Solution to Practice 3f

B6(a) First we need to find a polar form for $1-\sqrt{3}i$ and $-1+i$.

To find a polar form for $1-\sqrt{3}i$, we first find $r=\sqrt{1^2+(-\sqrt{3})^2}=\sqrt{1+3}=2$. Next, we need to find θ such that $\cos\theta=1/2$ and $\sin\theta=-\sqrt{3}/2$. Since our cosine value is positive and our sine value is negative, we know that θ is in the fourth quadrant, so we can use $\theta=5\pi/3$ as an argument. This means that $2(\cos(5\pi/3)+i\sin(5\pi/3))$ is a polar form for $1-\sqrt{3}i$.

To find a polar form for $-1+i$, we first find $r=\sqrt{(-1)^2+1^2}=\sqrt{1+1}=\sqrt{2}$. Next, we need to find θ such that $\cos\theta=-1/\sqrt{2}$ and $\sin\theta=1/\sqrt{2}$. Since our cosine value is negative and our sine value is positive, we know that θ is in the second quadrant, so we can use $\theta=3\pi/4$ as an argument. This means that $\sqrt{2}(\cos(3\pi/4)+i\sin(3\pi/4))$ is a polar form for $-1+i$.

We can use these polar forms to calculate the following:

$$\begin{aligned}(1-\sqrt{3}i)(-1+i) &= (2(\cos(5\pi/3)+i\sin(5\pi/3)))(\sqrt{2}(\cos(3\pi/4)+i\sin(3\pi/4)))\\ &= 2\sqrt{2}(\cos((5\pi/3)+(3\pi/4))+i\sin((5\pi/3)+(3\pi/4)))\\ &= 2\sqrt{2}(\cos(29\pi/12)+i\sin(29\pi/12))\end{aligned}$$

$$\begin{aligned}(1-\sqrt{3}i)/(-1+i) &= (2(\cos(5\pi/3)+i\sin(5\pi/3)))/(\sqrt{2}(\cos(3\pi/4)+i\sin(3\pi/4)))\\ &= (2/\sqrt{2})(\cos((5\pi/3)-(3\pi/4))+i\sin((5\pi/3)-(3\pi/4)))\\ &= \sqrt{2}(\cos(11\pi/12)+i\sin(11\pi/12))\end{aligned}$$

B6(b) First we need to find a polar form for $-\sqrt{3}+i$ and $-3-3i$.

To find a polar form for $-\sqrt{3}+i$, we first find $r=\sqrt{(-\sqrt{3})^2+1^2}=\sqrt{3+1}=2$. Next, we need to find θ such that $\cos\theta=-\sqrt{3}/2$ and $\sin\theta=1/2$. Since our cosine value is negative and our sine value is positive, we know that θ is in the second quadrant, so we can use $\theta=5\pi/6$ as an argument. This means that $2(\cos(5\pi/6)+i\sin(5\pi/6))$ is a polar form for $-\sqrt{3}+i$.

To find a polar form for $-3-3i$, we first find $r=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=3\sqrt{2}$. Next, we need to find θ such that $\cos\theta=-3/3\sqrt{2}=-1/\sqrt{2}$ and $\sin\theta=-3/3\sqrt{2}=-1/\sqrt{2}$. Since our cosine value is negative and our sine value is negative, we know that θ is in the third quadrant, so we can use $\theta=5\pi/4$ as an argument. This means that $3\sqrt{2}(\cos(5\pi/4)+i\sin(5\pi/4))$ is a polar form for $-3-3i$.

We can use these polar forms to calculate the following:

$$\begin{aligned}(-\sqrt{3}+i)(-3-3i) &= (2(\cos(5\pi/6)+i\sin(5\pi/6)))(3\sqrt{2}(\cos(5\pi/4)+i\sin(5\pi/4)))\\ &= (2)(3\sqrt{2})(\cos((5\pi/6)+(5\pi/4))+i\sin((5\pi/6)+(5\pi/4)))\\ &= 6\sqrt{2}(\cos(25\pi/12)+i\sin(25\pi/12))\end{aligned}$$

$$\begin{aligned}(-\sqrt{3}+i)/(-3-3i) &= (2(\cos(5\pi/6)+i\sin(5\pi/6)))/(3\sqrt{2}(\cos(5\pi/4)+i\sin(5\pi/4)))\\ &= (2/(3\sqrt{2}))(\cos((5\pi/6)-(5\pi/4))+i\sin((5\pi/6)-(5\pi/4)))\\ &= (\sqrt{2}/3)(\cos(-5\pi/12)+i\sin(-5\pi/12))\end{aligned}$$

B6(c) First we need to find a polar form for $1+3i$ and $-1-2i$.

