is the current seed of the pseudorandom number generator.

Evaluation Sequence:

For form $\square RL$
  Return **random-link**.
For form $\square RL \leftarrow B$
  If the **rank** of B is **greater-than one**, signal **rank-error**.
  If the **count** of B is not **one**, signal **length-error**.
  Set $B1$ to the **first-scalar** in B.
  If $B1$ is not a **near-integer**, signal **domain-error**.
  Set $B2$ to the **integer-nearest-to** $B1$.
  If $B2$ is **less-than one**, signal **domain-error**.
  If $B2$ is not in the **internal-value-set** for **random-link**, signal **limit-error**.
  Set **random-link** to $B2$.
  Return a **token** whose **class** is **committed-value** and whose **content** is B.

Additional Requirement:

The **internal-value-set** of **random-link** is **implementation-defined**.

The initial value of **random-link** in a **clear-workspace** is that member of the **internal-value-set** for **random-link** given by the **implementation-parameter initial-random-link**.

Note: *The system parameter* **random-link** *is used and set by* **roll** *and* **deal***.* **Roll** *gives a reference for a suitable algorithm.*

12.2.3 Print Precision

Z ← ⎕PP ← B

Z ← ⎕PP

Informal Description: Z is the current value of **print-precision**, which controls the number of significant positions in the output form produced by **monadic format** and **numeric-output-conversion**.

Evaluation Sequence:

For form ⎕PP
  Return **print-precision**.
For form ⎕PP ← B
  If the **rank** of B is **greater-than one**, signal **rank-error**.
  If the **count** of B is not **one**, signal **length-error**.
  If B is not **near-integer**, signal **domain-error**.
  Set B1 to the **integer-nearest-to** the **first-scalar** in B.
  If B1 is **less-than one**, signal **domain-error**.
  If B1 is not in the **internal-value-set** of **print-precision**, signal **limit-error**.
  Set **print-precision** to B1.
  Return a **token** whose **class** is **committed-value** and whose **content** is B.

Additional Requirements:

The **internal-value-set** of **print-precision** consists of the **integer** scalars in the **closed-interval-between one** and **print-precision-limit**.

The initial value of **print-precision** in a **clear-workspace** is that member of the **internal-value-set** for **print-precision** given by the **implementation-parameter initial-print-precision**.

Note: *The system parameter* **print-precision** *is used by* **monadic format** *and* **numeric-output-conversion**. **Print-precision-limit** *should be greater than or equal to* **full-print-precision**.

12.2.4 Index Origin

Z ← ⎕*IO* ← *B*

Z ← ⎕*IO*

Informal Description: *Z* is the current value of **index-origin**, which is the index associated with the first position of any axis of non-zero length.

Evaluation Sequence:

For form ⎕*IO*
  Return **index-origin**.
For form ⎕*IO* ← *B*
  If the **rank** of *B* is **greater-than one**, signal **rank-error**.
  If the **count** of *B* is not **one**, signal **length-error**.
  If *B* is not a **near-integer**, signal **domain-error**.
  Set *B*1 to the **integer-nearest-to** the **first-scalar** in *B*.
  If *B*1 is not in the **internal-value-set** of **index-origin**, signal **limit-error**.
  Set **index-origin** to *B*1.
  Return a **token** whose **class** is **committed-value** and whose **content** is *B*.

Additional Requirement:

The **internal-value-set** for **index-origin** is the **scalar integer** values **zero** and **one**.

The initial value of **index-origin** in a **clear-workspace** is that member of the **internal-value-set** for **index-origin** given by the **implementation-parameter initial-index-origin**.

Note: *The following primitive operations refer to* **index-origin***:* **deal**, **dyadic transpose**, **grade down**, **grade up**, **index generator**, **index of**, **indexed assignment**, **indexed reference**, **roll**, *and all functions that provide for axis specification when the form containing an axis is used.*

12.2.5 Latent Expression

$Z \leftarrow \square LX \leftarrow B$

$Z \leftarrow \square LX$

$Z \leftarrow \square LX[I] \leftarrow B$

Informal Description: Z is the current value of **latent-expression**, which is a **character** vector that is executed when a workspace is activated.

Evaluation Sequence:

For form $\square LX$
  Return **latent-expression**.
For form $\square LX \leftarrow B$
  If the **rank** of B is **greater-than one**, signal **rank-error**.
  If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.
  If B is not in the **internal-value-set** of **latent-expression**, signal **limit-error**.
  Set **latent-expression** to $,B$.
  Return a **token** whose **class** is **committed-value** and whose **content** is B.
For form $\square LX[I] \leftarrow B$
  Set LX to **latent-expression**.
  Evaluate $LX[I] \leftarrow B$.
  Set **latent-expression** to LX.
  Return a **token** whose **class** is **committed-value** and whose **content** is B.

Additional Requirement:

The **internal-value-set** for **latent-expression** is an **implementation-defined** subset of the set of all character vectors which includes the empty vector.

The initial value of **latent-expression** in a **clear-workspace** is that member of the **internal-value-set** for **latent-expression** given by the **implementation-parameter initial-latent-expression**.

13 Defined Functions

13.1 Introduction

Note: *Algorithms written in APL are called defined functions. A defined function consists of a header line and zero or more body lines. The header line indicates the name and syntax class of the function and gives a list of names to be localised and, in the case of argument names, initialised. Each body line consists of an optional label followed by an APL line to be evaluated. Body lines are evaluated in the order in which they occur in the defined function unless a statement beginning with a right arrow is evaluated. In this chapter, the term defined function also includes defined operator.*

A defined function is established in a workspace by the system function ⎕FX, by actions in function definition mode, or by the system command `)COPY`.

In this International Standard, a **defined-function** *is represented as an object with three attributes:* **canonical-representation***,* **trace-vector***, and* **stop-vector**. *A* **defined-function** *is called when its* **name** *occurs in a* **prefix** *of the* **current-stack** *matching one of the* **patterns** *in the* **phrase-table**.

A **defined-function** *is called by the operation* **call-defined-function**. *This operation prefixes the* **state-indicator** *with a new* **context**, **localises** *and initialises any local names in the* **header-line**, *and calls the evaluation sequence in the subsection* **defined-function-control**.

A **defined-function** *ends the evaluation if* **defined-function-control** *finds* **current-line-number** *set to a value not in the* **closed-interval-between one** *and* **last-line-number**. *At this point, the first* **item** *of the* **state-indicator** *is discarded and a* **token** *of class* **result** *is returned, via* **call-defined-function**, *to the* **phrase-evaluator** *that called it.*

The returned **token** *may be* **constant** *(for functions returning a value),* **nil** *(for functions that return no value),* **unwind** *(for functions that end through the evaluation of an escape arrow), or* **clear-state-indicator** *(as the result of an* `)SIC` *command).*

If a line in a **defined-function** *signals an error,* **immediate-execution** *is called to report the error and suspend the* **defined-function**. **Immediate-execution** *may return either a* **branch** *to indicate that evaluation of the* **defined-function** *should continue, or an* **escape** *or* **clear-state-indicator** *to indicate that the* **defined-function context** *should be removed from the* **state-indicator**.

13.2 Definitions

Note: *The following term defines the subset of* **arrays** *that can be used to create* **defined-functions**.

- **Proper-Defined-Function**: An **array** A is a **proper-defined-function** if
  The **type** of A is **character**.
  The **rank** of A is **two**.
  A has at least one **row**.
  The first **row** of A matches the **character-diagram header-line**.
  No **identifier** in the first row of A is the **distinguished-identifier** ⎕ or ⍞ or is a **system-function-name** or **niladic-system-function-name**.
  All **rows** after the first in A match the **character-diagram body-line**.

Note: *The following definitions give properties of arrays that are* **proper-defined-functions***.*

- **Niladic**: A, an **array** that is a **proper-defined-function**, is **niladic** if the **form** of the first **row** of A is **niladic-function-header**.
- **Monadic**: A, an **array** that is a **proper-defined-function**, is **monadic** if the **form** of the first **row** of A is **monadic-function-header**.
- **Ambivalent**: A, an **array** that is a **proper-defined-function**, is **ambivalent** if the **form** of the first **row** of A is **ambivalent-function-header**.
- **Monadic-Monadic-Operator**: A, an **array** that is a **proper-defined-function**, is **monadic-monadic-operator** if the **form** of the first **row** of A is **monadic-monadic-operator-header**.
- **Monadic-Dyadic-Operator**: A, an **array** that is a **proper-defined-function**, is **monadic-dyadic-operator** if the **form** of the first **row** of A is **monadic-dyadic-operator-header**.
- **Ambivalent-Monadic-Operator**: A, an **array** that is a **proper-defined-function**, is **ambivalent-monadic-operator** if the **form** of the first **row** of A is **ambivalent-monadic-operator-header**.
- **Ambivalent-Dyadic-Operator**: A, an **array** that is a **proper-defined-function**, is **ambivalent-dyadic-operator** if the **form** of the first **row** of A is **ambivalent-dyadic-operator-header**.
- **Value-Returning**: A, an **array** that is a **proper-defined-function**, is **value-returning** if, when **header-line** is threaded with the first **row** of A, the diagram **result** is threaded.

Note: *The following terms categorise* **defined-functions** *according to their presence on or absence from the* **state-indicator**.

- **Suspended**: N, a **simple-identifier**, is **suspended** if it is the **function-name** of a **context** that immediately follows an **immediate-execution context**.
- **Pendent**: N, a **simple-identifier**, is **pendent** if it is the **function-name** of a **context** that does not immediately follow an **immediate-execution context**.

– **Waiting**: *N*, a **simple-identifier**, is **waiting** if a **token** whose **class** is **defined-function-name** and whose **content** is the **content** of *N* occurs in the **stack** of some **context** in the **state-indicator** of the **active-workspace**, and is not the first **defined-function-name** in that **stack**.

Note: *For example, if F and G are defined function names, G is* **waiting** *during the evaluation of* (F 2) *in the line*

(F 2) G 3.

– **Editable**: *N*, a **simple-identifier**, is **editable** if the **current-referent** of *N* is **nil**, or if all the following are true:
  The **current-class** of *N* is **defined-function**, **defined-operator** or **niladic-defined-function**.
  The **number-of-items** in its **referent-list** is **one**.
  N is neither **pendent** nor **waiting**.
  N, if **suspended**, is **suspended** in only the **current-context**.

Note: *This definition requires that a* **conforming-implementation** *be capable of editing the top function on the state indicator as long as it has not been* **localised** *and occurs nowhere else on the state indicator. This minimal definition is included to be certain that all* **conforming-implementations** *be capable of interactive program correction.*

– **Globally-Erasable**: **Globally-Erasable** is defined for *N*, a **simple-identifier**, as follows:
  If no **context** of the **state-indicator** contains, in its **current-statement**, a **defined-name** with **content** that of *N*, then *N* is **globally-erasable**.
  Otherwise, let *L* be the last (most global) **context** in whose **local-name-list** *N* appears.
  If there is no such **context** then *N* is not **globally-erasable**.
  Otherwise, let *C* be the last (most global) **context** in whose **current-statement** a **defined-name** with **content** that of *N* appears.
  If the index of *L* is greater-than or equal to that of *C* then *N* is **globally-erasable**.

Note: *Informally, N is* **globally-erasable** *if it was localised before it was made waiting or pendent.*

– **Locally-Erasable**: **Locally-Erasable** is defined for *N*, a **simple-identifier**, as follows:
  If no **context** of the **state-indicator** contains, in its **current-statement**, a **defined-name** with **content** that of *N*, then *N* is **locally-erasable**.
  Otherwise, let *L* be the first (most local) **context** in whose **local-name-list** *N* appears.
  If there is no such **context** then *N* is not **locally-erasable**.
  Otherwise, let *C* be the first (most local) **context** in whose **current-statement** a **defined-name** with **content** that of *N* appears.
  If the index of *C* is greater than that of *L*, then *N* is **locally-erasable**.

Note: *Informally, N is* **locally-erasable** *if it has been localised since the last time it was made waiting or pendent. This definition is used in ⎕EX to specify that the expression* (⎕EX'F') F 1 *be the same as* 0 F 1*; it is used in ⎕FX to specify that the expression* (⎕FX 1 1⍴'F') F 1 *signal a* **domain-error**.

Note: *The following operations provide information about* **arrays** *that are* **proper-defined-functions**.

– **Function-Name** of A: For A a **proper-defined-function**, the **simple-identifier** that matches **function-name** when the **character-diagram header-line** is threaded with the first **row** of A.

– **Last-Line-Number** of A: For A a **proper-defined-function**, **negative-one plus** the **number-of-rows** in A.

– **Header-Name-List** of A: For A a **proper-defined-function**, a **list** of **identifiers** developed as follows:
Thread the **character-diagram header-line** with the first **row** of A.
Return a **list** of **tokens** consisting of **identifiers** that matched any of the **character-diagrams local-name**, **result-name**, **left-argument-name**, **right-argument-name right-operand-name**, or **left-operand-name**.

– **Label-Name-List** of A: For A a **proper-defined-function**, a **list** of **simple-identifiers** developed as follows:
Set Z to the **empty-list** of **tokens**.
For each **index** I in the **closed-interval-between one** and **last-line-number** of A,
Thread the **character-diagram body-line** with **row** I **plus one** of A.
If **labelled-line** was matched, append the **simple-identifier token** that matched the **character-diagram label-name** to Z as a new last **item**.
Return Z.

