

Pure Math 822 - Dilation Theory - Winter 2013

Instructor: Ken Davidson

Meeting Time: Tuesday and Thursday 9:00 – 10:30 in MC 5046

website: <http://www.math.uwaterloo.ca/~krdavids/PM822.html>

Course outline

This course is an introduction to dilation theory and abstract operator algebras. It has roots in the theory of completely positive maps and dilation theory, and Arveson's approach to studying nonself-adjoint algebras via the minimal enveloping C^* -algebra, the C^* -envelope.

1. Background on C^* -algebras and operator theory
2. Sz.Nagy and Ando dilation theorems, Commutant lifting theorem.
3. Completely positive maps, Stinespring's Theorem.
4. Arveson extension and dilation theorems.
5. C^* -envelope (Dritschel–McCullough, Arveson approach).
6. Completely bounded maps, Wittstock's theorem.
7. Completely bounded homomorphisms (Paulsen, Haagerup, Christensen)
8. Polynomially bounded operators, Pisier's counterexample.
9. Abstract operator algebras (Blecher-Ruan-Sinclair)
10. Universal operator algebras, factorization, Pisier's similarity degree.
11. Dilation theory and semicrossed products.

Required Background

The student needs a good course in functional analysis. Some further exposure to operator theory or operator algebras would be an asset.

Reference Texts

The main text for the course will be:

- V. Paulsen, *Completely bounded maps and operator algebras*, Cambridge University Press, 2002. (Online version available at CUP site.)

Other useful references are:

- B. Sz. Nagy and C. Foias (with H. Bercovici and L. Kerchy), *Harmonic analysis of operators on Hilbert space*, 2nd ed., Springer 2010. (Online version available through UW Library.)
- G. Pisier, *Completely bounded maps and similarity problems*, 2nd ed., Springer, 1996. (Online version available through UW Library.)
- K.R. Davidson and E. Katsoulis, *Dilation theory, commutant lifting and semicrossed products*, Documenta Math. **16** (2011), 781–868. (Available on my web site.)

Grading

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| 1. Problem Sets | 70 |
| 2. Talk and paper | 30 |