Second commutative square.

CO739, Winter 2020

Another square

Here's another commutative square.

$$\begin{tikzcd} h_n \arrow[d, mapsto] & \mathrm{NSym} \arrow[r] \arrow[d] & \mathrm{NCSym} \arrow[d] \\ h_n & \mathrm{Sym} \arrow[r] & \mathrm{Sym} \end{tikzcd}$$

with

$$\mathrm{NSym} \to \mathrm{Sym}$$
$$h_i \mapsto h_i$$

just make them commute, same as for the other square.

More maps

We have

$$\widetilde{\chi} : \mathrm{Sym} \to \mathrm{NCSym}$$

$$\underline{\underline{m_\lambda}} \mapsto \frac{1}{\#} \sum_{\substack{\text{sizes of parts} \\ \text{of } \pi \text{ give } \lambda}} m_\pi$$

eg

$$\widetilde{\chi}(m_{21}) = \frac{1}{3}\left(m_{\{1,2\},\{3\}} + m_{\{1,3\},\{2\}} + m_{\{2,3\},\{1\}}\right)$$

and

$$\chi : \mathrm{NCSym} \to \mathrm{Sym}$$

by forgetting the noncommutativity of variables.

Putting these maps together

Then we can define

$$I : \mathrm{NSym} \to \mathrm{NCSym}$$
$$h_n \mapsto \widetilde{\chi}(h_n)$$

$h_n \in \mathrm{NSym}$ $\quad$ $\tilde{\chi}(h_n) \in \mathrm{Sym}$ $\quad$ and extend as an alg. homomorphism

So

$h_n \longmapsto \tilde{\chi}(h_n) = \sum_{\pi \text{ set partn of } h_n} \frac{m_\pi}{\#}$

$\#$ of set partitions with same partition of sizes of parts.

$$\begin{tikzcd}
\mathrm{NSym} \arrow[r, "I"] \arrow[d] & \mathrm{NCSym} \arrow[d, "\chi"] \\
\mathrm{Sym} \arrow[r, "\mathrm{id}"'] & \mathrm{Sym}
\end{tikzcd}$$

$h_n \mapsto h_n$ (left), $\tilde{\chi}(h_n) \mapsto h_n$ (right)

Another page for diagrams that run over