Solution to Practice 3x

Note: The solutions to B4 will depend heavily on how you choose to row reduce the given matrix, and so your answer may not be the same as mine. Except for A^{-1}, which does not depend on the choice of row operations.

B4(a)(i) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 2 & 4 & 0 \end{bmatrix} \; R_3 - 2R_1 \sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \; (1/2)R_3$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} R_1 + R_3 \\ R_2 - 2R_3 \end{matrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; R_1 - 2R_2 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

The first row operation is $R_3 - 2R_1$, so $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$.

The second row operation is $(1/2)R_3$, so $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$.

The third row operation is $R_1 + R_3$, so $E_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

The fourth row operation is $R_2 - 2R_3$, so $E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.

The fifth row operation is $R_1 - 2R_2$, so $E_5 = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

B4(a)(ii) $E_2E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1/2 \end{bmatrix}$

$E_3E_2E_1 = E_3(E_2E_1) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1 & 0 & 1/2 \end{bmatrix}$

$E_4E_3E_2E_1 = E_4(E_3E_2E_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1 & 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/2 \\ 2 & 1 & -1 \\ -1 & 0 & 1/2 \end{bmatrix}$

$A^{-1} = E_5E_4E_3E_2E_1 = E_5(E_4E_3E_2E_1) = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1/2 \\ 2 & 1 & -1 \\ -1 & 0 & 1/2 \end{bmatrix} =$

$$\begin{bmatrix} -4 & -2 & 5/2 \\ 2 & 1 & -1 \\ -1 & 0 & 1/2 \end{bmatrix}$$

B4(a)(iii) $A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}E_5^{-1}$, so A is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B4(b)(i) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 4 & 1 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} \; 1/3R_2 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} \; R_3 - 4R_2$
$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The first row operation is $R_2 - 2R_1$, so $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The second row operation is $R_3 - R_1$, so $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

The third row operation is $(1/3)R_2$, so $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The fourth row operation is $R_3 - 4R_2$, so $E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix}$

B4(b)(ii) $E_2E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$E_3E_2E_1 = E_3(E_2E_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$A^{-1} = E_4E_3E_2E_1 = E_4(E_3E_2E_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & 0 \\ -1 & 0 & 1 \end{bmatrix} =$
$\begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & 0 \\ 5/3 & -4/3 & 1 \end{bmatrix}$

B4(b)(iii) $A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$, so A is

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

B4(c)(i) $\begin{bmatrix} 0 & 1 & 2 \\ -4 & -3 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{matrix} R_1 \updownarrow R_3 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ -4 & -3 & -3 \\ 0 & 1 & 2 \end{bmatrix} \; R_2 + 4R_1$

$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{matrix} R_1 - R_2 \\ \\ R_3 - R_2 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \; R_2 - R_3 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The first row operation is $R_1 \updownarrow R_3$, so $E_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

The second row operation is $R_2 + 4R_1$, so $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The third row operation is $R_1 - R_2$, so $E_3 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The fourth row operation is $R_3 - R_2$, so $E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

The fifth row operation is $R_2 - R_3$, so $E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

B4(c)(ii) $E_2E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 4 \\ 1 & 0 & 0 \end{bmatrix}$

$E_3E_2E_1 = E_3(E_2E_1) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 4 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -3 \\ 0 & 1 & 4 \\ 1 & 0 & 0 \end{bmatrix}$

$E_4E_3E_2E_1 = E_4(E_3E_2E_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & -3 \\ 0 & 1 & 4 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -3 \\ 0 & 1 & 4 \\ 1 & -1 & -4 \end{bmatrix}$

$A^{-1} = E_5E_4E_3E_2E_1 = E_5(E_4E_3E_2E_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & -3 \\ 0 & 1 & 4 \\ 1 & -1 & -4 \end{bmatrix} =$

$$\begin{bmatrix} 0 & -1 & -3 \\ -1 & 2 & 8 \\ 1 & -1 & -4 \end{bmatrix}$$

B4(c)(iii) $A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}E_5^{-1}$, so A is

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$