Solution to Practice 3v

B4(a) The inverse of a rotation by $\pi/4$ would be a rotation by $-\pi/4$, so the inverse of $R_{\pi/4}$ is $R_{-\pi/4}$, which has standard matrix $\begin{bmatrix} \cos -\pi/4 & -\sin -\pi/4 \\ \sin -\pi/4 & \cos -\pi/4 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$.

B4(b) This is the matrix of a shear in the x_2 direction by 2. To undo this action, we would need to shear in the x_2 direction by -2. Thus, the inverse of this matrix is $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$.

B4(c) This matrix represents the composition of a reflection over the x_1 axis and a shrink by a factor of 1/3 in the x_2 direction. (Note that it does not matter which order these transformations take place.) Thus, the inverse of this matrix will be the composition of the inverses of these transformations. A reflection over the x_1 axis is its own inverse. The inverse of a shrink by a factor of 1/3 in the x_2 direction is a stretch by a factor of 3 in the x_2 direction. Thus, the inverse of the given matrix is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$.

B4(d) This matrix is the composition of a reflection over the x_1x_2 plane, a reflection over the x_1x_3 plane, and a reflection over the x_2x_3 plane. (It does not matter which order these reflections occur.) Since a reflection is its own inverse, we see that this matrix is its own inverse. That is, the inverse of $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

B5(a) $S = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$. The inverse of S is a stretch in the x_2 direction by 1/3, so the matrix of S^{-1} is $\begin{bmatrix} 1 & 0 \\ 0 & 1/3 \end{bmatrix}$.

B5(b) From the equation of the line, we see that $\vec{n} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Thus, $||\vec{n}||^2 = 1^2 + 1^2 = 2$, $\vec{n} \cdot \vec{e}_1 = 1$ and $\vec{n} \cdot \vec{e}_2 = 1$. Using this, we get that:

$$\text{refl}_{\vec{n}}\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 2\text{proj}_{\vec{n}}\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 2(1/2)\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\text{refl}_{\vec{n}}\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - 2\text{proj}_{\vec{n}}\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - 2(1/2)\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Thus, the matrix for R is $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$. And, since reflections are their own inverse, we see that $R^{-1} = R = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

B5(c) $[(R \circ S)^{-1}] = [R \circ S]^{-1} = ([R][S])^{-1} = [S]^{-1}[R]^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} =$
$\begin{bmatrix} 0 & -1 \\ -1/3 & 0 \end{bmatrix}$

$[(S \circ R)^{-1}] = [S \circ R]^{-1} = ([S][R])^{-1} = [R]^{-1}[S]^{-1} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/3 \end{bmatrix} =$
$\begin{bmatrix} 0 & -1/3 \\ -1 & 0 \end{bmatrix}$