

## Second commutative square.

CO739, Winter 2020

## Another square

Here's another commutative square.

$$\begin{array}{ccc}
 h_n & \text{NSym} & \longrightarrow & \text{NCSym} \\
 \downarrow & \downarrow & & \downarrow \\
 h_n & \text{Sym} & \longrightarrow & \text{Sym}
 \end{array}$$

with

$$\text{NSym} \rightarrow \text{Sym}$$

$$h_i \mapsto h_i$$

just make them commute, same as for the other square.

## More maps

We have

$$\tilde{\chi} : \text{Sym} \rightarrow \text{NCSym}$$

$$\underline{m_\lambda} \mapsto \frac{1}{\#} \sum_{\substack{\text{sizes of parts} \\ \text{of } \pi \text{ give } \lambda}} m_\pi$$

eg  
 $\tilde{\chi}(m_{2,1})$   
 $= \frac{1}{3} (m_{\{1,2\},\{3\}} + m_{\{1,3\},\{2\}} + m_{\{2,3\},\{1\}})$

and

$$\chi : \text{NCSym} \rightarrow \text{Sym}$$

by forgetting the noncommutativity of variables.

# Putting these maps together

Then we can define

$$I : \text{NSym} \rightarrow \text{NCSym}$$

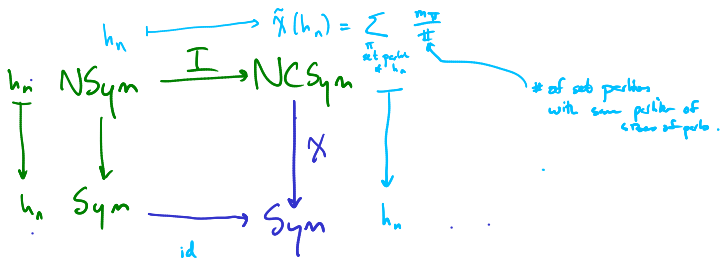
$$h_n \mapsto \tilde{\chi}(h_n)$$

$$\begin{array}{c} h_n \\ \uparrow \\ \text{NSym} \end{array}$$

$$\begin{array}{c} \tilde{\chi}(h_n) \\ \uparrow \\ \text{Sym} \end{array}$$

and extend  
as an alg. homomorphism

So



Another square relating some Hopf algebras we've seen

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## Another page for diagrams that run over