Solution to Practice 3b

B3(a) $\overline{2i} = -2i$

B3(b) $\overline{17} = 17$

B3(c) $\overline{4-8i} = 4+8i$

B3(d) $\overline{5+11i} = 5-11i$

Property (5) Let $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$ be complex numbers. Then we see that

$$\begin{aligned}
\overline{z_1 z_2} &= \overline{(x_1 + y_1 i)(x_2 + y_2 i)} \\
&= \overline{(x_1x_2 - y_1y_2) + (x_1y_2 + x_2y_1)i} \\
&= (x_1x_2 - y_1y_2) - (x_1y_2 + x_2y_1)i \\
&(x_1x_2 - (-y_1)(-y_2)) + (x_1(-y_2) + x_2(-y_1))i \\
&= (x_1 - y_1 i)(x_2 - y_2 i) \\
&= \overline{x_1 + y_1 i}\ \overline{x_2 + y_2 i}
\end{aligned}$$

Property (7) Let $z_1 = x_1 + y_1 i$ be a complex number. Then $z_1 + \overline{z_1} = (x_1 + y_1 i) + (x_1 - y_1 i) = (x_1 + x_1) + (y_1 - y_1)i = 2x_1 = 2\mathrm{Re}(z_1)$.

Property (8) Let $z_1 = x_1 + y_1 i$ be a complex number. Then $z_1 - \overline{z_1} = (x_1 + y_1 i) - (x_1 - y_1 i) = (x_1 + y_1 i) + (-x_1 + y_1 i) = (x_1 - x_1) + (y_1 + y_1)i = 2y_1 i = i2\mathrm{Im}(z_1)$.

Property(9) Let $z_1 = x_1 + y_1 i$ be a complex number. Then

$$\begin{aligned}
z_1\overline{z_1} &= (x_1 + y_1 i)(x_1 - y_1 i) \\
&= (x_1x_1 - (y_1)(-y_1)) + ((x_1)(-y_1) + x_1y_1)i \\
&= x_1^2 + y_1^2 + 0i \\
&= x_1^2 + y_1^2
\end{aligned}$$