– **Local-Name-List** of A: For A a **proper-defined-function**, a **list** of **identifiers** consisting of the **label-name-list** followed by the **header-name-list**.

– **Identifier-Matching** D **in** A: For A a **proper-defined-function** and D a **character-diagram**, a **simple-identifier token** developed as follows:
Thread the **character-diagram header-line** with the first **row** of A.
Return a **token** of **class simple-identifier** whose **content** is the **list** of **characters** that matched D.

Note: **Identifier-Matching** *is used to refer to the arguments and result of an arbitrary defined function.*

– **Function-Line** I of A: For A a **proper-defined-function**, a **list** of **characters** developed as follows:
Thread the **character-diagram body-line** with **row** I **plus one** of A.
Return the **list** of **characters** that matched the **character-diagram line** in **body-line**.

Note: *The following operations affect* **symbols**.

– **Localise** N:
Append a new first **item**, a **token** of **class nil**, to the **referent-list** of the **symbol-named-by** N.

– **Delocalise** N:
If the **current-class** of N is **shared-variable**, **retract** N.
Remove the first **item** from the **referent-list** of the **symbol-named-by** N.

– **Current-Canonical-Representation**: The **canonical-representation** of the **current-function** of the **current-context** in the **active-workspace**.

– **Current-Function-Line** I: **Function-Line** I of the **current-canonical-representation**.

– **Current-Last-Line-Number**: The **last-line-number** of the **current-canonical-representation**.

– **Current-Stop-Vector**: The **stop-vector** of the **current-function** of the **current-context** in the **active-workspace**.

– **Current-Trace-Vector**: The **trace-vector** of the **current-function** of the **current-context** in the **active-workspace**.

– **Current-Local-Names**: The **local-name-list** of the **current-canonical-representation**.

– **Current-Right-Argument-Name**: The **identifier-matching right-argument-name** in the **current-canonical-representation**.

– **Current-Left-Argument-Name**: The **identifier-matching left-argument-name** in the **current-canonical-representation**.

– **Current-Result-Name**: The **identifier-matching result-name** in the **current-canonical-representation**.

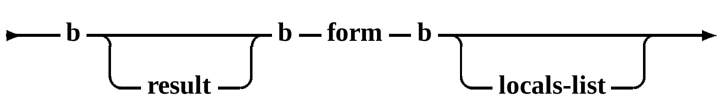
– **Current-Left-Operand-Name**: The **identifier-matching left-operand-name** in the **current-canonical-representation**.

– **Current-Right-Operand-Name**: The **identifier-matching right-operand-name** in the **current-canonical-representation**.

13.3 Diagrams

In these diagrams,
b stands for **permitted-blanks**.
r stands for **required-blanks**.

Header-Line



Example:

    Z ←A F B;C;⎕IO

Note: *The header line indicates the name and syntactic class of the* **defined-function**. *It also gives a list of identifiers to be* **localised** *when the* **defined-function** *is called.*

This International Standard does not provide for end-of-line comments in header lines.

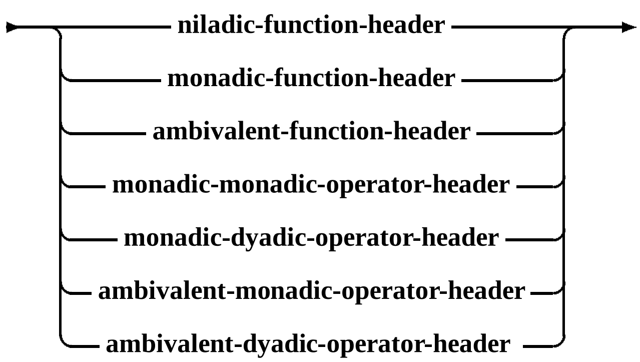
Result

►── **result-name** — **b** — **assignment-arrow** ───►

Result-Name

►── **simple-identifier** ───►

Form



Niladic-Function-Header

►── **function-name** ───►

Monadic-Function-Header

►── **function-name** — **r** — **right-argument-name** ───►

Ambivalent-Function-Header

►── **left-argument-name** — **r** — **function-name** — **r** — **right-argument-name** ───►

In the next four diagrams,

la stands for **left-argument-name**

ra stands for **right-argument-name**

mop stands for **monadic-operator-part**

dop stands for **dyadic-operator-part**

Monadic-Monadic-Operator-Header

►── **b** — **mop** — **b** — **ra** ──►

Monadic-Dyadic-Operator-Header

►── **b** — **dop** — **b** — **ra** ──►

Ambivalent-Monadic-Operator-Header

►── **la** — **b** — **mop** — **b** — **ra** ──►

Ambivalent-Dyadic-Operator-Header

►── **la** — **b** — **dop** — **b** — **ra** ──►

Monadic-Operator-Part

►── **(** — **b** — **left-operand-name** — **r** — **function-name** — **b** — **)** ──►

Dyadic-Operator-Part

►── **(** — **b** — **left-operand-name** — **r** — **function-name** — **r** — **right-operand-name** — **b** — **)** ──►

Right-Argument-Name

►── **simple-identifier** ──►

Left-Argument-Name

►── **simple-identifier** ──►

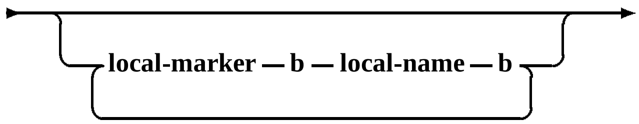
Right-Operand-Name

►── **simple-identifier** ──►

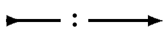
Left-Operand-Name

►── **simple-identifier** ──►

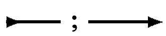
Locals-List



Colon



Local-Marker

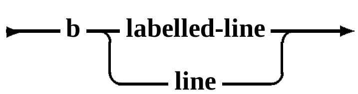


Function-Name

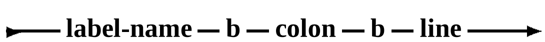
Local-Name

Label-Name

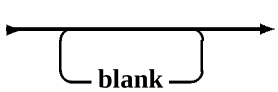
Body-Line



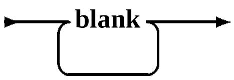
Labelled-Line



Permitted-Blanks



Required-Blanks



13.4 Operations

13.4.1 Call-Defined-Function

Z ← *DFN*

Z ← *DFN B*

Z ← *A DFN B*

Z ← **f** *DFN B*

Z ← *A* **f** *DFN B*

Z ← **f** *DFN* **g** *B*

Z ← *A* **f** *DFN* **g** *B*

Informal Description: *Z* is the value of the variable given as the result name in the header line of the defined function whose name is *DFN*. If there is no result name, *Z* is **nil**.

Evaluation Sequence:

Generate a new **context** in which
mode is **defined-function**,
current-line is the empty **list** of **characters**,
current-function is the **current-referent** of *DFN*,
current-line-number is **one**,
current-statement is the empty **list** of **tokens**, and
stack is the empty **list** of **tokens**.
Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.
Localise the **current-local-names**.
For form *Z*←*A* **f** *DFN* **g** *B*
If **current-function** is not **ambivalent-defined-dyadic-operator**, signal **valence-error**.
Set the **symbol-named-by current-left-argument-name** to *A*.
Set the **symbol-named-by current-left-operand-name** to **f**.
Set the **symbol-named-by current-right-operand-name** to **g**.
Set the **symbol-named-by current-right-argument-name** to *B*.
For form *Z*←**f** *DFN* **g** *B*

If **current-function** is not **monadic-defined-dyadic-operator**, signal **valence-error**.
Set the **symbol-named-by current-left-operand-name** to **f**.
Set the **symbol-named-by current-right-operand-name** to **g**.
Set the **symbol-named-by current-right-argument-name** to *B*.
For form *Z←A* **f** *DFN B*
If **current-function** is not **ambivalent-defined-monadic-operator**, signal **valence-error**.
Set the **symbol-named-by current-left-argument-name** to *A*.
Set the **symbol-named-by current-left-operand-name** to **f**.
Set the **symbol-named-by current-right-argument-name** to *B*.
For form *Z←***f** *DFN B*
If **current-function** is not **monadic-defined-monadic-operator**, signal **valence-error**.
Set the **symbol-named-by current-left-operand-name** to **f**.
Set the **symbol-named-by current-right-argument-name** to *B*.
For form *Z ←A DFN B*
If **current-function** is not **ambivalent**, signal **valence-error**.
Set the **symbol-named-by current-left-argument-name** to *A*.
Set the **symbol-named-by current-right-argument-name** to *B*.
For form *Z ←DFN*
If **current-function** is not **niladic**, signal **valence-error**.
For all **indices** *I* in the **closed-interval-between one** and **current-last-line-number**,
If **row** *I* **plus one** of **canonical-representation** matches the diagram **labelled-line**, set the **current-referent** of the **simple-identifier** that matches **label-name** to a **token** of **class label** and **content** the **numeric-scalar** with value *I*.
Set *Z* to **defined-function-control**.
Delocalise the **current-local-names**.
Remove the **first-item** in the **state-indicator** of the **active-workspace**.
If *Z* is **escape**, signal **unwind**.
Otherwise, return *Z*.

Note: Tokens *of* **class escape** *are converted to* **tokens** *of* **class unwind** *so that* **immediate-execution** *can distinguish an escape issued in* **immediate-execution** *mode from one issued in a* **defined-function**.

The **context** *attribute* **current-function** *includes the function itself plus its trace and stop vectors.*

13.4.2 Defined-Function-Control

Informal Description: This procedure controls the execution of a user defined function.

Evaluation Sequence:

Set Z to a **token** whose **class** is **nil**.
Let I stand for **current-line-number**.
Set I to **one**.
Repeat:
  If I is in the **current-stop-vector**, set **attention-flag** to **one**.
  If Z is an **error** or **attention-flag** is **one**,
    Set N to **immediate-execution** with Z.
    If N is **escape** or **clear-state-indicator**, return N.
    If N is **branch**, set I to the **content** of N.
  If I is not in the **closed-interval-between one** and **current-last-line-number**,
    If the **current-function** is not **value-returning**, return **nil**.
    If the **current-class** of the **current-result-name** is **variable** or **nil**, return the **current-referent** of the **current-result-name**.
    Otherwise, signal **value-error**.
  Set Z to **evaluate-line** of **current-function-line** I.
  If Z is **escape**, **unwind**, or **clear-state-indicator**, return Z.
  If Z is **branch**, set I to the **content** of Z.
  If Z is **value** or **nil**, set I to I plus **one**.
(End of repeated block)

Note: *The* **attention-flag** *is reset by the user facility* **enter**.

The **class** *of the* **token** *returned by* **evaluate-line** *for* →⍳0 *is* **nil**, *not* **branch**.

The **value-error** *signalled from* **defined-function-control** *is signalled in the line that invoked the defined function; the error is introduced to prevent* **conforming-programs** *from exiting a* **defined-function** *with the* **current-class** *of the* **result-name** *set to such classes as* **defined-function**, **niladic-defined-function**, *and* **shared-variable**.

13.4.3 Function Fix

Z ← ⎕*FX B*

Informal Description: *Z* is, for *B* the **canonical-representation** of a defined function or operator having **function-name** *N*, a **character** vector holding *N*. As a side effect, the defined function that *B* represents is established as the current referent of *N*. Rows of *B* that are all blanks give rise to blank lines in the established function.

Evaluation Sequence:

If the **rank** of *B* is not **two**, signal **rank-error**.
If *B* is an **empty array**, signal **length-error**.
If any **item** of the **ravel-list** of *B* is not a **character**, signal **domain-error**.
If *B* is not a **proper-defined-function**, signal **domain-error**.
If **identifiers** are duplicated in the **local-name-list** of *B*, signal **domain-error**.
Let *N* stand for the **simple-identifier** that matches **function-name** in the first **row** of *B*.
If the **current-class** of *N* is not **defined-operator**, **defined-function**, **niladic-defined-function**, or **nil**, signal **domain-error**.
If *N* is not **locally-erasable**, signal **domain-error**.
Set the **current-referent** of *N* to a **defined-function** or **defined-operator** for which
  canonical-representation is set from *B*,
  stop-vector is ⍳0,
  trace-vector is ⍳0.
Return a **character vector** *Z* such that the **length** of *Z* is the **number-of-items** in the **content** of *N* and the **ravel-list** of *Z* is the **content** of *N*.

13.4.4 Character Representation

Z ← ⎕*CR B*

Informal Description: *Z* is a **character** matrix representation of the defined function or operator named by *B*.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If the **length** of *B* is **zero**, signal **length-error**.
If *B* does not match the **character-diagram identifier-row**, signal **domain-error**.
If the **current-class** of *B* is not **defined-operator**, **defined-function** or **niladic-defined-function**, signal **domain-error**.
Otherwise return a **token** whose **class** is **constant** and whose **content** is the **canonical-representation** of the **current-content** of *B*.

Additional Requirement:

The formal model of APL used in this International Standard assumes that the internal representation of a defined function is precisely the **character** matrix satisfying the requirements for a **proper-defined-function** that was supplied as an argument to **function fix**. Representation of defined functions in this way in an implementation is neither required nor suggested.

