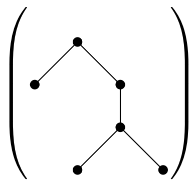
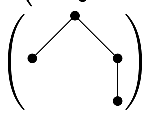
COMBINATORIAL HOPF ALGEBRAS, WINTER 2020, ASSIGNMENT 2

DUE THURSDAY MARCH 5 IN CLASS

(1) (a) Calculate the following things in the Connes-Kreimer Hopf algebra of rooted trees.

(i) Δ 

(ii) S 

(b) In the tree-based toy model calculate the renormalized Feynman rules applied to the tree $B_+(\bullet\bullet)$. You can use maple or another computer algebra system to actually do the integrals.

(2) Let l_n be the rooted tree which is a path with n vertices with the root at one end. Call this the *ladder* on n vertices, and let $l_0 = 1$.

(a) Prove that $K[l_1, l_2, \ldots]$ is a sub Hopf algebra of the Connes-Kreimer Hopf algebra.

(b) Prove that $[x^n] \log\left(\sum_{i=0}^{\infty} l_i x^i\right)$ is primitive in the Connes-Kreimer Hopf algebra. *In fact these characterize exactly the primitive elements made of ladders. There are some algebraic characterizations of all primitives, but to my knowledge, not nice combinatorial understandings of primitives beyond the ladders.*

(3) Find all Hochschild 1-cocyles for the Hopf algebra of polynomials. (Use $\psi_L = \Delta$ and $\psi_R = \mathrm{id} \otimes 1$, as we do for Connes-Kreimer.)