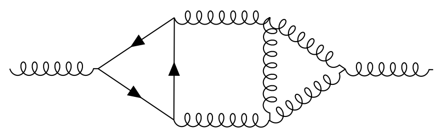
COMBINATORIAL HOPF ALGEBRAS, WINTER 2020, ASSIGNMENT 3

DUE THURSDAY APRIL 2 AT CLASS TIME

You can email me your assignment to submit it (take photos of any handwritten parts, like the students do for crowdmark).

(1) (a) Calculate the coproduct of the following graph in QCD (use the power counting weights we discussed in class for QCD)



(b) We discussed two maps R, evaluation of an integral at a fixed value (take the specific case of the toy model on trees for concreteness) and more briefly we discussed *minimal subtraction* where $R(\sum_{i=-L}^{\infty} c_i\epsilon^i) = \sum_{i=-L}^{-1} c_i\epsilon^i$. Both these maps R are Rota-Baxter maps. Check this and determine the weight of the Rota-Baxter map in each case.

(2) Consider scalar ϕ^k theory, that is a combinatorial physical theory with a single half edge type, a single edge type consisting of two half edges and with power counting weight 2, and a single vertex type of degree k with power counting weight 0.

We said a combinatorial physical theory is renormalizable if the superficial degree of divergence only depends on the multiset of external edges.

For what dimension of space time is ϕ^k theory renormalizable (give D as a function of k)? For which values of k is this dimension of space time an integer?

(3) We gave S and Δ in Sym for all the bases we discussed except for the Schur functions. Find formulas for $S(s_\lambda)$ and $\Delta(s_\lambda)$.

It will probably be helpful for you to make use of *skew schur functions*. Say $\nu \le \mu$, for two partitions ν and μ when $\nu_i \le \mu_i$. This means that the shape of ν sits inside the shape of μ (both aligned at the top left). The *skew shape* μ/ν consists of the boxes of μ that are not boxes of ν when the shapes are aligned at their top lefts. We can fill skew shapes just as we filled partition shapes (weakly increasing in rows and strictly increasing in columns), and analogously define the *skew schur function* $s_{\mu/\nu}$ to be the sum over fillings of the shape μ/ν, where each filling contributes the monomial $\prod x_i^{\text{number of } i\text{s in the filling}}$.