The preservation of numeric literals and blanks as entered is desirable in the character representation, but it is not required by this International Standard. ⎕*CR* ⎕*FX* ⎕*CR* ⎕*FX* *X* shall be the same as ⎕*CR* ⎕*FX X*, if *X* is a **proper-defined-function**.

If **numeric-literals** are stored as **numbers** in **defined-functions**, they are to be formatted with **print-precision** set to **full-print-precision**.

13.5 Function Editing

Note: *The function editing facilities described here are included to provide the APL programmer with a standard method for entering and correcting defined functions.*

13.5.1 Evaluate-Function-Definition-Request

Evaluate-Function-Definition-Request *Q*.

Note: *This subsection provides minimal facilities for the establishment and alteration of* **defined-functions**.

An identifier can be edited by the function editing facilities only if it is is **editable**. *The header line of a* **defined-function** *can be altered only if the* **function-name** *is* **locally-erasable**.

In **function-definition mode**, *lines of the function being edited are temporarily associated with numbers in* **decimal-rational** *form by an unspecified mechanism. When a function is closed, the order of the lines given by the line number associated with each line is maintained.*

The display of a function should be the same as the entries required to add the function to a **clear-workspace**.

This operation is called by **immediate-execution** *when it recognises a* **function-definition-line**.

Evaluation Sequence:

Thread **opening-request** with *Q*; if *Q* does not match **opening-request**, signal **definition-error**.
Rethread **opening-request** with *Q*, taking the following actions:
When **creation-request** is matched,
Set *G* to the **list** of **characters** that matched **creation-request**.
Set *N* to the **simple-identifier** that matched **function-name** in *G*.
If the **current-class** of *N* is not **nil** and *G* does not match **identifier-row**, signal **definition-error**.
When **change-request** is matched,
Set *N* to the **simple-identifier** that matched the diagram **subject-function** in **change-request**.
Set *G* to the **list** of **characters** that matched **initial-request**.
If the **current-class** of *N* is **nil**, set *M* to a new **defined-function** for which
Canonical-Representation is `0 0⍴' '`
Stop-Vector is `⍳0`
Trace-Vector is `⍳0`.
Otherwise,
If *N* is not **editable**, signal **definition-error**.
Set *M* to the **current-referent** of *N*.
Generate a new **context** in which
mode is **function-definition**,
current-line is *G*,
current-function is *M*,
current-line-number is the **number-of-rows** in the **canonical-representation** of *M*,
current-statement is the empty **list** of **tokens**, and
stack is the empty **list** of **tokens**.
Append the **context** as a new first **item** to the **state-indicator**.
Repeat:
Set *Z* to **evaluate-editing-request**.
If *Z* is **command-complete**,
Remove the **first-item** in the **state-indicator** of the **active-workspace**.
Return *Z*.
If *Z* is an **error**, **display** *Z*.
Set **current-prompt** to **function-definition-prompt** of **current-line-number**.
Set **current-line** to **read-keyboard**.

(End of repeated block)

Additional Requirement:

The prompt in definition mode is a line number enclosed in brackets. Line numbers are always displayed in **decimal-rational** form. The line number of the header is always **zero**; it is not affected by **index-origin**.

13.5.2 Evaluate-Editing-Request

Evaluate-Editing-Request

Informal Description: This subsection acts on a single line of input to the editor.

Evaluation Sequence:

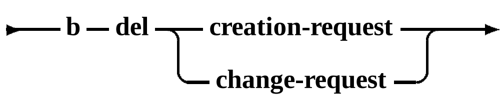
Let C stand for the **current-canonical-representation**.
Let N stand for the **simple-identifier** that matches **function-name** in **function-line**.
Let S stand for the **current-stop-vector**.
Let T stand for the **current-trace-vector**.
Thread **general-request** with **current-line**; if **general-request** does not match **current-line**, signal **definition-error**.
Otherwise, rethread **general-request** with **current-line**, taking the following actions:
When **positioning-request** is threaded,
Set L to the **numeric-input-conversion** of the **list** of **characters** that matched **line-number** in **positioning-request**.
If L is a **negative-number**, signal **definition-error**.
If L is **greater-than definition-line-limit**, signal **limit-error**.
Set **current-line-number** to L.
When **deletion-request** is threaded,
Set L to the **numeric-input-conversion** of the **list** of **characters** that matched **line-number** in **deletion-request**.
If L is not a **positive-number**, signal **definition-error**.
If L is **greater-than definition-line-limit**, signal **limit-error**.
Set **current-line-number** to L.
Delete from C the row associated with L, if it exists.
Delete from S and T the item associated with L, if it exists.
When **display-request** is threaded, **display function-display** of C.
When **function-line** is threaded, if **function-line** does not match **permitted-blanks**,
If **current-line-number** is **zero**,
If **function-line** does not match **header-line**, signal **definition-error**.
If N is not **locally-erasable**, signal **definition-error**.
Set the **row** of C associated with **current-line-number** to the **list** of **characters** that matched **function-line**.
If **current-line-number** is not an **integer**, or is **greater-than current-last-line-**

number, set the **items** of S and T associated with **current-line-number** to **zero**.
Set **current-line-number** to **next-definition-line** of **current-line-number**.
When **end-definition** is matched,
If C is not a **proper-defined-function**, signal **definition-error**.
If **identifiers** are duplicated in the **local-name-list** of C, signal **definition-error**.
Set the **current-referent** of N to a **defined-function** for which
Canonical-Representation is C.
Stop-Vector is S.
Trace-Vector is T.
Return **command-complete**.

13.5.3 Diagrams

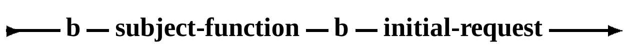
In these diagrams,
b stands for **permitted-blanks**.

Opening-Request

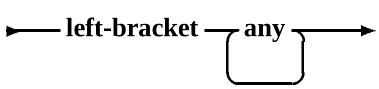


Creation-Request

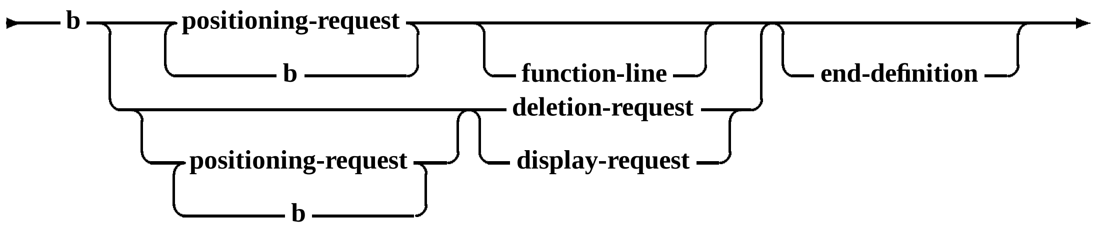
Change-Request



Initial-Request



General-Request



Positioning-Request

►— **left-bracket — b — line-number — right-bracket** —►

Deletion-Request

►— **left-bracket — b — delta — b — line-number — right-bracket** —►

Display-Request

►— **left-bracket — b — quad — b — right-bracket** —►

Function-Line

►— **body-line** —►

End-Definition

►— **b — del — b** —►

Line-Number

►— **decimal-rational — b** —►

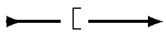
Subject-Function

►— **simple-identifier** —►

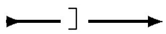
Delta

►— ∆ —►

Left-Bracket



Right-Bracket



14 Shared Variables

14.1 Informal Introduction

This section gives an informal introduction to the **Shared-Variable-Protocol**. The evaluation sequences which follow this introduction determine the required behaviour of **shared-variables** exactly.

The **Shared-Variable-Protocol** is an **optional-facility**. The **Shared-Variable-Protocol** is described in this document as an interface between cooperating **sessions**, but it may also be used to provide an interface between a **session** and an **auxiliary processor**. An **auxiliary processor** is a program, written in any programming language, that typically provides access to facilities in the underlying operating system, such as file systems.

A **shared-variable** can be shared by precisely two partners, each either a **session** or an **auxiliary processor**.

Conforming-programs that use the **Shared-Variable-Protocol** should document this fact, since the **Shared-Variable-Protocol** is not a **defined-facility** of this International Standard.

Operations

There are conceptually four operations involved in the **Shared-Variable-Protocol**: **offer, retract, set, and reference.**

Sharing

Offer is an operation provided to permit **sessions** to share **shared-variables**. A given **shared-variable** can be shared between at most two **sessions**. Once a **shared-variable** has been shared, assignments made to it in one **session** are visible in the other **session**.

In the evaluation sequences for the **Shared-Variable-Protocol**, sharing is accomplished by appending **tokens** of **class shared-variable** to the **shared-variable-list** which can be accessed by all **active-workspaces** in the **system**.

Any **Shared-Variable-Protocol** operation may cause a **session** to be delayed while another **session** changes a **shared-variable** or adds an item to the **shared-variable-list**.

Sharing begins when one **session** offers to share a **symbol** whose **current-class** is **nil** or **variable**. The **system** adds a new **shared-variable** to the **shared-variable-list** and returns the **shared-variable** to the offering **session**. The **shared-variable** then replaces the **current-referent** of the offered **symbol**.

There are two forms of offers. A **specific-offer** identifies a particular **session** as the partner; the **general-offer** does not.

Two offers match if both **sessions** use the same **shared-name** and:
At least one of the offers is a **specific-offer**.
Either **specific-offer** identifies the other **session**.

Consequently, two **general-offers** do not match.

Once both **sessions** have shared the **shared-variable**, the **shared-variable** is the **current-referent** of a **symbol** in both **active-workspaces**, and a change made to the **shared-variable** by one **session** will be visible to the other **session**.

Note that two **sessions** can share more than one **shared-variable**, that a given **session** can share **shared-variables** with more that one other **session**, and that each **shared-variable** can be shared by only two **sessions**.

Retracting

Two **sessions**, coupled by a **shared-variable**, decouple when either **retracts** the **shared-variable**. This leaves the **shared-variable** in the same state it would have been had the remaining **session** made a **specific-offer** to the **session** that **retracted**. If the remaining **session retracts**, the **shared-variable** becomes inaccessible to the **sessions**.

Degree of Coupling

All **symbols** in the **symbol-table** of an **active-workspace** have a **degree of coupling**. This is **zero** if the **current-referent** is not a **shared-variable**, **one** if the **current-referent** is a **shared-variable** that is offered to another **session**, and **two** if **current-referent** is a **shared-variable** shared with another **session**.

Naming

For any session, there are two **identifiers** associated with a given **shared-variable**. The first is the **name** of the **symbol** whose **current-referent** is the **shared-variable**; the second is the **shared-name** of the **shared-variable**. The **shared-name** is the name used in matching offers to share.

Only the **name** of the **symbol** is required for invoking any of the **Shared-Variable-Protocol** system functions other than **Shared Variable Offer**.

Setting, Referencing, and Using

Each partner can **set** and **reference** the **shared-variable**.

A **shared-variable** is **set** when the operation **shared-variable-assignment** or **shared-variable-indexed-assignment** is called. A **shared-variable** is **referenced** when the operation **shared-variable-reference** is called. A **shared-variable** is used when it is either **set** or **referenced**. Both indexed assignment and indexed reference of **shared-variables** are possible.

Synchronisation

Synchronisation of **sessions** sharing a **shared-variable** is achieved by **access control**. With each **shared-variable** is associated an **access-control-vector**, which is designated here as the *ACV*. The *ACV* is a four element Boolean vector that is accessible to a **session** through the **Shared-Variable-Access-Control-Inquiry** operation.

The order of the elements in the *ACV* is relative to the viewing **session**. When **session** A, sharing a **shared-variable** with **session** B, views the *ACV*, its elements have the following meanings:

– If `ACV[1]` is **one**, once A has set the **shared-variable** B must use it before A can set it again.

– If `ACV[2]` is **one**, once B has set the **shared-variable** A must use it before B can set it again.

– If `ACV[3]` is **one**, once A has used the **shared-variable** B must set it before A can reference it again.

– If `ACV[4]` is **one**, once B has used the **shared-variable** A must set it before B can reference it again.

If a **session** attempts to use a **shared-variable** when not permitted to by the above rules, it is delayed until the partner has performed the required intermediate operation.

The rules can be summarised in a three-state state diagram. Let A be the **session** that made the initial offer. Then the states are as follows:

– State 0: The **shared-variable** has been **referenced** by the partner of the **session** that last set it.

– State 1: The **shared-variable** was last **set** by A and has not been **referenced** by B since then.

– State 2: The **shared-variable** was last **set** by B and has not been **referenced** by A since then.

State 0 is the state entered when the **shared-variable** is initially offered. Note that only an explicit assignment is considered a **set** to the **shared-variable**. The act of transferring the **current-content** of a **symbol** to the **shared-value** of a **shared-variable** when an offer

Action attempted:		SET by A			SET by B			REF by A			REF by B		
Current state:	0		+				+	3			4		
	1		1				+		3		+		
	2		+				2	+					4
Next state:		0	1	2	0	1	2	0	1	2	0	1	2

Figure 2: Shared Variable Access Rules.

is made is not a **set**. The state is not affected by retraction and re-sharing. **Figure 2** summarises the state transitions permitted. In the figure, the action is always permitted if there is a + in the row corresponding to the input state. The column in which the + appears determines the output state. A digit in the row indicates that the corresponding element in the *ACV* as seen by A must be zero.

