# Assignment 7

[2pt] 1. Prove that $\langle p, q\rangle = p(1)q(1) + p(2)q(2) + p(3)q(3)$ defines an inner product on $P_2$.

[1pt] 2. Prove that $\langle \vec{x}, \vec{y}\rangle = x_1y_1 - x_2y_2$ does not define an inner product on $\mathbb{R}^2$.

[4pt] 3. On $M(2,2)$, define the inner product $\langle A, B\rangle = \operatorname{tr}(B^T A)$. Use the Gram-Schmidt Procedure to determine an orthonormal basis for

$$\mathbb{S} = \operatorname{Span}\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

and use this basis to determine $\operatorname{proj}_{\mathbb{S}} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

[3pt] 4. Let $A = \begin{bmatrix} -2 & 2 & -1 \\ 2 & 1 & -2 \\ -1 & -2 & -2 \end{bmatrix}$. Find an orthogonal matrix $P$ and a diagonal matrix $D$ such that $P^T AP = D$.