Solution to Practice 3n

B4(a) First, we need to row reduce A to its reduced row echelon form:

$$\begin{bmatrix} 1 & 1 & i \\ i & i & 1 \\ 1+i & 1+i & -1-i \end{bmatrix} \begin{array}{l} \\ R_2 - iR_1 \\ R_3 + (-1-i)R_1 \end{array} \sim \begin{bmatrix} 1 & 1 & i \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ (1/2)R_2 \\ \\ \end{array}$$
$$\sim \begin{bmatrix} 1 & 1 & i \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} R_1 - iR_2 \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = M$$

The non-zero rows of M form a basis for the rowspace of A: $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

The columns of A that correspond to the columns of M that contain a leading 1 form a basis for the columnspace of A: $\left\{ \begin{bmatrix} 1 \\ i \\ 1+i \end{bmatrix}, \begin{bmatrix} i \\ 1 \\ -1-i \end{bmatrix} \right\}$

To find the nullspace, we note that the general solution to $A\vec{z} = \vec{0}$ is the same as the general solution to $M\vec{z} = \vec{0}$, which is the general solution to this system of equations:

$$\begin{aligned} z_1 + z_2 &= 0 \\ z_3 &= 0 \end{aligned}$$

If we set the variable z_2 equal to the parameter α, then we see that the general solution to this system is

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} -\alpha \\ \alpha \\ 0 \end{bmatrix} = \alpha \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

As such, we see that $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a basis for the nullspace of A.

B4(b) First, we need to row reduce B to its reduced row echelon form:

$$\begin{bmatrix} 1+i & 2-i \\ 1 & -i \\ -1+i & 2i \end{bmatrix} \begin{array}{l} R_1 \updownarrow R_2 \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & -i \\ 1+i & 2-i \\ -1+i & 2i \end{bmatrix} \begin{array}{l} \\ R_2 - (-1-i)R_1 \\ R_3 + (1-i)R_1 \end{array}$$
$$\sim \begin{bmatrix} 1 & -i \\ 0 & 1 \\ 0 & -1+i \end{bmatrix} \begin{array}{l} R_1 + iR_2 \\ \\ R_3 + (1-i)R_2 \end{array} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = M.$$

The non-zero rows of M form a basis for the rowspace of B: $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

The columns of B that correspond to the columns of M that contain a leading 1 form a basis for the columnspace of B: $\left\{ \begin{bmatrix} 1+i \\ 1 \\ -1+i \end{bmatrix}, \begin{bmatrix} 2-i \\ -i \\ 2i \end{bmatrix} \right\}$

To find the nullspace, we note that the general solution to $B\vec{z} = \vec{0}$ is the same as the general solution to $M\vec{z} = \vec{0}$, which is the general solution to this system of equations:

$$\begin{aligned} z_1 &= 0 \\ z_2 &= 0 \end{aligned}$$

So we see that only solution to the system is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$. As such, the emptyset is a basis for the nullspace of B.

B4(c) First, we need to row reduce C to its reduced row echelon form:

$$\begin{bmatrix} i & -1 & 2 & i \\ 2i & 6 & 8 & -4i \end{bmatrix} \begin{matrix} -iR_1 \\ (i/2)R_2 \end{matrix} \sim \begin{bmatrix} 1 & i & -2i & 1 \\ -1 & 3i & 4i & 2 \end{bmatrix} \; R_2 + R_1$$
$$\sim \begin{bmatrix} 1 & i & -2i & 1 \\ 0 & 4i & 2i & 3 \end{bmatrix} \; (-i/4)R_2 \sim \begin{bmatrix} 1 & i & -2i & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{4}i \end{bmatrix} \; R_1 - iR_2$$
$$\sim \begin{bmatrix} 1 & 0 & -\frac{5}{2}i & \frac{1}{4} \\ 0 & 1 & \frac{1}{2} & -\frac{3}{4}i \end{bmatrix} = M$$

The non-zero rows of M form a basis for the rowspace of C: $\left\{ \begin{bmatrix} 1 \\ 0 \\ -\frac{5}{2}i \\ \frac{1}{4} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ \frac{1}{2} \\ -\frac{3}{4}i \end{bmatrix} \right\}$.

The columns of C that correspond to the columns of M that contain a leading 1 form a basis for the columnspace of A: $\left\{ \begin{bmatrix} i \\ 2i \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \end{bmatrix} \right\}$

To find the nullspace, we note that the general solution to $C\vec{z} = \vec{0}$ is the same as the general solution to $M\vec{z} = \vec{0}$, which is the general solution to this system of equations:

$$\begin{aligned} z_1 - \tfrac{5}{2}iz_3 + \tfrac{1}{4}z_4 &= 0 \\ z_2 + \tfrac{1}{2}z_3 - \tfrac{3}{4}iz_4 &= 0 \end{aligned}$$

If we set the variable z_3 equal to the parameter α and the variable z_4 equal to the parameter β, then we see that the general solution to this system is

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} (5i/2)\alpha - (1/4)\beta \\ (-1/2)\alpha + (3i/4)\beta \\ \alpha \\ \beta \end{bmatrix} = \alpha \begin{bmatrix} 5i/2 \\ -1/2 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -1/4 \\ 3i/4 \\ 0 \\ 1 \end{bmatrix}$$

As such, we see that $\left\{ \begin{bmatrix} 5i/2 \\ -1/2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1/4 \\ 3i/4 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for the nullspace of C.

B4(d) First, we need to row reduce D to its reduced row echelon form:

$$\begin{bmatrix} 0 & i & 2-i \\ 1+4i & i & 3 \\ 1 & 0 & i \end{bmatrix} \begin{matrix} R_1 \updownarrow R_3 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & i \\ 1+4i & i & 3 \\ 0 & i & 2-i \end{bmatrix} R_2 + (-1-4i)R_1$$
$$\sim \begin{bmatrix} 1 & 0 & i \\ 0 & i & 7-i \\ 0 & i & 2-i \end{bmatrix} \begin{matrix} \\ \\ R_3 - R_2 \end{matrix} \sim \begin{bmatrix} 1 & 0 & i \\ 0 & i & 7-i \\ 0 & 0 & -5 \end{bmatrix} \begin{matrix} \\ -iR_2 \\ (-1/5)R_3 \end{matrix}$$
$$\sim \begin{bmatrix} 1 & 0 & i \\ 0 & 1 & -1-7i \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} R_1 - iR_3 \\ R_2 + (1+7i)R_3 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = M$$

The non-zero rows of M form a basis for the rowspace of D: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

The columns of D that correspond to the columns of M that contain a leading 1 form a basis for the columnspace of D: $\left\{ \begin{bmatrix} 0 \\ 1+4i \\ 1 \end{bmatrix}, \begin{bmatrix} i \\ i \\ 0 \end{bmatrix}, \begin{bmatrix} 2-i \\ 3 \\ i \end{bmatrix} \right\}$

To find the nullspace, we note that the general solution to $D\vec{z} = \vec{0}$ is the same as the general solution to $M\vec{z} = \vec{0}$, which is the general solution to this system of equations:

$$\begin{aligned} z_1 &= 0 \\ z_2 &= 0 \\ z_3 &= 0 \end{aligned}$$

So we see that only solution to the system is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. As such, the emptyset is the basis for the nullspace of D.