Solution to Practice 3w

B1(a) $E = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, so

$$EA = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 10 & 1 & 19 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix}$$

which is the same as

$$\begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} \begin{matrix} R_1 + 4R_2 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 10 & 1 & 19 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix}$$

B1(b) $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, so

$$EA = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 \\ 2 & 0 & 5 \\ 2 & 1 & -1 \end{bmatrix}$$

which is the same as

$$\begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} \begin{matrix} R_1 \updownarrow R_3 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & -3 & -2 \\ 2 & 0 & 5 \\ 2 & 1 & -1 \end{bmatrix}$$

B1(c) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, so

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ -6 & 0 & -15 \\ 1 & -3 & -2 \end{bmatrix}$$

which is the same as

$$\begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} \; -3R_2 \sim \begin{bmatrix} 2 & 1 & -1 \\ -6 & 0 & -15 \\ 1 & -3 & -2 \end{bmatrix}$$

B1(d) $E = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, so

$$EA = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix}$$

which is the same as

$$\begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} \begin{matrix} 2R_1 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 4 & 2 & -2 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix}$$

B1(e) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$, so

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$$

which is the same as

$$\begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} \begin{matrix} \\ \\ R_3 - 2R_1 \end{matrix} \sim \begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$$

B1(f) $E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, so

$$EA = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 5 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{bmatrix}$$

which is the same as

$$\begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & -3 & -2 \end{bmatrix} \begin{matrix} R_1 \updownarrow R_2 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 2 & 0 & 5 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{bmatrix}$$

B2(a) $\begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B2(b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B2(c) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

B2(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

B2(e) $\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B2(f) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

B3(a) This is not an elementary matrix, because is takes two row operations ($R_1 \updownarrow R_2$, then $R_2 \updownarrow R_3$; others are possible) to row reduce this matrix to I.

B3(b) This is an elementary matrix, corresponding to the row operation $R_3 + 3R_2$.

B3(c) This is not an elementary matrix, because it takes two row operations ($-R_1$ and $R_2 \updownarrow R_3$) to row reduce this matrix to I.

B3(d) This is not an elementary matrix. It is not even row equivalent to I.

B3(e) This is an elementary matrix, corresponding to the row operation $2R_1$.

B3(f) This is an elementary matrix, corresponding to the row operation $R_2 - R_1$.