For example, if the state of a **shared-variable** is `0` and the *ACV* is `0 0 1 0`, **session** A can assign a new value to the variable (SET by A), but must wait until the variable is in state `2` to refer to it (REF by A).

Note that all digits lie along the diagonal in these matrices, indicating that only actions that do not change the state can be blocked by the *ACV*. That is, the *ACV* can only prevent a given session from referencing or setting a **shared-variable** twice in a row.

Other Rules

Retraction of all the **shared-variables** in a **session** takes place when a new **workspace** is loaded, the **active-workspace** is cleared, or the **session** ends. A **shared-variable** is automatically **retracted** when the **shared-variable** is erased, expunged, or replaced by a `)`*COPY* command, or, if it is local, when the function to which it is local terminates execution. The state of sharing is not saved in a library workspace.

The shared variable mechanism must appear to the user to be consistent with one in which there exists a single copy of each **shared-variable**. Specifically, this means:

– When a **shared-variable** is initially offered, its value, if any, becomes the shared-value of the **shared-variable**.

– When an offered **shared-variable** is matched, it retains its value if it has one. If it does not have a value, it takes on the value (if any) in the matcher's workspace.

– When a **shared-variable** is retracted, its value in the retractor's workspace is the value last set by either partner before the retraction.

Note that, since there is conceptually a single copy across a period of sharing, no use is implied in the above rules for making the value of a **shared-variable** available.

It must appear to the APL user that **sessions** communicating via **shared-variables** are independently scheduled. Any synchronisation between the **sessions** must appear to be by means of the delays caused by accessing **shared-variables**. The procedures used to describe the **Shared-Variable-Protocol** are written as though synchronisation conflicts do not occur when two **sessions** are accessing the **shared-variable-list** or the same **shared-variable**.

A **limit-error** may occur during the retraction of an active **shared-variable** if there is not enough room in a **workspace** for the **shared-value** of the **shared-variable**. When the expected system action is to ignore the variable (as in the case of *)CLEAR* or delocalisation), a **limit-error** is not signalled.

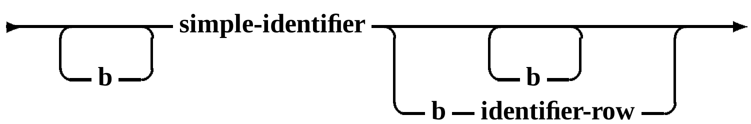
14.2 Definitions

– **Identifier-Pair-Array**: A **character array** of **rank two** in which each **row** matches the **character-diagram identifier-pair-row**.

14.3 Diagrams

In the following, **b** stands for **blank**.

Identifier-Pair-Row



14.4 Operations

14.4.1 Primary-Name

Primary-Name in A: For A, a **character vector** that matches **identifier-pair-row**, the first **simple-identifier** matched in **identifier-pair-row**.

14.4.2 Surrogate-Name

Surrogate-Name in A: For A, a **character vector** that matches **identifier-pair-row**, the last **simple-identifier** matched in **identifier-pair-row**.

14.4.3 Degree-of-Coupling

Degree-of-Coupling of N: For **symbol** N, a **number** defined as follows:
If the **current-class** of N is not **shared-variable**, **zero**.
Otherwise, using the **current-referent** of N, the sum of **session-A-active** and **session-B-active**.

14.4.4 Access-Control-Vector

Access-Control-Vector of N: An operation that, for N a **shared-variable**, is defined as follows:
Using N,
Let $ACVA$ stand for **session-A-ACV** and $ACVB$ stand for **session-B-ACV**.
If **this-session** is **session-A**, return $ACVA \vee ACVB$.
If **this-session** is **session-B**, return `(`$ACVA \vee ACVB$`)[2 1 4 3]`.

14.4.5 Offer

Offer S **to** P **with value** V: An operation defined as follows:
Let X stand for the **item** in the **shared-variable-list** with the smallest **index** such that
shared-name is S.
session-A is **this-session** or, if P is not **general-offer**, **general-offer**.
session-A-active is **zero**.
session-B is P.
session-B-active is **one**.
or
shared-name is S.
session-A is P.
session-A-active is **one**.
session-B is **this-session** or, if P is not **general-offer**, **general-offer**.
session-B-active is **zero**.
If such a **shared-variable** exists,
If **shared-value** is **nil**, set **shared-value** to V.
If **session-A-active** is **zero**,
Set **session-B-event** to **one**.
Set **session-A-active** to **one**.
Set **session-A** to **this-session**.
If **session-B-active** is **zero**,
Set **session-A-event** to **one**.
Set **session-B-active** to **one**.
Set **session-B** to **this-session**.
Return X.
Otherwise, create a new **shared-variable**, Y, as follows:
shared-name is S.

session-A is **this-session**.
session-A-active is **one**.
session-A-ACV is `0 0 0 0`.
session-B is *P*.
session-B-active is **zero**.
session-B-ACV is `0 0 0 0`.
shared-value is *V*.
state is **zero**.
session-A-event is **zero**.
session-B-event is **one**.
Append *Y* to the **shared-variable-list** as a new last **item**.
Return *Y*.

14.4.6 Retract

Retract *N*: An operation that, for *N* a **shared-variable**, is defined as follows:
Using *N*,
If **session-A** is **this-session**, set **session-A-active** to **zero**.
If **session-B** is **this-session**, set **session-B-active** to **zero**.
Signal-event.
Clear-event.

14.4.7 Shared-Variable-Reset

Shared-Variable-Reset
For each item in the **shared-variable-list**,
If **session-A** is **this-session**, set **session-A-active** to **zero**.
If **session-B** is **this-session**, set **session-B-active** to **zero**.
For each **symbol** *N* of the **symbol-table** of the **active-workspace**,
For each **token** *T* in the **referent-list** of *N*,
If *T* is a **shared-variable**, set *T* to a **token** whose **class** is **nil**.

Note: *In the model, a* **shared-variable** *is never reused. It is effectively discarded once* **session-A-active** *and* **session-B-active** *are both* **zero**.

14.4.8 Report-State

Report-State for *N*: An operation that, for *N* a **shared-variable**, is defined as follows:
Using *N*,
If **state** is **zero**, return `0 0 1 1`.
Otherwise,
If **state** is **one** and **session-A** is **this-session**,
or if **state** is **two** and **session-B** is **this-session**,

Return `1 0 1 0`.
Otherwise return `0 1 0 1`.

14.4.9 Signal-Event

Signal-Event for *N*: An operation that, for *N* a **shared-variable**, is defined as follows:
Using *N*,
If **session-A** is **this-session**, set **session-B-event** to **one**.
If **session-B** is **this-session**, set **session-A-event** to **one**.

14.4.10 Clear-Event

Clear-Event for *N*: An operation that, for *N* a **shared-variable**, is defined as follows:
Using *N*,
If **session-A** is **this-session**, set **session-A-event** to **zero**.
If **session-B** is **this-session**, set **session-B-event** to **zero**.

14.5 Shared Variable Forms

14.5.1 Shared Variable Reference

Z ← *SHV*

Informal Description: Return the value of a shared variable.

Evaluation Sequence:

Using the **current-content** of *SHV*,
Let *ACV* stand for **access-control-vector**.
Let *ASV* stand for **report-state**.
Set *A* to *ACV*^*ASV*.
If *A*[2] or *A*[3] is **one**, **signal-event**.
Wait until at least one of the following is true:
ACV[3] is **zero**.
State is **two** and **session-A** is **this-session**.
State is **one** and **session-B** is **this-session**.
If either of the following is true, set **State** to **zero**:
State is **two** and **session-A** is **this-session**.
State is **one** and **session-B** is **this-session**.
Clear-event.
Return **Shared-Value**.

Note: *This operation is all that is required for* **shared-variable-indexed-reference***, since the* **phrase-evaluator** *calls* **indexed reference** *with a value, not a name.*

14.5.2 Shared Variable Assignment

$Z \leftarrow SHV \leftarrow B$

Informal Description: Z is B. As a side effect, the value B is assigned to the shared variable SHV.

Evaluation Sequence:

Using the **current-content** of SHV,
  Let ACV stand for **access-control-vector**.
  Let ASV stand for **report-state**.
  Set A to ACV^ASV.
  If A[1] or A[4] is **one**, **signal-event**.
  Wait until at least one of the following is true:
    ACV[1] is **zero**.
    State is not **one** and **session-A** is **this-session**.
    State is not **two** and **session-B** is **this-session**.
  Set **shared-value** to B.
  If **session-A** is **this-session**, set **state** to **one**.
  If **session-B** is **this-session**, set **state** to **two**.
  Clear-event.
  Return a **token** whose **class** is **committed-value** and whose **content** is B.

14.5.3 Shared Variable Indexed Assignment

`Z ← SHV[I] ← B`

Informal Description: *Z* is *B*. As a side effect, elements of the array *SHV* selected by the position and values of the arrays in the **index-list** *I* are replaced by elements of the array *B*.

Evaluation Sequence:

Using the **current-content** of *SHV*,
- Let *ACV* stand for **access-control-vector**.
- Let *ASV* stand for **report-state**.
- Set *A* to *ACV∧ASV*.
- If *A*`[1]` or *A*`[4]` is **one**, **signal-event**.
- Wait until at least one of the following is true:
  - *ACV*`[1]` is **zero**.
  - **session-A** is **this-session** and **state** is not **one**.
  - **session-B** is **this-session** and **state** is not **two**.
- If the **current-class** of **shared-value** is **nil**, signal **value-error**.
- Set *C* to **shared-value**.
- Search the **form-table** for `Z ←V[I] ←B`.
- Call the corresponding evaluation sequence, passing *C* as the value of *V*.
- Set *Z* to the **token** it returns.
- If *Z* is an **exception**, return *Z*.
- Otherwise,
  - Set **shared-value** to *C*.
  - If **session-A** is **this-session**, set **state** to **one**.
  - If **session-B** is **this-session**, set **state** to **two**.
  - **Clear-event**.
  - Return *Z*.

Note: *Indexed assignment of a shared variable constitutes a* **set***, not a* **reference***, of the* **shared-variable**.

14.6 Shared Variable System Functions

14.6.1 Shared Variable Access Control Inquiry

`Z ← ⎕SVC B`

Informal Description: Each row of *Z* is the **access-control-vector** of the shared variable named by the corresponding row of *B*.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
Set *B*1 to (¯2↑1 1,ρ*B*)ρ*B*.
If any **item** of the **ravel-list** of *B*1 is not a **character**, signal **domain-error**.
Set *Z* to ((1↑ρ*B*1),4)ρ0.
For every *I* in ⍳1↑ρ*B*1, evaluated with **index-origin** set to **one**,
  Let *B*2 stand for **row** *I* of *B*1.
  If *B*2 matches **identifier-row**,
    Set *N* to the **simple-identifier** in *B*2.
    If the **current-class** of *N* is **shared-variable**,
      Set **row** *I* of *Z* to **access-control-vector** of the **current-referent** of *N*.
  Otherwise, signal **domain-error**.
Return ((¯1↓ρ*B*),4)ρ*Z*.

14.6.2 Shared Variable Query

Z ← ⎕*SVQ B*

Informal Description: *Z* is one of the following:

An array representing sessions offering to share variables with this session.

An array representing the names of shared variables offered to this session by another session.

Evaluation Sequence:

If **session-identification-type** is **character**, and *B* is the **general-offer**,
  Set *Z* to `0 0⍴' '`.
  Using each **item** of the **shared-variable-list**,
    If
      session-A is **this-session** and
      session-A-active is **zero** and
      session-B-active is **one**,
        Set *S* to **session-B**.
        Set *Z* to `((⍴Z)⌈0,⍴S),[1]((¯1↑⍴Z)⌈⍴S)↑S`, evaluated with **index-origin** set to **one**.
    If
      session-A-active is **one** and
      session-B is **this-session** and
      session-B-active is **zero**,
        Set *S* to **session-B**.
        Set *Z* to `((⍴Z)⌈0,⍴S),[1]((¯1↑⍴Z)⌈⍴S)↑S`, evaluated with **index-origin** set to **one**.
  Return *Z*.
If **session-identification-type** is **numeric**, and *B* is **zero**, signal **domain-error**.
If **session-identification-type** is **numeric**, and *B* is `⍳0`,
  Set *Z* to `⍳0`.
  Using each **item** of the **shared-variable-list**,
    If all of the following are true,
      session-A is **this-session**.
      session-A-active is **zero**.
      session-B-active is **one**.
    append **session-B** as a new last element to the **vector** *Z*.
    If all of the following are true,
      session-A-active is **one**.
      session-B is **this-session**.
      session-B-active is **zero**.
    append **session-A** as a new last element to the **vector** *Z*.
  Return `((Z⍳Z)=⍳⍴Z)/Z`.
Otherwise,
  If *B* is not a **session-identification**, signal **domain-error**.
  If *B* is **this-session**, signal **domain-error**.

