If you go to "view options" and turn on "annotate"
you should be able to write on my screen-share
TRY IT!

NSym.

CO739, Winter 2020

Don't forget course evals : evaluate.uwaterloo.ca
last I checked none had done it

NSym as an algebra

noncomm. polys

As an algebra NSym $= K\langle h_1, h_2, \ldots\rangle$, noncommutative polynomials.

Take h_i to have degree i.

eg $\deg(h_2 h_1 h_2 h_3) = 8$

Let $h_\alpha = h_{\alpha_1} h_{\alpha_2} \cdots h_{\alpha_k}$ where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k)$ is a composition. These are noncomm. monomials in the h_i

Then $\{h_\alpha\}_{\alpha \text{ composition}}$ gives a vector space basis for NSym.

So $\dim \mathrm{NSym}_n = 2^{n-1}$.

NSym as a bialgebra

NSym is a bialgebra with $\Delta(h_n) = \sum_{k=0}^{n} h_k \otimes h_{n-k}$, and extended as an algebra homomorphism.

It is graded and connected so it is a Hopf algebra.

by degree

We have es too

same relation that connects es and hs in honest-to-goodness symmetric functions

Can define $e_1, e_2, \ldots$ recursively by

$$\sum_{i=0}^{n} (-1)^i e_i h_{n-i} = \begin{cases} 1 & n > 0 \\ 0 & n = 0 \end{cases}$$

Get that $\mathrm{NSym} = K\langle e_1, e_2, \ldots \rangle$ and $\Delta(e_n) = \sum_{k=0}^{n} e_k \otimes e_{n-k}$.

Idea: Like Sym but the h_i don't commute
Just define it that way (what we did above)
And don't worry about an interpretation as functions
on the x_is.