Solution to Practice 3d

(1) Since $1+4i$ is a root of p_1, $1-4i$ is also a root of p_1. Which means that both $(x-1-4i)$ and $(x-1+4i)$ are factors of p_1. This means that their product $(x-1-4i)(x-1+4i)$ is a factor of p_1. In fact, we see that $p_1(x)=(x-1-4i)(x-1+4i)$.

(2) Since $3-4i$ is a root of p_2, $3+4i$ is also a root of p_2. Which means that both $(x-3+4i)$ and $(x-3-4i)$ are factors of p_2. This means that their product $(x-3+4i)(x-3-4i)$ is a factor of p_2. Multiplying we get

$$(x-3+4i)(x-3-4i) = x^2-3x-4ix-3x+9+12i+4ix-12i-16i^2 = x^2-6x+25$$

Dividing $p_2(x) = x^3-9x^2+43x-75$ by $x^2-6x+25$ gives us $x-3$. So $p_2(x)=(x-3+4i)(x-3-4i)(x-3)$.

(3) Since $-2-2i$ is a root of p_3, $-2+2i$ is also a root of p_3. Which means that both $(x+2+2i)$ and $(x+2-2i)$ are factors of p_3. This means that their product $(x+2+2i)(x+2-2i)$ is a factor of p_3. Multiplying we get

$$(x+2+2i)(x+2-2i) = x^2+2x-2ix+2x+4-4i+2ix+4i-4i^2 = x^2+4x+8$$

Dividing $p_3(x)=x^4+8x^3+19x^2+12x-40$ by x^2+4x+8 gives us x^2+4x-5. And since $x^2+4x-5=(x-1)(x+5)$, we see that $p_3(x)=(x+2+2i)(x+2-2i)(x-1)(x+5)$.

(4) Since $5i$ is a root of p_4, $-5i$ is also a root of p_4. Which means that both $(x-5i)$ and $(x+5i)$ are factors of p_4. This means that their product $(x-5i)(x+5i)$ is a factor of p_4. Multiplying we get

$$(x-5i)(x+5i) = x^2+5ix-5ix-25i^2 = x^2+25$$

Dividing $p_4(x)=x^4-2x^3+30x^2-50x+125$ by x^2+25 gives us x^2-2x+5. To find the roots of this quadratic equation, we use the quadratic formula:

$$\begin{aligned} \frac{-b\pm\sqrt{b^2-4ac}}{2a} &= \frac{2\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)} \\ &= \frac{2\pm\sqrt{4-20}}{2} \\ &= \frac{2\pm\sqrt{-16}}{2} \\ &= \frac{2\pm 4i}{2} \\ &= 1+2i \text{ and } 1-2i \end{aligned}$$

And so we see that $1+2i$ and its conjugate $1-2i$ are also roots of p_4. And since $(x-1-2i)(x-1+2i) = x^2-2x+5$, we see that $p_4(x) = (x-5i)(x+5i)(x-1-2i)(x-1+2i)$.