To find a polar form for $1+3i$, we first find $r=\sqrt{1^2+3^2}=\sqrt{1+9}=\sqrt{10}$. Next, we need to find θ such that $\cos\theta = 1/\sqrt{10}$ and $\sin\theta = 3/\sqrt{10}$. Since our cosine value is positive and our sine value is positive, we know that θ is in the first quadrant, so we can use $\theta = \cos^{-1}(1/\sqrt{10}) \approx 1.25$ as an argument. This means that $\sqrt{10}(\cos(1.25)+i\sin(1.25))$ is a polar form for $1+3i$.

To find a polar form for $-1-2i$, we first find $r=\sqrt{(-1)^2+(-2)^2}=\sqrt{1+4}=\sqrt{5}$. Next, we need to find θ such that $\cos\theta = -1/\sqrt{5}$ and $\sin\theta = -2/\sqrt{5}$. Since our cosine value is negative and our sine value is negative, we know that θ is in the third quadrant. So we can use $\theta = -\cos^{-1}(-1/\sqrt{5}) \approx -2.03$ as an argument. This means that $\sqrt{5}(\cos(-2.03)+i\sin(-2.03))$ is a polar form for $-1-2i$.

We can use these polar forms to calculate the following:

$$\begin{aligned}(1+3i)(-1-2i) &= (\sqrt{10}(\cos(1.25)+i\sin(1.25)))(\sqrt{5}(\cos(-2.03)+i\sin(-2.03)))\\ &= (\sqrt{10})(\sqrt{5})(\cos(1.25-2.03)+i\sin(1.25-2.03))\\ &= 5\sqrt{2}(\cos(-0.78)+i\sin(-0.78))\end{aligned}$$

$$\begin{aligned}(1+3i)/(-1-2i) &= (\sqrt{10}(\cos(1.25)+i\sin(1.25)))/(\sqrt{5}(\cos(-2.03)+i\sin(-2.03)))\\ &= (\sqrt{10}/\sqrt{5})(\cos(1.25+2.03)+i\sin(1.25+2.03))\\ &= \sqrt{2}(\cos(3.28)+i\sin(3.28))\end{aligned}$$

B6(d) First we need to find a polar form for $-2+i$ and $4-i$.

To find a polar form for $-2+i$, we first find $r=\sqrt{(-2)^2+1^2}=\sqrt{4+1}=\sqrt{5}$. Next, we need to find θ such that $\cos\theta = -2/\sqrt{5}$ and $\sin\theta = 1/\sqrt{5}$. Since our cosine value is negative and our sine value is positive, we know that θ is in the second quadrant. So we can use $\theta = \cos^{-1}(-2/\sqrt{5}) \approx 3.68$ as an argument. This means that $\sqrt{5}(\cos(2.68)+i\sin(2.68))$ is a polar form for $-2+i$.

To find a polar form for $4-i$, we first find $r=\sqrt{4^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}$. Next, we need to find θ such that $\cos\theta = 4/\sqrt{17}$ and $\sin\theta = -1/\sqrt{17}$. Since our cosine value is positive and our sine value is negative, we know that θ is in the fourth quadrant. So we can use $\theta = \sin^{-1}(-1/\sqrt{17}) \approx -0.24$ as an argument. This means that $\sqrt{17}(\cos(-0.24)+i\sin(-0.24))$ is a polar form for $-1-2i$.

We can use these polar forms to calculate the following:

$$
\begin{aligned}
(-2+i)(4-i) &= (\sqrt{5}(\cos(2.68)+i\sin(2.68)))(\sqrt{17}(\cos(-0.24)+i\sin(-0.24))) \\
&= (\sqrt{5})(\sqrt{17})(\cos(2.68-0.24)+i\sin(2.68-0.24)) \\
&= \sqrt{85}(\cos(2.44)+i\sin(2.44))
\end{aligned}
$$

$$
\begin{aligned}
(-2+i)/(4-i) &= (\sqrt{5}(\cos(2.68)+i\sin(2.68)))/(\sqrt{17}(\cos(-0.24)+i\sin(-0.24))) \\
&= (\sqrt{5}/\sqrt{17})(\cos(2.68+0.24)+i\sin(2.68+0.24)) \\
&= \sqrt{5/17}(\cos(2.92)+i\sin(2.92))
\end{aligned}
$$