

CO 430/630 LECTURE 18 SUMMARY

WINTER 2026

SUMMARY

Today you worked on the non-intersecting path worksheet and then groups which finished started on the cyclic sieving worksheet.

The non-intersecting path worksheet is the only worksheet whose content may directly appear on the final exam (the other worksheets are still useful practice with course material, but the new things developed in them will not directly be on the exam.) If you missed the worksheet talk to a friend in the class or look up the Lindström-Gessel-Viennot lemma. I will also bring a few more copies and there will be some time to work further on them on Monday.

Two extra notes on that worksheet. The determinants are possible to simplify, though not with techniques from this class. It turns out that for the number of ways to have m non-crossing lattice paths from $(0, 0)$ to (k, ℓ) the determinant evaluates to

$$\prod_{a=1}^k \prod_{b=1}^{\ell} \prod_{c=1}^m \frac{a+b+c-1}{a+b+c-2}$$

and for the final example, the plane partitions, the same arguments can be augmented to give the generating function with respect to the number of cubes by replacing $\binom{2n}{n-i+j}$ with

$$q^{(j-1)(j-i)} \begin{bmatrix} 2n \\ n-i+j \end{bmatrix}_q.$$

NEXT TIME

Presentations begin, but there will also be some time to continue working on the worksheets.

REFERENCES

Ask your friends for the worksheets if you missed them. You can also find the Lindström Gessel Viennot lemma in many sources. The version in Stanley is quite short.