COMBINATORIAL HOPF ALGEBRAS LECTURE 1 SUMMARY

WINTER 2020

Summary

Today we began with a summary of the main parts of the course. One thread will be versions of the definition of a combinatorial Hopf algebra. The 0th version is that a combinatorial Hopf algebra is when you have some combinatorial objects that you like along with a way to put objects together, giving a product, and a way to take them apart, giving a coproduct, that are compatible in the way that we'll discuss soon.

The three main parts of the course (roughly equal in number of weeks spent) will be

- Introduction to Hopf algebras in combinatorics, including background and definitions, word Hopf algebras, incidence Hopf algebras, how the antipode is essentially Möbius inversion.
- Renormalization by Hopf algebras, including the Connes-Kreimer Hopf algebra, Feynman diagrams and Feynman integrals, and the universal property of the Connes-Kreimer Hopf algebra.
- Symmetric functions and friends as Hopf algebras, including Sym, QSym, ..., universal property of Sym and the 3rd version of the definition of combinatorial Hopf algebra.

Then we went over some administrative things and you helped decide on some details of the course. This is all in the syllabus which is posted on the website `http://www.math.uwaterloo.ca/~kayeats/teaching/co739_w20.html`. Finally everyone shared some information on their background and what they are hoping to learn from the course. I hope that many of you who attended will sign up for the course and that many of you who don't sign up will still attend. Also, if you didn't come today you are still very welcome to join the course. If you think it's interesting then please spread the word! Wouldn't it be nice to get some people from the physics side too?

Next time

Next class we will define two Hopf algebras on words (and then we'll actually define Hopf algebras and related stuff next week).

References

Some references on Hopf algebras from a combinatorial point of view include

- "Hopf Algebras in Combinatorics" by Darij Grinberg and Victor Reiner, arXiv:1409.8356
- "Hopf algebras, from basics to applications to renormalization" by Dominique Manchon, arXiv:math/0408405v2
- "Incidence Hopf algebras" by Schmitt, `https://doi.org/10.1016/0022-4049(94)90105-8`

- Federico Ardila's lectures on youtube, `http://tinyurl.com/ardilahopf`, with a summary and index compiled by Sara Billey `http://math.sfsu.edu/federico/Clase/hopfindex.pdf`.