Set *Z* to the **empty character array** of **rank two**.
Using each **item** of the **shared-variable-list**,
If all of the following are true,
session-A is **this-session**.
session-A-active is **zero**.
session-B is *B*.
session-B-active is **one**.
append **shared-name** as a new last row to the **identifier-array** *Z*.
If all of the following are true,
session-A is *B*.
session-A-active is **one**.
session-B is **this-session**.
session-B-active is **zero**.
append **shared-name** as a new last row to the **identifier-array** *Z*.
Return *Z*.

14.6.3 Shared Variable Degree of Coupling

`Z ← ⎕SVO B`

Informal Description: *Z* is a numeric vector in which each element is the degree of coupling of the symbol named by the corresponding row of *B*.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
Set `B1` to `(¯2↑1 1,ρB)ρB`.
If any **item** of the **ravel-list** of `B1` is not a **character**, signal **domain-error**.
Set *Z* to `(1↑ρB1)ρ0`.
For every *I* in `⍳1↑ρB1`,
Let `B2` stand for **row** *I* of `B1`.
If `B2` matches **identifier-row**,
Set *N* to the **primary-name** in `B2`.
Set **item** *I* of the **ravel-list** of *Z* to **degree-of-coupling** of the **symbol-named-by** *N*.
Otherwise, signal **domain-error**.
Return *Z*.

Note: Degree-of-coupling *is a property of* **symbols**, *not* **shared-variables**.

14.6.4 Shared Variable Offer

Z ← *A* ⎕*SVO* *B*

Informal Description: *Z* is a numeric vector representing the **degree-of-coupling** of the identifiers in the rows of the **identifier-pair-array** *B* after the operation. As a side effect, dyadic ⎕*SVO* shares variables with another **session**. *B* is an **identifier-pair-array** representing the names of variables to be shared. *A* is a **session-identification**, representing the **session** with which the names in *B* are to be shared.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
If any **item** of the **ravel-list** of *B* is not a **character**, signal **domain-error**.
If *A* is not a valid **session-identification**, signal **domain-error**.
If **this-session** is *A*, signal **domain-error**.
Set *B*1 to (¯2↑1 1,ρ*B*)ρ*B*.
For every *I* in ⍳1↑ρ*B*1,
  Let *B*2 stand for **row** *I* of *B*1.
  If *B*2 does not match **identifier-pair-row**, signal **domain-error**.
  Otherwise,
    Set *N* to the **primary-name** in *B*2.
    Set *S* to the **surrogate-name** in *B*2.
    If the **current-class** of *N* is neither **shared-variable**, **variable** nor **nil**, signal **domain-error**.
    If the **current-class** of *N* is **variable** or **nil**,
      Set *V* to the **current-referent** of *N*.
      Set *Y* to **offer** *S* **to** *A* **with value** *V*.
      Set the **current-referent** of *N* to *Y*.
Return ⎕*SVO* *B*.

Note: *This operation does not allow for multiple* **session-identifications**.

14.6.5 Shared Variable Retraction

`Z ← ⎕SVR B`

Informal Description: Elements of *Z* hold the degree of coupling that the identifier represented in the corresponding row of the **identifier-array** *B* had, before this retraction. As a side effect, ⎕*SVR* retracts any shared-variables named in *B*.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
Set *B*1 to `(¯2↑1 1,⍴B)⍴B`.
Set *Z* to ⎕*SVO B*1.
If *Z* is an **error**, return *Z*.
For every *I* in `⍳1↑⍴B1`,
  Let *B*2 stand for **row** *I* of *B*1.
  If *B*2 matches **identifier-row**,
    Set *N* to the **simple-identifier** in *B*2.
    If the **current-class** of *N* is **shared-variable**,
      Set *SV* to the **current-referent** of *N*.
      Set the **current-referent** of *N* to **shared-value** of *SV*.
      Retract *SV*.
  Otherwise, signal **domain-error**.
  Return *Z*.

14.6.6 Shared Variable Access Control Set

Z ← *A* ⎕*SVC* *B*

Informal Description: *Z* is the **access-control-vector** of each of the shared variables represented by the **identifier-array** *B* when ⎕*SVC* completes. As a side effect, ⎕*SVC* sets to *A* the contribution this session makes to the combined access control vector of the shared variables.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
Set *B*1 to (¯2↑1 1,ρ*B*)ρ*B*.
If the **rank** of *A* is **greater-than two**, signal **rank-error**.
If the first **item** of (¯1↑4,ρ*A*) is not **four**, signal **length-error**.
If *A* is a **scalar** or a **vector**, set *A*1 to ((1↑ρ*B*1),4)ρ*A*.
Otherwise, set *A*1 to *A*.
If 1↑ρ*A*1 is not the same as 1↑ρ*B*1, signal **length-error**.
If any **item** of the **ravel-list** of *B*1 is not a **character**, signal **domain-error**.
If any **item** of the **ravel-list** of *A*1 is not a **near-Boolean**, signal **domain-error**.
Set *A*2 to the **Boolean-array-nearest-to** *A*1.
For every *I* in ⍳1↑ρ*B*1,
  Let *B*2 stand for **row** *I* of *B*1.
  If *B*2 matches **identifier-row**,
    Set *N* to the **simple-identifier** in *B*2.
    If the **current-class** of *N* is **shared-variable**,
      Let *ACV* stand for **row** *I* of *A*2.
      Using the **current-content** of *N*,
        If **session-A** is **this-session**, set **session-A-ACV** to *ACV*.
        If **session-B** is **this-session**, set **session-B-ACV** to *ACV*[2 1 4 3].
        Signal-event.
  Otherwise, signal **domain-error**.
Return ⎕*SVC* *B*.

14.6.7 Shared Variable State Inquiry

`Z ← ⎕SVS B`

Informal Description: Each row of `Z` is a boolean 4-element representation of the **state** of the shared variable named by the corresponding row of `B`, as seen by **this-session**. It may be combined with the **access-state-vector** to show which accesses are currently possible for the **shared-variable**.

Evaluation Sequence:

If the **rank** of `B` is **greater-than two**, signal **rank-error**.
Set `B1` to `(¯2↑1 1,⍴B)⍴B`.
If any **item** of the **ravel-list** of `B1` is not a **character**, signal **domain-error**.
Set `Z` to `((1↑⍴B1),4)⍴0`.
For every `I` in `⍳1↑⍴B1`, evaluated with **index-origin** set to **one**,
  Let `B2` stand for **row** `I` of `B1`.
  If `B2` matches **identifier-row**,
    Set `N` to the **simple-identifier** in `B2`.
    If the **current-class** of `N` is **shared-variable**,
      Set **row** `I` of `Z` to **report-state** of the **current-referent** of `N`.
  Otherwise, signal **domain-error**.
Return `((¯1↓⍴B),4)⍴Z`.

14.6.8 Shared Variable Event

⎕SVE ← B

Z ← ⎕SVE

Informal Description: Assignment to the system variable starts a timer. The specified number of seconds is the maximum time the program will wait for a **shared-variable-event**.

Reference to the system variable suspends program execution until a **shared-variable-event** occurs, or until maximum time has elapsed. Then *Z* is the remaining time in the timer, and all outstanding **shared-variable-events** are cleared.

A **shared-variable-event** occurs whenever a new offer to share a variable occurs, or whenever certain operations are performed on a shared variable by the partner (see the various Evaluation Sequences in this chapter).

Evaluation Sequence:

- For form *⎕SVE ← B*
  - Set *T0* to **current-time** in seconds.
  - If *B* is not a **scalar**, signal **rank-error**.
  - If any **item** of the **ravel-list** of *B* is not a **nonnegative-number**, signal **domain-error**.
  - Set **event-time** to *T0* **plus** *B*.
- For form *Z ← ⎕SVE*
  - Wait until at least one of the following is true:
    - **current-time** is not **less-than event-time**.
    - For any item *N* in the **shared-variable-list**,
      - Using *N*,
        - If **session-A** is **this-session**, **session-A-event** is **one**.
        - If **session-B** is **this-session**, **session-B-event** is **one**.
  - Return **event-time minus current-time**.
  - For each item *N* in the **shared-variable-list**, **clear-event** for *N*.

15 Formatting and Numeric Conversion

15.1 Introduction

The mapping of arrays of numbers into arrays of characters is referred to as formatting.

Note: *The form, field width, and number of digits in a formatted number can be specified explicitly through the use of* **dyadic format***, or left to be chosen by the system.* **Conforming-programs** *that depend upon a particular selection of formatting parameters should employ* **dyadic format***.*

15.2 Numeric Conversion

Note: *Conversion involves the transformation of* **lists** *of* **characters** *to and from* **numbers***. This conversion is performed by the* **implementation-algorithms numeric-input-conversion** *and* **numeric-output-conversion***.*

The accuracy and rounding properties of **numeric-input-conversion** and **numeric-output-conversion** are interdependent. The most significant property of these **implementation-algorithms** is the following:

For any **numeric scalar** X, when **print-precision** is set to **full-print-precision**, X shall be the same as ⍎⍕X.

15.2.1 Numeric-Input-Conversion

Numeric-Input-Conversion of C.

Informal Description: Numeric input conversion converts numeric values represented in decimal notation as **lists** of **characters** into equivalent **numbers**—numeric quantities whose format is **implementation-defined**.

In the following, C stands for a **real-number**, or the **real-part** or **imaginary-part** of a **complex-number**.

Evaluation Sequence:

Let L and G be respectively the most negative and the most positive abstract numeric values represented by all **numeric-scalar-literals** producible by **numeric-output-conversion** for all **numbers** and all valid values of **print-precision**.
Let T be the set of all **lists** of **characters** that match the diagram **numeric-scalar-literal** and have abstract numeric values lying in the closed interval between L and G.
If C is not an element of T, signal **limit-error**.
Otherwise,
  Let **c** stand for the abstract numeric value of C.
  If **c** is the same as some **number** N, return N.
  If **c** is greater than **positive-number-limit**, return **positive-number-limit**.
  If **c** is less-than **negative-number-limit**, return **negative-number-limit**.
  Otherwise, **c** lies between two **numbers**. Return one that permits **numeric-input-conversion** to satisfy the following property:
    For any two **numeric-scalar-literals** $C1$ and $C2$, if the abstract numeric value of $C1$ is less than the abstract numeric value of $C2$, then **numeric-input-conversion** of $C1$ is not **greater-than numeric-input-conversion** of $C2$.

Examples:

```
      1-1.000000000000000000000000000000000001
0
      1-0.999999999999999999999999999999999999
0
      0.00000000000000000001E20
1
      0E999
0
      1E¯9999 = 0
1
      2.5E4-25000
0
```

Note: *In many programming languages, the syntactic form of a literal is used to determine the datatype of the corresponding number. This is not so in APL. Therefore, it is important that the input conversion routine not make distinctions between different forms of the same abstract numeric value.* `2.5E4` *should produce the same* **number** *as* `25000` *does and* `002` *should produce the same number as* `2` *does.*

Every **number** *can be obtained by converting some* **numeric-scalar-literal**.

The set T in the evaluation sequence above may include **numbers** *with* **real-part** *or* **imaginary-part** *having abstract numeric values less than* **negative-number-limit** *or greater than* **positive-number-limit**, *due to rounding by* **numeric-output-conversion**.

There is no **number** N *for which the expression* `⍎⍕N` *signals a* **limit-error**.

15.2.2 Numeric-Output-Conversion

E **Numeric-Output-Conversion** of N.

E **Numeric-Output-Conversion** of N **to** P **places**.

Informal Description: Numeric output conversion converts numeric values represented as **numbers**—numeric quantities whose format is **implementation-defined**—into the same numeric quantities represented in decimal notation as **lists** of **characters**.

For E a **character-diagram**, and N and P **numbers**, numeric output conversion returns a **numeric-scalar-literal** that matches E and represents the abstract numeric value of the **real-part** of N either to P decimal places, if P is specified, or else to **print-precision** places.

Note: *Any* **imaginary-part** *of the numeric input to* **Numeric-Output-Conversion** *is simply ignored; only the* **real-part** *of a* **complex-number** *is used.*

Evaluation Sequence:

If E is **minimal-decimal-exponential**,
Return a **list** of **characters** whose abstract numeric value is a good approximation of N.

Note: *The choice of approximation is* **implementation-defined**, *but can be characterised as follows:*
Let DS *be the set whose members* D *are* **lists** *of* **characters** *that match* **minimal-decimal-exponential** *and have at most* **print-precision** *digits to the left of the* **exponent-marker**.
Let DT *be the subset of* DS *for which* $|N-⍎D$ *is minimal.*
Let DM *be the subset of* DT *whose members have the fewest digits.*
Choose the member of DM *whose abstract numeric value has the largest magnitude.*

If E is **fixed-decimal**,
Let DF be the set whose members are **lists** of **characters** that match **fixed-decimal** and have P digits to the right of the units digit.
Return a member D of DF for which $|N-⍎D$ is minimal.

If E is **decimal-exponential**,
Let DE be the set whose members are **lists** of **characters** that match diagram **decimal-exponential** and have P digits to the left of the **exponent-marker**.
Select a member D of DE for which $|N-⍎D$ is minimal.
Let W stand for **exponent-field-width**.
Return $(W+D⍳\text{'}E\text{'})↑D$.

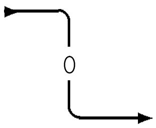
Note: *When* P *is not specified,* E *is always* **minimal-decimal-exponential**.

