Solution to Practice 3h

B8(a) First we note that a polar form for 32 is $32e^{i0}$. This means that the fifth roots of 32 are

$k = 0: \quad 32^{1/5}e^{i0/5} = 2e^{i0}$

$k = 1: \quad 32^{1/5}e^{i2\pi(1)/5} = 2e^{i2\pi/5}$

$k = 2: \quad 32^{1/5}e^{i2\pi(2)/5} = 2e^{i4\pi/5}$

$k = 3: \quad 32^{1/5}e^{i2\pi(3)/5} = 2e^{i6\pi/5}$

$k = 4: \quad 32^{1/5}e^{i2\pi(4)/5} = 2e^{i8\pi/5}$

B8(b) First we note that a polar form for $81i$ is $32e^{i\pi/2}$. This means that the fifth roots of $81i$ are

$k = 0: \quad 81^{1/5}e^{i(\pi/2)/5} = 3^{4/5}e^{i\pi/10}$

$k = 1: \quad 81^{1/5}e^{i(\pi/2+2\pi(1))/5} = 3^{4/5}e^{i\pi/2}$ (Sidenote: This shows that i is a fifth root of i, which makes sense since $(i)(i)(i)(i)(i) = (-1)(-1)(i) = i$.)

$k = 2: \quad 81^{1/5}e^{i(\pi/2+2\pi(2))/5} = 3^{4/5}e^{i9\pi/10}$

$k = 3: \quad 81^{1/5}e^{i(\pi/2+2\pi(3))/5} = 3^{4/5}e^{i13\pi/10}$

$k = 4: \quad 81^{1/5}e^{i(\pi/2+2\pi(4))/5} = 3^{4/5}e^{i17\pi/10}$

B8(c) First need to find a polar form for $-\sqrt{3}+i$. We start by calculating $r = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2$. Next, we need to find θ such that $\cos\theta = -\sqrt{3}/2$ and $\sin\theta = 1/2$. Since the cosine is negative and the sine is positive, we want θ to be in the second quadrant, so we can use $\theta = 5\pi/6$. This means that the cube roots of $-3\sqrt{3}+i$ are

$k = 0: \quad 2^{1/3}e^{i(5\pi/6)/3} = 2^{1/10}e^{i5\pi/18}$

$k = 1: \quad 2^{1/3}e^{i(5\pi/6+2\pi(1))/3} = 2^{1/10}e^{i17\pi/18}$

$k = 2: \quad 2^{1/3}e^{i(5\pi/6+2\pi(2))/3} = 2^{1/10}e^{i29\pi/18}$

B8(d) First need to find a polar form for $4+i$. We start by calculating $r = \sqrt{4^2+1^2} = \sqrt{16+1} = \sqrt{17}$. Next, we need to find θ such that $\cos\theta = 4/\sqrt{17}$ and $\sin\theta = 1/\sqrt{17}$. Since the cosine is positive and the sine is positive, we want θ to be in the first quadrant. This means we can use either $\sin^{-1}$ or $\cos^{-1}$ in our calculator to get $\theta \approx .245$. This means that the cube roots of $4+i$ are

$k = 0: \quad \sqrt{17}^{1/3}e^{i(.245)/3} = 17^{1/6}e^{i(.082)}$

$$k = 1: \quad \sqrt{17}^{1/3} e^{i(.245+2\pi(1))/3} = 17^{1/6} e^{i(2.176)}$$

$$k = 2: \quad \sqrt{17}^{1/3} e^{i(.245+2\pi(2))/5} = 17^{1/6} e^{i(4.270)}$$