PM 352 **Assignment 5** **Due Monday March 22.**

Evaluate the following integrals using the residue theorem. Be careful to specify the curves that you integrate over, and explain why extraneous terms go to 0.

1. $\displaystyle\int_0^{\pi/2} \frac{dx}{a+\sin^2 x}$ for $a > 0$.

2. $\displaystyle\int_0^{\infty} \frac{x^2}{(x^2+a^2)^3}\,dx$ for $a > 0$.

3. $\displaystyle\int_0^{\infty} \frac{\cos x}{a^2+x^2}\,dx$ for $a > 0$.

4. $\displaystyle\int_0^{\infty} \frac{x^{1/3}}{1+x^2}\,dx$

5. $\displaystyle\int_0^{\infty} \frac{\log x}{(1+x^2)^2}\,dx$

6. $\displaystyle\int_0^{\infty} \frac{dx}{1+x^n}$ for $n > 1$. Use Fig.2.

Bonus Problems. Please hand in separately.

A. $\displaystyle\int_0^{\infty} \frac{\cos x}{\cosh x}\,dx$ Use Fig.3.

B. Compute $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$.

Extra Problems. Don't hand in.

C. $\displaystyle\int_0^{\infty} \frac{\log x}{x^2-1}\,dx$ Use Fig.1.

D. $\displaystyle\int_0^{\infty} \frac{\sin^2 x}{x^2}\,dx$.

E. $\displaystyle\int_0^{\infty} \frac{\cos x}{a^2-x^2}\,dx$ for $a = (2n+1)\pi/2$, $n \geq 1$. Use Fig.4.

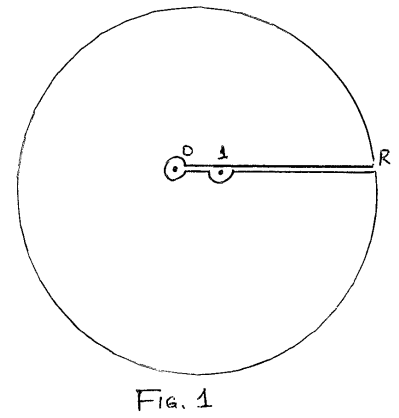

Fig. 1

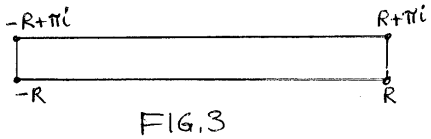

Fig.3

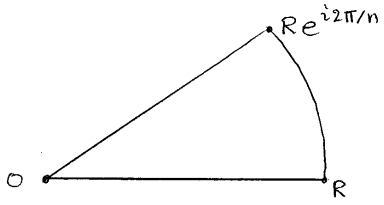

Fig. 2

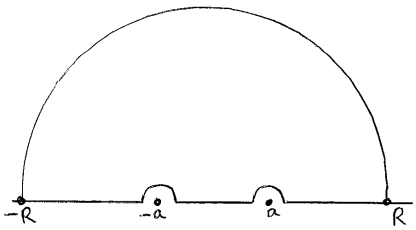

Fig.4