Fixed-decimal *and* **decimal-exponential** *cases are used only by* **dyadic format**. *They are permitted, but not required, to return results of arbitrarily high precision. They should, however, return accurate results if no more than* **full-print-precision** *significant digits are requested.*

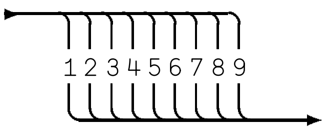
15.3 Diagrams

Note: *The following* **character-diagrams** *characterise the outputs of both monadic and dyadic format as subsets of the set of* **lists** *of* **characters** *that match* **numeric-scalar-literal***.*

Zero-Digit

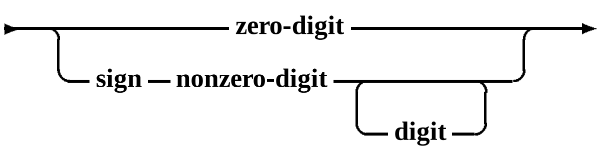


Nonzero-Digit



Sign

Decimal-Integer



Examples:

```
¯44 0 622 1066 1215 1415 1685 1750 1776 1812 1867 1944
```

In the following diagrams,

– **b** stands for **blank**.

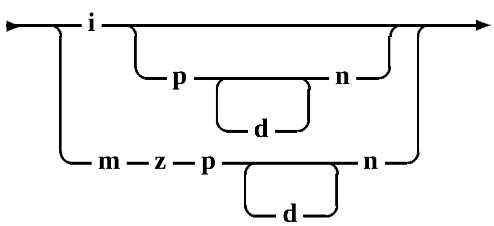
– **d** stands for **digit**.

– **e** stands for **exponent-marker**.

– **i** stands for **decimal-integer**.

- **m** stands for **overbar**.
- **n** stands for **nonzero-digit**.
- **p** stands for **dot**.
- **s** stands for **sign**.
- **z** stands for **zero-digit**.

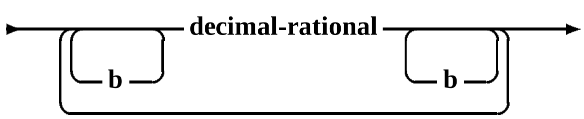
Decimal-Rational



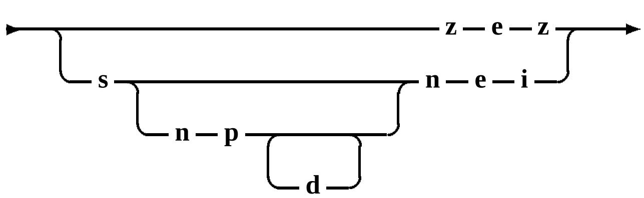
Examples:

```
98.6 212 ¯2.001 ¯0.00000000001
```

Decimal-Rational-Row



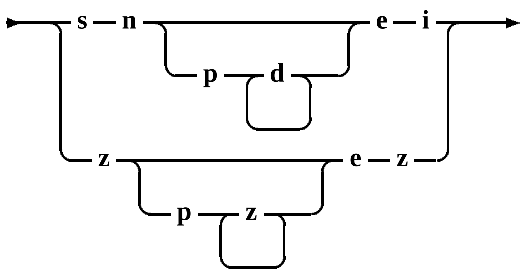
Minimal-Decimal-Exponential



Examples:

```
¯1.234567E¯890 1E1 ¯1.1111E1 1E¯11111 0E0
```

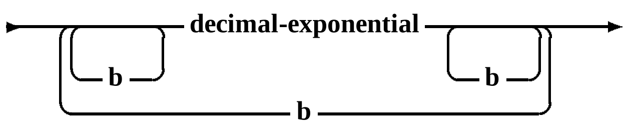
Decimal-Exponential



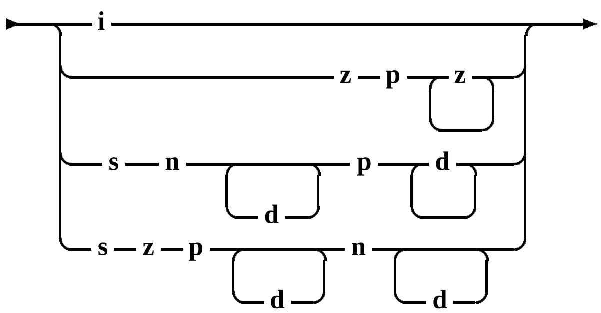
Examples:

```
¯1.23456700000E¯890 0.00000000E0 ¯1.0000E¯10
```

Decimal-Exponential-Row



Fixed-Decimal



Examples:

```
0 ¯1 12 0.00000000 12.30 0.1235 ¯0.1235 0.1 ¯0.00010000
```

15.4 Operations

15.4.1 Monadic Format

```
Z ← ⍕ B
```


Informal Description: *Z* is a **character** array. If *B* is a **character** array, then *Z* is *B*. If *B* is a numeric array, then *Z* is a **character** array arranged so that each row of *Z* is a decimal representation of the corresponding row of *B*. The output form chosen for a column of *B* is determined by **print-precision** and the values of elements in that column. Uses **print-precision**

Evaluation Sequence:

If **print-precision** is **nil**, signal **implicit-error**.
If the **type** of *B* is **character**, return *B*.
If *B* is **empty**, return an **array** *Z0* such that the **type** of *Z0* is **character**, the **shape-list** of *Z0* is the **shape-list** of *B*, and the **ravel-list** of *Z0* is the **empty-list**.
If the **rank** of *B* is **less-than two**,
return `,⍕(¯2↑1 1,ρB)ρB`.
If the last **item** of the **shape-list** of *B* is not **one**,
return `(⍕((¯1↓ρB),1)↑B),' ',⍕((-ρρB)↑1)↓B`.
Otherwise, choose *U*, *E*, and *W*, and return an **array** *Z* for which the **type** of *Z* is **character**, the **shape-list** of *Z* is `(¯1↓ρB),W`, and one of the following statements is true:
Every **row** of *Z* is the **decimal-exponential-row** equivalent of the **minimal-decimal-exponential numeric-output-conversion** of the corresponding **row** of *B*, and has its **exponent-marker** at position *E* and its first digit at position *U*.
Every **row** of *Z* is the **decimal-rational-row** equivalent of the **minimal-decimal-exponential numeric-output-conversion** of the corresponding **row** of *B*, and has its units digit at position *U*.
Every **row** of *Z* is formatted independently for each element *B1* of *B*. The results are padded with blanks to width *W*.
If *B1* is a **character**, use *B1*.
If *B1* is a **number**, and the **imaginary-part** of *B1* is 0, use `(⍕B1)`.
If *B1* is a **number**, and the **imaginary-part** of *B1* is not 0, then
Let *B2* be the **real-part** of *B1*.
Let *B3* be the **imaginary-part** of *B1*.
Let *CI* be the character representation of the **Complex-Marker**.
Use `((⍕B2), CI, ⍕B3)`.

Note: *This form of output must match the diagram* **numeric-scalar-literal**.

Note: *The quantities W, U and E and the choice of formatting form are not standardised here, since this is currently impractical. However, the following algorithm is recommended for all future implementations of APL:*
If any of the following conditions holds for any element X of B
X is **greater-than positive-counting-number-limit**.
X is **less-than negative-counting-number-limit**.
The **decimal-rational** *equivalent of the* **minimal-decimal-exponential numeric-output-conversion** *of X would have more than* **print-precision** *significant digits to the left of the decimal point.*
The **decimal-rational** *equivalent of the* **minimal-decimal-exponential numeric-output-conversion** *of X would have more than five zero digits to the right of the decimal point and to the left of the first nonzero digit.*

then select **decimal-exponential** *form.*
Otherwise, select **decimal-rational** *form.*
If **decimal-exponential** *form is selected,*
Let `S` *be* **one** *if any element of* `B` *is a* **negative-number**.
Let `M` *be the maximum number of* **characters** *to the left of the* **exponent-marker** *in the* **minimal-decimal-exponential numeric-output-conversion** *of any element of* `B`.
Let `N` *be the maximum number of* **characters** *to the right of the* **exponent-marker** *in the* **minimal-decimal-exponential numeric-output-conversion** *of any element of* `B`.
Set `U` *to* `S+1`.
Set `E` *to* `M+1`.
Set `W` *to* `M+1+N`.
If **decimal-rational** *form is selected,*
Let `M` *be the maximum number of* **characters** *to the left of the units position in the* **decimal-rational** *equivalent of the* **minimal-decimal-exponential numeric-output-conversion** *of any number in* `B`.
Let `N` *be the maximum number of* **characters** *to the right of the units position in the* **decimal-rational** *equivalent of the* **minimal-decimal-exponential numeric-output-conversion** *of any number in* `B`.
Set `U` *to* `M+1`.
Set `W` *to* `M+1+N`.

Note: *There is no uniformity in the display of an array containing mixed types, nor in the display of a column containing complex numbers. Suggested methods for displaying these are as follows:*
For an array containing mixed types, format each simple array, and provide enough space in each column to contain the widest element.
For a column containing at least one complex number, right-justify each number in the column, making no attempt to align other numeric elements.

Examples:

```
      ⎕PP←10
      '|',(⍕5 ¯6 7 8 9 10∘.÷⍳5),'|'
| 5  2.5  1.666666667  1.25  1   |
|¯6 ¯3   ¯2           ¯1.5  ¯1.2 |
| 7  3.5  2.333333333  1.75  1.4 |
| 8  4    2.666666667  2     1.6 |
| 9  4.5  3            2.25  1.8 |
|10  5    3.333333333  2.5   2   |
```



```
      D←0.7 0.8 0.9, 7 8 9÷10
      ⍕D
0.7 0.8 0.9 0.7 0.8 0.9
      D-⍎⍕D
0 0 0 0 ¯1.387778781E¯17 0
      ⎕PP←999
      ⍕D
0.7 0.8 0.9 0.7 0.79999999999999999 0.9
      D-⍎⍕D
0 0 0 0 0 0
      ⎕PP←4
      M←0.78901×(10*⍳6)∘.×1 1E¯10 1E¯6 0.01 0.1
      ⍕M
 7.89E0 7.89E¯10 0.00000789    0.0789 7.89E¯1
 7.89E1 7.89E¯9  0.0000789     0.789  7.89E0
 7.89E2 7.89E¯8  0.000789      7.89   7.89E1
 7.89E3 7.89E¯7  0.00789      78.9    7.89E2
 7.89E4 7.89E¯6  0.0789      789      7.89E3
 7.89E5 7.89E¯5  0.789      7890      7.89E4

      ⍕5⍴M
7.89 7.89E¯10 0.00000789 0.0789 0.789
      ⍕1 5⍴M
7.89 7.89E¯10 0.00000789 0.0789 0.789
```

Note: *When* **print-precision** *has a value in its* **internal-value-set** *greater than or equal to* **full-print-precision***, the result produced for* N *is unique and is unaffected by increases in the value of* **print-precision***. At such settings of* **print-precision***,* N *is the same as* ⍎⍕N *for any* **numeric scalar** N*.*

15.4.2 Dyadic Format

`Z ← A ⍕ B`

Informal Description: `Z` is a **character** array representing the numbers in `B` formatted according to the specifications in `A`. `A` consists of pairs of integers. One pair is associated with each column of `B`. The first integer in the pair specifies the field width—the number of columns in the character result—for the formatted numeric values.

If `A` has only two elements then all elements of `B` are formatted according to `A`.

Note: *For complex elements of* `B`*, the precision specifier applies to each part of the representation.*

A non-negative second integer indicates that the field is to have **fixed-decimal** form and gives the number of digits to the right of the decimal point, with zero indicating no decimal places and no decimal point. A negative second integer indicates that the field is to have decimal exponential form; the absolute value specifies the number of digits in the mantissa.

Evaluation Sequence:

If the **rank** of `A` is **greater-than one**, signal **rank-error**.
If `⍴,A` is not `2×¯1↑1,⍴B`
  If `⍴A` is **two**, return `((2×¯1↑⍴B)⍴A)⍕B`.
  Otherwise, signal **length-error**.
If any **item** of the **ravel-list** of `A` is not a **near-integer**, signal **domain-error**.
If any **item** of the **ravel-list** of `B` is not a **real-number**, signal **domain-error**.
Set `A1` to the **integer-array-nearest-to** `A`.
If `B` is **empty**,
  Set `W` to `+/((⍴A1)⍴1 0)/A1`.
  Return `((¯1↓⍴B),W)⍴' '`.
If `⍴A1` is not **two**, return `((2↑A1)⍕((¯1↓⍴B),1)↑B),(2↓A1)⍕((-⍴⍴B)↑1)↓B`.
If `B` is not a **scalar**,
  If `A1[1]` is not a **positive-integer**, signal **domain-error**.
  Return a **character array** `Z` such that the **shape-list** of `Z` is `(¯1↓⍴B),A1[1]` and the **ravel-list** of `Z` has the property that each **vector-item along-axis** `⍴⍴Z` of `Z` is `A1 ⍕B0`, where `B0` is the corresponding (scalar) **item** of `(¯1↓⍴B)⍴B`.
If `B` is a **scalar**,
  If `A1[2]` is not **less-than zero**, set `R` to the **fixed-decimal numeric-output-conversion** of `B` to `A1[2]` places.
  Otherwise, set `R` to the **decimal-exponential numeric-output-conversion** of `B` to `|A1[2]` places.
  If `A1[1]` is **less-than** `⍴R`, signal **domain-error**.
  Otherwise, return `(-A1[1])↑R`.

