Assignment 4

[3pt] 1. Determine the matrix of the linear mapping $L : M(2,2) \to P_3$ defined by $L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = b + cx + dx^2 + ax^3$ with respect to the basis
$\mathcal{B} = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\right\}$ for $M(2,2)$ and the basis
$\mathcal{C} = \{1 + x, 1 - x, x^2 + x^3, x^2 - x^3\}$ for P_3.

[4pt] 2. Let $\begin{bmatrix} -1 & 3 \\ -3 & 2 \end{bmatrix}$ be the standard matrix of a linear mapping $L : \mathbb{R}^2 \to \mathbb{R}^2$. Determine the matrix of L with respect to the basis $\mathcal{B} = \left\{\begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix}\right\}$, and use it to determine $[L(\vec{x})]_\mathcal{B}$ if $[\vec{x}]_\mathcal{B} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$.

[3pt] 3. Define an explicit isomorphism to establish that the spaces P_2 and $\mathbb{D}_3$ (the space of 3×3 diagonal matrices) are isomorphic. Prove that your map is an isomorphism.