PMATH 990: Topics in Pure Mathematics
Elliptic Methods in Geometric Analysis (FALL 2026)

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- **Office Hours:** TBD
- **Lecture Room:** TBD
- **Lecture Times:** TBD

(Tentative) course description: We study the main tools used in elliptic theory (Sobolev spaces, embedding theorems, Rellich lemma, elliptic regularity) on $\mathbb{R}^n$ and explain the passage from these local results to Banach spaces of sections of vector bundles over compact oriented Riemannian manifolds. Then we consider the basic linear problem, namely the *Hodge decomposition theorem* for various Laplacians.

Prerequisites and plan: Prior exposure to both functional analysis and measure theory is very helpful, but as long as the student is willing to accept certain results from functional analysis and measure theory as "black boxes", they should be able to follow. The first half of the course will focus on differential analysis on $\mathbb{R}^n$. In the second half of the course we will adapt this machinery to the setting of vector bundles over compact oriented Riemannian manifolds. The lectures in the second half will be designed based on the background of the audience, specifically in smooth manifold theory and Riemannian geometry.

Topics to be covered include (tentative and subject to change):

- Convolution; mollification; Schwartz class functions; distributions; Fourier transform.
- Sobolev spaces ($W^{k,p}$); Sobolev Embedding Theorem; Kondrakov–Rellich Theorem.
- Calderon–Zygmnund inequality; Sobolev elliptic regularity.
- Laplacians; elliptic operators; elliptic complexes.
- Compact operators; Fredholm operators; the Fredholm alternative.
- Elliptic operators on vector bundles over compact oriented Riemannian manifolds.
- The Hodge decomposition theorem; applications.

Textbook

There is no official textbook. I will be using many different sources for this course. Some of these are:

- Aubin; *Some Nonlinear Problems in Riemannian Geometry*; Springer-Verlag
- Evans; *Partial Differential Equations*; American Mathematical Society
- Gilbarg and Trudginer; *Elliptic Partial Differential Equations of Second Order*; Springer-Verlag
- Gilkey; *Invariance Theory, the Heat Equation, and the Atiyah-Singer Index Theorem*, freely (and legally) available here: `http://www.emis.de/monographs/gilkey/`
- Griffiths and Harris; *Principles of Algebraic Geometry*; John Wiley & Sons
- Lawson and Michelsohn; *Spin Geometry*; Princeton University Press
- McDuff and Salamon; *J-holomorphic curves and symplectic topology*; American Mathematical Society
- Melrose; *Graduate Analysis*; lecture notes from an M.I.T. course
- Rosenberg; *The Laplacian on a Riemannian Manifold*; Cambridge University Press
- Roe; *Elliptic operators, topology and asymptotic methods*; Chapman & Hall/CRC
- Warner; *Foundations of Differentiable Manifolds and Lie Groups*; Springer-Verlag
- Wells; *Differential Analysis on Complex Manifolds*; Springer-Verlag

Marking scheme: There will be six assignments, *one due every two weeks*. The assignments will be lengthy and often technical. *Do not leave them for the last minute. Start them right away.*

You are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems. Your course mark will be determined by the following scheme:

- Assignments: 100% (six assignments at 16.67% each)

NOTE: For information on academic offences and accessibility services, please see the detailed version of the course outline available at TBD