Examples:

– In the following examples, **exponent-field-width** is 3.

```
      D ← ¯1 ¯0.1 0 0.1 1 ∘.× 5⍴0.5
      '|',(9 ¯3 7 ¯1 6 3 5 1 3 0 ⍕ D),'|'
| ¯5.00E¯1 ¯5E¯1 ¯0.500 ¯0.5 ¯1 |
| ¯5.00E¯2 ¯5E¯2 ¯0.050 ¯0.1  0 |
|  0.00E0   0E0   0.000  0.0  0 |
|  5.00E¯2  5E¯2  0.050  0.1  0 |
|  5.00E¯1  5E¯1  0.500  0.5  1 |

      ⍴ 1 0 2 0 4 0 8 0⍕0 4⍴1
0 15
      31 0 ⍕ 2*100
1267650600228229401496703205376
      31 20 ⍕ .07
0.07000000000000000666

      4 1 ⍕ ¯.99 ¯.89 0 7.5 11.5
¯1.0¯0.9 0.0 7.511.5
```

Note: *Like other axis lengths, the field width* `A[1]` *is limited only by* **index-limit**.

16 Input and Output

16.1 Introduction

Note: *A user interacts with an APL system through a* **session***, an abstraction that represents a hypothetical machine capable of carrying out the evaluation sequences in this International Standard.*

The protocol for the use of a session takes the form of a dialogue: the user makes an **entry** *and passes control to the APL system. The system processes the entry, produces a response, and returns control to the user.*

The user makes entries on a keyboard, and obtains responses by seeing them presented on a display-device. The combination of a keyboard and a display-device is intended to represent, abstractly, a terminal. This International Standard does not require the use of any particular terminal, keyboard, display or method of encoding **characters***. The ISO 2022.2 APL character encoding reproduced in Annex A is widely used.*

The display-device is considered a window into an unbounded sequence of past entries and responses. The mapping between this unbounded sequence and the display-device is not specified by this International Standard.

The system obtains entries as **tokens** *of* **class constant** *or* **interrupt** *by calling the session operation* **read-keyboard***. It presents responses by calling the session operation* **display** *with a* **token** *of class* **result***. The transformations from entry to token and from token to response are not specified by this International Standard, but certain desirable characteristics of these transformations are given as comments.*

Because the dialogue protocol described here alternates between user and system, the session is always in one of two logical **keyboard-states***,* **open-keyboard** *or* **locked-keyboard***, depending upon whether the user is making an entry or the system is producing a response. The user can request a change in* **keyboard-state** *from* **locked-keyboard** *to* **open-keyboard** *by using the* **signal-attention** *facility. In addition, the user can force a change in* **keyboard-state** *by signalling* **interrupt***.*

16.2 Definitions

16.2.1 User Facilities

A user should have the following facilities.

– **Edit-Actions**: A collection of **implementation-defined** facilities that permit the user to make entries.

Note: *Typical* **edit-actions** *include inserting, deleting, replacing and superimposing graphic symbols within the current entry and combining display-lines from the presentation-space with the current entry.*

– **Enter**: A facility, available to the user when the terminal is in **open-keyboard** state, for changing the **keyboard-state** to **locked-keyboard** and returning to the caller of **read-keyboard** either an **interrupt** or a **character vector**. **Enter** sets the value of the session attribute **attention-flag** to **zero**.

– **Signal-Attention**: A facility, available to the user when the terminal is in **locked-keyboard** state, for setting the value of the session attribute **attention-flag** to **one**.

– **Signal-Interrupt**: A facility, available to the user at all times, for interrupting any evaluation sequence, causing it to return an **interrupt token** to the current caller of **evaluate-line**. Any **atomic** operation so interrupted has no effect on the state of the **active-workspace**.

Note: *Signal-interrupt can be issued in* **open-keyboard** *state and in* **locked-keyboard** *state, although the particular means of invoking the operation may be different for the two states. Transmitting the superimposition of the graphics for* O U *and* T *is a typical means of signalling* **interrupt** *in* **open-keyboard** *state.*

16.2.2 Implementation Algorithms

– **Read-Keyboard**: An operation, available to the system when the session is in **locked-keyboard** state, that returns either an **interrupt token** or a **token** of **class constant** and **content** a **character vector**.

Note: **Read-keyboard** *should* **display current-prompt**, *then change the* **keyboard-state** *to* **open-keyboard** *to permit the user to use* **edit-actions** *to enter a* **list** *of* **characters** *or to signal an* **interrupt**.

A concept long honoured in APL is that of visual fidelity: the graphic symbols that appear on the display as the result of **edit-actions** *should correspond exactly to those returned to the system as the result of* **read-keyboard**.

– **Presentation Space**: an unbounded sequence of past entries and responses.

– **Display** T: An operation, available when the session is in **locked-keyboard** state, for presenting the **token** T on the **display-device**. T is either a **value** or an **error**.

Note: *The following is a suggested, but not required, evaluation sequence for* **display** `T`*:*

If **quote-quad-prompt** *is not the* **empty character vector***,*

Append to the **presentation-space** *as a new last item a* **display-line** *whose graphic symbols represent the* **characters** *in* **quote-quad-prompt***.*

Set **quote-quad-prompt** *to the* **empty character vector***.*

If `T` *is an* **error***, append to the* **presentation-space** *as a new last item an indication of*

The **class** *of the* **error***.*

The point in **current-line** *at which evaluation was stopped.*

If the **mode** *is* **defined-function***, the* **function-name** *and* **current-line-number** *of the* **current-context***.*

If `T` *is a* **value***,*

Let `A` *stand for the* **content** *of* `T`*.*

If `A` *is* **numeric***,* **display** `⍕A`*.*

Otherwise,

If `A` *is a* **scalar** *or* **vector***, append to the* **presentation-space** *as a new last item a* **display-line** *whose graphic symbols represent the* **characters** *in* `A`*.*

Otherwise,

Set `I` *to* **zero***.*

Set `M` *to* `((×/¯1↓⍴A),¯1↑⍴A)⍴A`*.*

Repeat the following `1↑⍴M` *times:*

Set `I` *to* `I+1`*.*

Display row `I` *of* `M`*.*

Append `(¯1+⍴⍴A)|+/∧\0=⌽(¯1↓⍴A)⊤I` *blank* **display-lines** *to the end of the* **presentation-space***.*

(End of repeated block)

16.2.3 Prompts

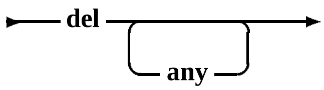
Note: *The following* **implementation-defined arrays** *are used to indicate to the user the type of entry required.*

- **Indent-Prompt**: An **implementation-defined character array**, typically a **vector** of six **blanks**.
- **Quad-Prompt**: An **implementation-defined character array**, whose first row typically begins with ⎕: and whose second row is the **indent-prompt**.
- **Function-Definition-Prompt**: An **implementation-defined character array**, typically a **vector** beginning with [**nn**] where **nn** is a **line-number**.

16.3 Diagrams

System-Command-Line

Function-Definition-Line



16.4 Operations

16.4.1 Immediate-Execution

Immediate-Execution with X.

Informal Description: **Immediate-execution** mode is the primal mode, the first evaluation sequence processed by the system to enter into a dialogue with a user. X is either an **error token** or **nil**.

The system also calls **immediate-execution** when an **error** or the **attention-flag** causes a defined function to be suspended. In this case, **immediate-execution** displays a status indication before prompting for input.

Evaluation Sequence:

Display X.
Repeat:
  Set **current-prompt** to **indent-prompt**.
  Set E to **read-keyboard**.
  If E is a **constant**,
    Let Q stand for the **content** of E.
    If Q matches **system-command-line**, set T to **evaluate-system-command** Q.
    If Q matches **function-definition-line**, set T to **evaluate-function-definition-request** Q.
    Otherwise,
      Generate a new **context** in which
        mode is **immediate-execution**,
        current-line is Q,
        current-function is `0 0⍴' '`,
        current-line-number is **one**,

current-statement is the empty **list** of **tokens**, and
stack is the empty **list** of **tokens**.

Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.

Set T to **evaluate-line**.

Remove the **first-item** in the **state-indicator** of the **active-workspace**.

If T is a **branch**, a **clear-state-indicator**, or an **escape** and the **state-indicator** of the **active-workspace** is not an **empty-list**, return T.

If T is an **error** or a **constant**, **display** T.

(End of repeated block)

16.4.2 Quad Input

`Z ← ⎕`

Informal Description: *Z* is an array provided by the user in response to a prompt.

Evaluation Sequence:

Repeat:
  Set **current-prompt** to **quad-prompt**.
  Set *E* to **read-keyboard**.
  If *E* is an **interrupt**, return *E*.
  Let *Q* stand for the **content** of *E*.
  If some element of *Q* is not **blank**,
    Generate a new **context** in which
      mode is **quad-input**,
      current-line is *Q*,
      current-function is `0 0⍴' '`,
      current-line-number is **one**,
      current-statement is the empty **list** of **tokens**, and
      stack is the empty **list** of **tokens**.
    Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.
    Set *Z* to **evaluate-line**.
    Remove the **first-item** in the **state-indicator**.
    If *Z* is **escape**, signal **unwind**.
    If *Z* is **unwind** or **clear-state-indicator**, return *Z*.
    If *Z* is a **value**, return a **token** with **class constant** and **content** that of *Z*.
    If *Z* is an **error**, **display** *Z*.
    Otherwise, **display value-error**. (see note)
(End of repeated block)

Note: *The* **display value-error** *line in the above evaluation sequence is introduced to permit certain* **consistent-extensions**. *In the evaluation sequence, errors are* **displayed** *rather than* **signalled**, *since signalling would terminate the Repeat loop and force Return.*

Quad-input returns only **tokens** *of class* **escape** *and* **unwind** *(from* `→`*),* **clear-state-indicator** *(from* `)SIC`*),* **constant** *(from* `2*.5`*) and* **interrupt**. *It reports an error and reprompts for all other* **results**. *Note that it reprompts without an error report if the input was empty or all blank.*

16.4.3 Quote Quad Input

$Z \leftarrow$ ⍞

Informal Description: Z is a **character vector**. Input in response to ⍞ is treated as a **character** value. Z is a vector whose length is the number of positions from the left margin up to the rightmost **character** of the input, including explicitly entered trailing blanks.

Evaluation Sequence:

Set **current-prompt** to **quote-quad-prompt**.
Set **quote-quad-prompt** to the **empty character vector**.
Set E to **read-keyboard**.
Return E.

Note: *The behaviour required here differs from that of some existing systems, in which a single character response to* ⍞ *is returned as a scalar.*

16.4.4 Quad Output

$Z \leftarrow$ ⎕ $\leftarrow B$

Informal Description: Z is B. As a side effect, the array B is displayed on the terminal.

Evaluation Sequence:

Display B.
Return a **committed-value** with **content** B.

16.4.5 Quote Quad Output

```
Z ← ⍞ ← B
```

Informal Description: *Z* is *B*, and **quote-quad-prompt** is set to *B*.

Evaluation Sequence:

If any **item** of the **ravel-list** of *B* is not a **character**, signal **domain-error**.
If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If the **count** of *B* is **greater-than quote-quad-output-limit**, signal **limit-error**.
If **quote-quad-prompt** is not empty, signal **limit-error**.
Set **quote-quad-prompt** to *B*.
Return a **committed-value** with **content** *B*.

Example:

```
∇Z←PROMPT X
[1] ⍞←X←,⍕X
[2] Z←(⍴X)↓⍞
∇
      PROMPT '1 CLEANSPACE DATE? '
1 CLEANSPACE DATE? 1966-11-27
1966-11-27
```

Note: *Quote-quad output is difficult to standardise because it is implemented in several different ways in existing systems, and, in each system, is heavily used. There is, however, sufficient commonality to permit the inclusion of this restricted form of quote-quad output. Quote-quad output on a* **conforming-implementation** *should support the prompt function shown in the example.*

17 System Commands

17.1 Introduction

Note: *Any line of input in* **immediate-execution** *whose first non-blank* **character** *is a right parenthesis is considered to be a system command.*

17.2 Definitions

– **Global-Referent of** T: For T a **classified-name**, the **last-item** in the **referent-list** of the **symbol-named-by** T.

– **Global-Context** The last **context** in the **state-indicator** of the **active-workspace**.

– **Library-Workspace-Named** A: For **workspace-identifier** A, the **item** of the **library** whose **owner** is **this-owner**, and whose **workspace-name** is A.

– **Already-Exists**: A **workspace already-exists** if its **existential-property** is **present**.

– **Does-Not-Exist**: A **workspace does-not-exist** if its **existential-property** is **absent**.

