COMBINATORIAL HOPF ALGEBRAS LECTURE 13 SUMMARY

WINTER 2020

Summary

Today we proved the theorem that we left off with last time. You can find the proof in section 2.4.3 of the reference. Hongdi pointed out that any bialgebra map between Hopf algebras is a Hopf algebra map, so the very last part is unnecessary.

Then we gave a sketch of the point of renormalization and started to work with a toy model of renormalization. The point is that you have some combinatorial objects (Feynman graphs in the actual QFT case), you associate an integral to each one, these integrals diverge, so you have to fix them and you can use the Hopf algebra for this, but then you actually want to take infinite sums of these and they also diverge.

Here's the toy model. Let $\mathcal{H}$ be the Connes-Kreimer Hopf algebra. Define

$$F_s(B_+(f)) = \int_0^\infty \frac{F_z(f)}{s+z} dz$$

where s is a parameter with $s > 0$ and $F_s(1) = 1$, $F_s(t_1 \cdots t_k) = F_s(t_1) \cdots F_s(t_k)$. We did a few examples together.

References

Erik Panzer's masters thesis arXiv:1202.3552, sections 2.4 and 3.1.