Solution to Practice 3k

1(a)

$$\begin{aligned}L(\alpha\vec{z}+\vec{w}) &= L(\alpha z_1+w_1,\alpha z_2+w_2)\\ &= (0,\alpha z_1+w_1,\alpha z_2+w_2)\\ &= (0,\alpha z_1,\alpha z_2)+(0,w_1,w_2)\\ &= \alpha(0,z_1,z_2)+(0,w_1,w_2)\\ &= \alpha L(\vec{z})+L(\vec{w})\end{aligned}$$

To find $[L]$, we first compute:

$L(1,0)=(0,1,0)$ and $L(0,1)=(0,0,1)$

so $[L]=\begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}$

1(b)

$$\begin{aligned}L(\alpha\vec{z}+\vec{w}) &= L(\alpha z_1+w_1,\alpha z_2+w_2,\alpha z_3+w_3,\alpha z_4+w_4)\\ &= (\alpha z_1+w_1,\alpha z_2+w_2,\alpha z_3+w_3)\\ &= (\alpha z_1,\alpha z_2,\alpha z_3)+(w_1,w_2,w_3)\\ &= \alpha(z_1,z_2,z_3)+(w_1,w_2,w_3)\\ &= \alpha L(\vec{z})+L(\vec{w})\end{aligned}$$

To find $[L]$, we first compute:

$$\begin{aligned}&L(1,0,0,0)=(1,0,0) &\qquad& L(0,1,0,0)=(0,1,0)\\ &L(0,0,1,0)=(0,0,1) && L(0,0,0,1)=(0,0,0)\end{aligned}$$

so $[L]=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}$

1(c)

$$\begin{aligned}L(\alpha\vec{z}+\vec{w}) &= L(\alpha z_1+w_1,\alpha z_2+w_2)\\ &= ((1+i)(\alpha z_1+w_1),(1-i)(\alpha z_1+w_1)+(1+i)(\alpha z_2+w_2))\\ &= ((1+i)(\alpha z_1)+(1+i)w_1,(1-i)(\alpha z_1)+(1-i)w_1+(1+i)(\alpha z_2)+(1+i)w_2)\\ &= ((1+i)(\alpha z_1),(1-i)(\alpha z_1)+(1+i)(\alpha z_2))+((1+i)w_1,(1-i)w_1+(1+i)w_2)\\ &= \alpha((1+i)z_1,(1-i)z_1+(1+i)z_2)+((1+i)w_1,(1-i)w_1+(1+i)w_2)\\ &= \alpha L(\vec{z})+L(\vec{w})\end{aligned}$$

To find $[L]$, we first compute:

$L(1,0)=(1+i,1-i)$ and $L(0,1)=(0,1+i)$

so $[L] = \begin{bmatrix} 1+i & 0 \\ 1-i & 1+i \end{bmatrix}$

2(a) Many counterexamples are possible. I choose to show that M does not preserve scalar multiplication, by letting $\alpha = (2 + i3)$ and $\mathbf{v}_1 = 1 + i$ (and $\mathbf{v}_2 = 0$).

We see that $M(1 + i) = 1$, so $(2 + 3i)M(1 + i) = (2 + 3i)(1) = 2 + 3i$.

We also see that $M((2+3i)(1+i)) = M(2+2i+3i+3i^2) = M(-1+5i) = -1$.

So we have that $M(\alpha\mathbf{v}_1) \neq \alpha M(\mathbf{v}_1)$.

2(b) Note that this mapping is not even a linear mapping on the reals. So an easy counterexample is to see that $M(1, 1) = (1, 1)$, $M(2, 2) = (4, 4)$, $M(1, 1) + M(2, 2) = (1, 1) + (4, 4) = (5, 5)$, but $M((1, 1) + (2, 2)) = M(3, 3) = (9, 9)$, so $M(1, 1) + M(2, 2) \neq M((1, 1) + (2, 2))$.