Third commutative square.

CO739, Winter 2020

Noncommutative Connes-Kreimer

Let $\mathcal{H}$ be Connes-Kreimer

(Recall $\mathcal{H} = K[\mathcal{T}]$. $\mathcal{T}$ rooted trees

$$\Delta(t) = \sum_{\substack{C \subseteq V(t) \\ \text{antichain}}} \left(\prod_{v \in C} t_v\right) \otimes \left(t \setminus \prod_{v \in C} t_v\right)$$

)

Could just as well order our forests

$$N\mathcal{H} = K\langle\widetilde{\mathcal{T}}\rangle$$

rooted trees with at each vertex a linear order on the children of that vertex.

but then we need to replace rooted trees $\mathcal{T}$ with plane rooted trees $\widetilde{\mathcal{T}}$. Why?

ordered . . .

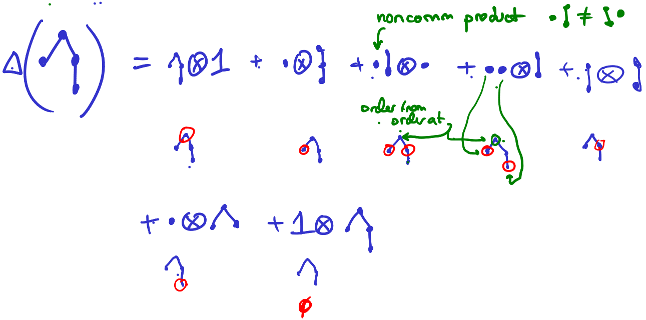
so cutting off subtrees you have an order for these subtrees

Also then B_+ of a monomial gives a tree with exactly the correct structure.

Operations for Noncommutative Connes-Kreimer

Because then the same operations work.

eg of Δ



order of children of root comes from order of product.

Yet another square

Then we get one final commutative square

$$\begin{tikzcd} e_n \arrow[r, mapsto] & \ell_n \\ NSym \arrow[r] \arrow[d] & N\mathcal{H} \arrow[d] \\ Sym \arrow[r] & \mathcal{H} \\ e_n \arrow[r, mapsto] & \ell_n \end{tikzcd}$$

still have ladders in plane trees

forget order on trees

(or you could use $h_n \mapsto \ell_n$)

where $\ell_n =$  n vertices

$$\Delta(\ell_n) = \sum_{i=0}^{n} \ell_i \otimes \ell_{n-i}$$

Extra space