Math 249, Winter 2013
Assignment 7

Due Wednesday, April 3, in class.

1. The 4×4 Knight's Move graph, denoted $KM(4,4)$ has vertex set $[4] \times [4]$ and $\{(s,t),(u,v)\}$ is an edge iff $(s-u)^2 + (t-v)^2 = 5$.

   Prove that $KM(4,4)$ is not planar.

2. Prove that if G is a planar graph with girth at least 6, then G is 3-colourable.

3. Let G be a graph with $p \geq 3$ vertices and q edges. Prove that in every drawing of G, there must be at least $q - 3p + 6$ pairs of edges that cross.

4. Find a graph G with the following three properties:
   - G is planar;
   - G has a cycle of length 3;
   - there is no planar embedding of G with a face of degree 3.

   Prove that your answer is correct.

   [*Hint:* Try to find an operation on planar graphs that eliminates of faces of degree 3.]

5. Let G be connected planar graph. Let P be a planar embedding of G, and let P^* be the dual of P. As we know, P^* depends on the choice of planar embedding P, and there can be more than one. Does girth(P^*) also depend on P, or can it be determined solely from G? Explain.