Assignment 12

[1pt] 1. Compute $\left\langle \begin{bmatrix} 2+i \\ 4i \\ 1-3i \\ 0 \end{bmatrix}, \begin{bmatrix} 1+3i \\ 2-i \\ 5 \\ 3-2i \end{bmatrix} \right\rangle$

[3pt] 2. Let $\vec{v}_1 = \begin{bmatrix} 1+i \\ 1+2i \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1+3i \\ -2i \end{bmatrix}$.

(a) Verify that $\vec{v}_1$ and $\vec{v}_2$ are orthogonal.

(b) Let $\mathbb{S} = \text{Span}\{\vec{v}_1, \vec{v}_2\}$ and $\vec{u} = \begin{bmatrix} 3 \\ 3+2i \end{bmatrix}$. Find $\text{proj}_{\mathbb{S}}\vec{u}$.

[1pt] 3. Let $A = \begin{bmatrix} 1+i & 0 & 2+5i \\ -3i & 14 & 2+6i \\ 7-3i & 8-i & 1+9i \end{bmatrix}$. What is A^*?

[1pt] 4. Let $B = \begin{bmatrix} (2/3)-(1/3)i & (-1/3)+(2/3)i \\ (2/3)i & (-5/3) \end{bmatrix}$. Is B unitary? If not, explain why the columns of B fail to be an orthonormal basis for $\mathbb{C}^2$.

[4pt] 5. Let $C = \begin{bmatrix} 1 & 1+2i \\ 1-2i & -3 \end{bmatrix}$. Then C is Hermitian. Find a <u>unitary</u> matrix U and a diagonal matrix D such that $U^*CU = D$.