Duality 000

## Duality between QSym and NSym.

CO739, Winter 2020

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## Duality

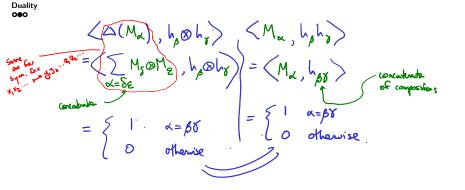
## Proposition

QSym and NSym are graded dual Hopf algebras

proof: Use the pairing  

$$QSym \times NSym \longrightarrow K$$
  
 $\langle M_{x}, h_{\beta} \rangle = \begin{cases} 1 & x=\beta \\ 0 & x\neq\beta \end{cases}$   
Now just need to verify  
 $\langle \Delta(M_{x}), h_{\beta} \otimes h_{\gamma} \rangle = \langle M_{x}, h_{\beta}h_{\gamma} \rangle$   
So let's compute to show this

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## **Observations**

This duality is compatible with the self-duality of Sym.

Specifically, the map

is dual to the map

Sym  $\hookrightarrow$  QSym  $K < \{h_1, h_2, \dots > K \in [h_1, h_2, \dots]$ NSym  $\rightarrow$  Sym  $h_n \mapsto h_n$  but now the  $h_n$ 's commute

Because the pairing is the same Note Sym is both commutate and cocommutate cound is given QSym is commutative but is not cocommutate of indexinguitate NSym is not commutative but is cocommutative manifests = 200