Solution to Practice 2f

B1(a) We have $\text{proj}_{\mathbb{S}}\vec{x} = \frac{\vec{x}\cdot\vec{v}_1}{||\vec{v}_1||^2}\vec{v}_1 + \frac{\vec{x}\cdot\vec{v}_2}{||\vec{v}_2||^2}\vec{v}_2$, so we perform the following calculations:

$$\begin{array}{ll} \vec{x}\cdot\vec{v}_1 = 4+0+2+5 = 11 & ||\vec{v}_1||^2 = 1^2+0^2+(-1)^2+1^2 = 3 \\ \vec{x}\cdot\vec{v}_2 = 4+3+2-10 = -1 & ||\vec{v}_2||^2 = 1^2+1^2+(-1)^2+(-2)^2 = 7 \end{array}$$

So

$$\text{proj}_{\mathbb{S}}\vec{x} = \frac{11}{3}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} - \frac{1}{7}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 74/21 \\ -1/7 \\ -80/21 \\ 71/21 \end{bmatrix}$$

B1(b) We have $\text{proj}_{\mathbb{S}}\vec{x} = \frac{\vec{x}\cdot\vec{v}_1}{||\vec{v}_1||^2}\vec{v}_1 + \frac{\vec{x}\cdot\vec{v}_2}{||\vec{v}_2||^2}\vec{v}_2$, so we perform the following calculations:

$$\begin{array}{ll} \vec{x}\cdot\vec{v}_1 = 8+3+0+5 = 16 & ||\vec{v}_1||^2 = 2^2+1^2+0^2+1^2 = 6 \\ \vec{x}\cdot\vec{v}_2 = -4+3-2+5 = 2 & ||\vec{v}_2||^2 = (-1)^2+1^2+1^2+1^2 = 4 \end{array}$$

So

$$\text{proj}_{\mathbb{S}}\vec{x} = \frac{8}{3}\begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 29/6 \\ 19/6 \\ 1/2 \\ 19/6 \end{bmatrix}$$

B1(c) We have $\text{proj}_{\mathbb{S}}\vec{x} = \frac{\vec{x}\cdot\vec{v}_1}{||\vec{v}_1||^2}\vec{v}_1 + \frac{\vec{x}\cdot\vec{v}_2}{||\vec{v}_2||^2}\vec{v}_2 + \frac{\vec{x}\cdot\vec{v}_3}{||\vec{v}_2||^3}\vec{v}_3$, so we perform the following calculations:

$$\begin{array}{ll} \vec{x}\cdot\vec{v}_1 = 4+0-2+5 = 7 & ||\vec{v}_1||^2 = 1^2+0^2+1^2+1^2 = 3 \\ \vec{x}\cdot\vec{v}_2 = 0+3+2+5 = 10 & ||\vec{v}_2||^2 = 0^2+1^2+(-1)^2+1^2 = 3 \\ \vec{x}\cdot\vec{v}_3 = 4+3+0-5 = 2 & ||\vec{v}_3||^2 = 1^2+1^2+0^2+(-1)^2 = 3 \end{array}$$

So

$$\text{proj}_{\mathbb{S}}\vec{x} = \frac{7}{3}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} + \frac{10}{3}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -1 \\ 5 \end{bmatrix}$$

B1(d) We have $\mathrm{proj}_{\mathbb{S}}\vec{x} = \frac{\vec{x}\cdot\vec{v}_1}{||\vec{v}_1||^2}\vec{v}_1 + \frac{\vec{x}\cdot\vec{v}_2}{||\vec{v}_2||^2}\vec{v}_2 + \frac{\vec{x}\cdot\vec{v}_3}{||\vec{v}_2||^3}\vec{v}_3$, so we perform the following calculations:

$$\begin{array}{ll} \vec{x}\cdot\vec{v}_1 = 4+3-2+0 = 5 & ||\vec{v}_1||^2 = 1^2+1^2+1^2+0^2 = 3 \\ \vec{x}\cdot\vec{v}_2 = 4+0+2+5 = 11 & ||\vec{v}_2||^2 = 1^2+0^2+(-1)^2+1^2 = 3 \\ \vec{x}\cdot\vec{v}_3 = 0+3+2-5 = 0 & ||\vec{v}_3||^2 = 0^2+1^2+(-1)^2+(-1)^2 = 3 \end{array}$$

So

$$\mathrm{proj}_{\mathbb{S}}\vec{x} = \frac{5}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \frac{11}{3}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \frac{0}{3}\begin{bmatrix} 0 \\ 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 16/3 \\ 5/3 \\ -2 \\ 11/3 \end{bmatrix}$$