

PM 822 Seminar Topics

Pick one of these papers or get my approval for something else. The talks will be 25 minutes long, and should survey the ideas and prove a few key things. Talks will be in the last week of March or the first week of April.

- (1) W.B. Arveson, *Subalgebras of  $C^*$ -algebras III*, Acta Math. **181** (1998), 159–228.  
He proves a dilation theorem for a row contraction of commuting operators, and computes the  $C^*$ -envelope.
- (2) C.A. Berger, *A strange dilation theorem*, 1965, unpublished.  
See Sz.Nagy–Foiaş chapter I.11. A  $\rho$ -dilation of  $T \in \mathcal{B}(\mathcal{H})$  is a unitary  $U$  on a larger space  $\mathcal{K}$  such that  $T^n = \rho P_{\mathcal{H}} U^n|_{\mathcal{H}}$ . These operators are characterized.  $\rho = 2$  yields Berger’s theorem.
- (3) J.W. Bunce, *Models for  $n$ -tuples of noncommuting operators*, J. Func. Anal. **57** (1984), 21–30.  
If  $T_i$  are operators such that  $\sum_i T_i T_i^* \leq I$ , then they may be simultaneously dilated to isometries with pairwise orthogonal range. The minimal dilation is unique (G. Popescu, *Isometric dilations for infinite sequences of noncommuting operators*, Trans. Amer. Math. Soc. **316** (1989), 523–536).
- (4) E. Christensen, *Extensions of derivations II*, Math. Scand. **50** (1982), 111–122.  
All derivations from a  $C^*$ -algebra  $\mathfrak{A}$  into  $\mathcal{B}(\mathcal{H})$  are inner if and only if they are completely bounded if and only if for all  $T \in \mathcal{B}(\mathcal{H})$ , one has  $\text{dist}(T, \mathfrak{A}') \leq K \|\delta_T|_{\mathfrak{A}}\|$ .
- (5) K.R. Davidson and E.G. Katsoulis, *Semicrossed products of the disc algebra*, Proc. Amer. Math. Soc. **140** (2012), 3479–3484.  
Computes the  $C^*$ -envelope of the semicrossed product of the disc algebra by an endomorphism.
- (6) R.G. Douglas and V.I. Paulsen, *Completely bounded maps and hypo-Dirichlet algebras*, Acta Sci. Math. (Szeged) **50** (1986), 143–157.  
If  $X$  is a finitely connected domain in  $\mathbb{C}$  with nice boundary and  $T$  has  $X$  as a spectral set, then  $T$  is similar to an operator which has a normal  $\partial X$ -dilation.
- (7) E.T.A. Kakariadis, *The Shilov Boundary for Operator Spaces*, arXiv:1204.4495.  
He provides a simpler proof of Hamana’s Theorem that every operator system is contained in a unique injective envelope. This yields the original proof of the existence of the  $C^*$ -envelope as a corollary.
- (8) P. Muhly and B. Solel, *On the uniqueness of operator algebra structures*, Indiana Univ. Math. J. **46** (1997), 575–591.  
A subalgebra of the  $n \times n$  matrices containing the diagonal algebra has a unique operator algebra structure if and only if every contractive representation is completely contractive.
- (9) V.I. Paulsen, S.C. Power and J.D. Ward, *Semidiscreteness and dilation theory for nest algebras*, J. Funct. Anal. **102** (1988), 76–87.  
The dilation theory for the upper triangular matrices is extended to infinite dimensions for weak- $*$  continuous representations by approximating infinite nest algebras by finite nest subalgebras.
- (10) G. Pisier, *Joint similarity problems and the generation of operator algebras with bounded length*, Int. Equations & Oper. Theory **31** (1998), 353–370.  
There are two commuting operators which are each separately similar to a contraction, but their product is not polynomially bounded; whence they are not jointly similar to contractions. Similar results for group representations.
- (11) D. Sarason, *Generalized interpolation in  $H^\infty$* , Trans. Amer. Math. Soc. **127** (1967), 179–203.  
He solves the Nevanlinna-Pick interpolation problem: given  $z_1, \dots, z_n \in \mathbb{D}$  and scalars (or matrices)  $w_1, \dots, w_n$ , when is there an analytic function  $f(z)$  on  $\mathbb{D}$  bounded by 1 so that  $f(z_i) = w_i$ ? by applying the commutant lifting theorem. But the CLT did not yet exist—he proved the first important case.
- (12) R.R. Smith, *Completely bounded module maps and the Haagerup tensor product*, J. Funct. Anal. **102** (1991), 156–175.  
If  $\mathfrak{A}$  and  $\mathfrak{B}$  are  $C^*$ -algebras with cyclic vector, then every bounded bimodule map from an  $\mathfrak{A}$ - $\mathfrak{B}$  bimodule into  $\mathcal{B}(\mathcal{H})$  is completely bounded. Also a nice structure theorem is obtained for  $\mathfrak{A}$ - $\mathfrak{B}$  bimodule maps of the compact operators into  $\mathcal{B}(\mathcal{H})$ .
- (13) J.W. Bunce, *The similarity problem for representations of  $C^*$ -algebras*, Proc. Amer. Math. Soc. **81** (1981), 409–414.  
Every representation of a nuclear  $C^*$ -algebra is similar to a  $*$ -representation. Also proven by E. Christensen, Amer. J. Math. **103** (1981), 817–833.