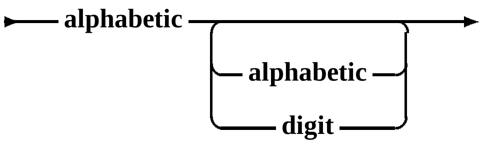
Note: *The model of libraries used in this International Standard assumes that a* **workspace** *object exists in the* **library** *for all possible* **workspace-identifiers**. **Workspaces** *that have been dropped or that have never been saved are distinguished from those that have been saved by the setting of their* **existential-property***: the* **existential-property** *of saved* **workspaces** *is* **present***, while that of unsaved* **workspaces** *is* **absent***.*

– **Attempt-to-Erase** A: For A, a **simple-identifier**, an operation defined as follows:
If A is **globally-erasable**,
  If the **current-class** of A is **shared-variable**, **retract** A.
  Set A to **nil**.
  Return **command-complete**.
Otherwise, signal **not-erased**.

Copy A **from** W: For **simple-identifier** A, and **workspace-identifier** W, an operation defined as follows:
Let GA stand for the **global-referent** of A in the **active-workspace**.
Let GB stand for the **global-referent** of A in the **library-workspace-named** W.
Set Z to **attempt-to-erase** GA.
If Z is an **exception**, signal **not-copied**.
If GB is a **shared-variable**, set GA to the **shared-value** of GB.
Otherwise, set GA to GB.
Return **command-complete**.

17.3 Diagrams

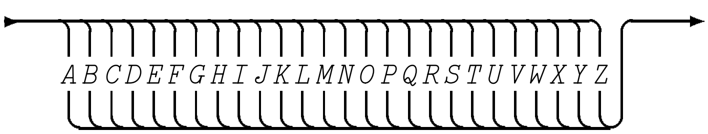
Workspace-Identifier



Example:

CLEANSPACE

Alphabetic



17.4 Operations

17.4.1 Evaluate-System-Command

Evaluate-System-Command Q.

Note: *The forms of system commands are not regular enough to permit much generalisation. Therefore, the forms themselves are used as the key to the evaluation sequences.*

Evaluation Sequence:

Find an entry that matches Q in the **system-command evaluation sequences**.

If no such entry is found, signal **incorrect-command**.
Otherwise, call the corresponding evaluation sequence.
Return the **token** it returns.

17.5 Diagrams and Evaluation Sequences

In the following,

- **p** stands for **permitted-blanks**,
- **r** stands for **required-blanks**,
- **w** stands for **workspace-identifier**, and
- **a** stands for **simple-identifier**.

Clear Active Workspace

►—— **p** —) — **p** — *CLEAR* — **p** ——►

Call **Shared-Variable-Reset**.
Set the **active-workspace** to the **clear-workspace**.
Set **comparison-tolerance** to **initial-comparison-tolerance**.
Set **event-message** to **initial-event-message**.
Set **event-type** to **initial-event-type**.
Set **index-origin** to **initial-index-origin**.
Set **latent-expression** to **initial-latent-expression**.
Set **print-precision** to **initial-print-precision**.
Set **quote-quad-prompt** to the **empty character vector**.
Set **random-link** to **initial-random-link**.
Return **command-complete**.

Copy Library Workspace Object

►—— **p** —) — **p** — *COPY* — **r** — **w** — **r** — **a** — **p** ——►

If the **library-workspace-named w does-not-exist**, signal **not-found**.
If the **global-referent** of **a** in **w** is not a **defined-function**, **defined-operator**, **niladic-defined-function**, **variable**, or **shared-variable**, signal **incorrect-command**.
Set Z to **copy a from w**.
Return Z.

Copy Library Workspace

►—— **p** —) — **p** — *COPY* — **r** — **w** — **p** ——►

If the **library-workspace-named w does-not-exist**, signal **not-found**.
For each **symbol** *S* of the **library-workspace-named w**
Let *A* stand for the **name** of *S*.
If the **global-referent** of *A* in **w** is a **defined-function**, **defined-operator**, **niladic-defined-function**, **variable**, or **shared-variable**,
Set *Z* to **copy** *A* **from w**.
If *Z* is an **exception**, return *Z*.
Return **command-complete**.

Drop Library Workspace

►── **p** ──) ── **p** ── *DROP* ── **r** ── **w** ── **p** ──►
If the **library-workspace-named w does-not-exist**, signal **not-found**.
Set the **existential-property** of the **library-workspace-named w** to **absent**.
Return **command-complete**.

Erase Global Referent

►── **p** ──) ── **p** ── *ERASE* ── **r** ── **a** ── **p** ──►
Let *GA* stand for the **global-referent** of **a** in the **active-workspace**.
If the **global-referent** of **a** is not a **defined-function**, **defined-operator**, **niladic-defined-function**, **variable**, or **shared-variable**, signal **incorrect-command**.
Set *Z* to **attempt-to-erase** *GA*.
Return *Z*.

List Global Function Names

►── **p** ──) ── **p** ── *FNS* ── **p** ──►
For each **symbol** *A* whose **global-referent** is a **defined-function** or a **niladic-defined-function**, **display** the **name** of *A*.
Return **command-complete**.

List Library Directory

►── **p** ──) ── **p** ── *LIB* ── **p** ──►
For each **workspace** *W* whose **existential-property** is **present** and whose **owner** is **this-owner**, **display** the **workspace-name** of *W*.
Return **command-complete**.

Load Library Workspace

►── **p** ──) ── **p** ── *LOAD* ── **r** ── **w** ── **p** ──►
If the **library-workspace-named w does-not-exist**, signal **not-found**.
Set the **active-workspace** to the **library-workspace-named w**.

Call **Shared-Variable-Reset**.
If **latent-expression** is not **nil**,
Generate a new **context** in which
mode is **immediate-execution**,
current-line is the **ravel-list** of **latent-expression**,
current-function is `0 0ρ' '`,
current-line-number is **one**,
current-statement is the empty **list** of **tokens**, and
stack is the empty **list** of **tokens**.
Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.
Set *T* to **evaluate-line**.
Remove the first **item** from the **state-indicator** of the **active-workspace**.
Return *T*.
Otherwise, return **command-complete**.

Note: Shared-variable-reset *has the effect of* **retracting** *and* **expunging** *all variables that were shared in the* **library-workspace***. This means that* **conforming-programs** *cannot depend upon the values of variables that were shared when a* `)SAVE` *command was issued.*

List Global Names and their Name Class

►— **p** —) — **p** — *NMS* — **p** ——►
For each **symbol** *A* whose **global-referent** is a **variable**, **defined-function**, **niladic-defined-function**, or **defined-operator**, **display** the **name** of *A* appended with a dot followed by the **name-class** (for example `FN.3`, `OP.4`).
Return **command-complete**.

List Global Operator Names

►— **p** —) — **p** — *OPS* — **p** ——►
For each **symbol** *A* whose **global-referent** is a **defined-operator**, **display** the **name** of *A*.
Return **command-complete**.

Save Active Workspace

►— **p** —) — **p** — *SAVE* — **p** ——►
Set *W* to the **workspace-name** of the **active-workspace**.
If *W* is **clear-workspace-identifier**, signal **not-saved**.
Replace the **library-workspace-named** *W* with the **active-workspace**.
Set the **existential-property** of the **library-workspace-named** *W* to **present**.
Return **command-complete**.

Note: *See* `)LOAD` *for a discussion of shared variable values.*

Save Active Workspace with Name

►── **p** ─) ─ **p** ─ *SAVE* ─ **r** ─ **w** ─ **p** ───►

If the **workspace-name** of the **active-workspace** is not **w**,
  If the **library-workspace-named w already-exists**, signal **not-saved**.
  Otherwise, set the **workspace-name** of the **active-workspace** to **w**.
Replace the **library-workspace-named w** with the **active-workspace**.
Set the **existential-property** of the **library-workspace-named w** to **present**.
Return **command-complete**.

Note: *See*)*LOAD for a discussion of shared variable values.*

List State Indicator

►── **p** ─) ─ **p** ─ *SI* ─ **p** ───►

Let *N* stand for the **number-of-items** in the **state-indicator** of the **active-workspace**.
Set *I* to **zero**.
Repeat:
  Set *I* to *I* **plus one**.
  If *I* is **greater-than** *N*, return **command-complete**.
  If the **mode** of **item** *I* of the **state-indicator** is **defined-function** or **defined-operator**, **display** the **function-name** and **current-line-number** of **item** *I* of the **state-indicator**, along with an indication of whether the **function-name** is **suspended**.
(End of repeated block)

Clear State Indicator

►── **p** ─) ─ **p** ─ *SIC* ─ **p** ───►

Return **clear-state-indicator**.

List State Indicator and Local Names

►── **p** ─) ─ **p** ─ *SINL* ─ **p** ───►

Let *N* stand for the **number-of-items** in the **state-indicator** of the **active-workspace**.
Set *I* to **zero**.
Repeat:
  Set *I* to *I* **plus one**.
  If *I* is **greater-than** *N*, return **command-complete**.
  If the **mode** of **item** *I* of the **state-indicator** is **defined-function** or **defined-operator**, **display** the **function-name**, **current-line-number**, and **local-name-list** of **item** *I* of the **state-indicator**, along with an indication of whether the **function-name** is **suspended**.
(End of repeated block)

List Global Variable Names

⟶ **p** —) — **p** — *VARS* — **p** ⟶

For each **symbol** *A* whose **global-referent** is a **variable**, **display** the **name** of *A*.
Return **command-complete**.

List Workspace Identification

⟶ **p** —) — **p** — *WSID* — **p** ⟶

Display the **workspace-name** of the **active-workspace**.
Return **command-complete**.

Change Workspace Identification

⟶ **p** —) — **p** — *WSID* — **r** — **w** — **p** ⟶

Set the **workspace-name** of the **active-workspace** to **w**.
Return **command-complete**.

Annex A
(normative)

Component Files

Informal Description: This annex describes a minimal set of functions which a conforming implementation must provide. There are, therefore, many facilities (such as shared libraries, access control, etc.) which are intentionally excluded from this annex, although it is expected that they will be available on most systems.

In particular, file names are severely restricted and access is provided only to a default library, in order to ensure compatible behaviour, on all systems, of the functions defined here.

This annex does not addresss the migration of the component files themselves, although it is presumed that at the very least one will be able to retrieve the objects into a workspace, migrate the workspace, and recreate the file, using these functions.

The user interface (arguments, function name, and result) are described here. When an error is signalled it implies that suspension will occur in the caller (that is, by means of ⎕*ES*).

A.1 Definitions of arguments and results

– *A*—Any array

– *CID*—Component id, must be a positive integer

– *FH*—File handle, an integer

– *FHL*—A list of file handles

– *FL*—File names, a character matrix with rows giving names of files

– *FNM*—File name, a character vector composed of 1–8 characters, the first chosen from *A-Z*, and any others chosen from *A-Z* and 0-9

– *NFNM*— New file name, see *FNM* above

A.2 Definition of functions

– *FL←CF_LIST A*

If *A* is an empty vector, *FL* is a character matrix containing the names of files existing in the default library.

If *A* is not empty, an error will be signalled. An implementation may provide a consistent extension where *A* is a library identifier.

If there are no component files in the library, *FL* will be an empty character matrix.

– *FH←CF_CREATE FNM*

Creates a file with name *FNM* and returns a numeric handle for the file. If the operation is unsuccessful, an error will be signalled.

– *Z←CF_ERASE FNM* (*Z* is 0 0ρ0)

Erase the file specified in *FNM*. If successful an empty numeric matrix will be returned. Success implies that the file no longer exists (either it has been erased or it never existed). If the operation is unsuccessful an error will be signalled.

– *Z←NFNM CF_RENAME FNM* (*Z* is 0 0ρ0)

The file named by *FNM* is renamed *NFNM*. If this is successful an empty numeric matrix is returned. If the operation is unsuccessful an error is signalled.

– *FH←[FH] CF_OPEN FNM*

The file named by *FNM* is opened and a file handle is returned. The left argument is optional. The presence of the left argument indicates that the file is to be opened with the specified file handle. If the operation is unsuccessful an error is signalled.

Note: *If a left argument is present but the file cannot be opened with the specified FH, the open will be unsuccessful.*

– *Z←CF_CLOSE FHL* (*Z* is 0 0ρ0)

Each file listed in *FHL* is closed. The presence of the result indicates that all files named in *FHL* are now closed. An error will be signalled if the close is unsuccessful (for example, an invalid *FH* or unable to close the file).

– *FHL←CF_INUSE*

The result *FHL* is a list of file handles (*FH*) that are currently valid. It will be empty if there are no valid file handles current.

– *Z← A CF_WRITE FH,CID* (*Z* is 0 0ρ0)

Array *A* is written to component *CID* in file *FH*. The presence of the result indicates success. An error will be signalled if the operation is unsuccessful.

– *CID← A CF_APPEND FH*

  Array *A* is appended to the file *FH* and *CID* is its component id in the file. An error is signalled if the operation is unsuccessful.

– *A←CF_READ FH,CID*

  The result *A* is component *CID* of file *FH*. An error is signalled if the operation is unsuccessful.

– *CID←CF_NEXT FH*

  The result is the component id that will be returned for the next successful use of *CF_APPEND* with file *FH*. An empty file will return a *CID* of one. An error is signalled if the operation is unsuccessful.

A.3 Errors

The functions described here must be written so that they do not suspend. Any errors encountered or detected must be passed to the context from which the function was invoked.

Bibliography

The following publications are referred to in this International Standard to provide general background and guidance for the benefit of implementers. The International Standard does not rely upon the content of these documents for its interpretation, and implementers are under no obligation to consult these documents or to use the information contained in them.

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