Solution to Practice 3t

I will not be showing the "check by multiplication" steps for B1, but you should nevertheless do so.

B1(a) $\left[\begin{array}{rr|rr} 1 & 4 & 1 & 0 \\ -2 & 7 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 + 2R_1 \end{array} \sim \left[\begin{array}{rr|rr} 1 & 4 & 1 & 0 \\ 0 & 15 & 2 & 1 \end{array}\right] \begin{array}{l} \\ (1/15)R_2 \end{array}$
$\sim \left[\begin{array}{rr|rr} 1 & 4 & 1 & 0 \\ 0 & 1 & 2/15 & 1/15 \end{array}\right] \begin{array}{l} R_1 - 4R_2 \\ \end{array} \sim \left[\begin{array}{rr|rr} 1 & 0 & 7/15 & -4/15 \\ 0 & 1 & 2/15 & 1/15 \end{array}\right].$

So $\begin{bmatrix} 1 & 4 \\ -2 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} 7/15 & -4/15 \\ 2/15 & 1/15 \end{bmatrix}$

B1(b) $\left[\begin{array}{rrr|rrr} 1 & -1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 5 & 0 & 1 & 0 \\ 2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 2R_1 \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 4 & -1 & -3 & 1 & 0 \\ 0 & 4 & -1 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_2 \end{array}$
$\sim \left[\begin{array}{rrr|rrr} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 4 & -1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{array}\right]$

We can see at this point that $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 5 \\ 2 & 2 & 3 \end{bmatrix}$ does not have reduced row echelon form I, so it is not invertible.

B1(c) $\left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 2 & 5 & 0 & 1 & 0 \\ 1 & -1 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -2 & 5 & -2 & 1 & 0 \\ 0 & -3 & 3 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ (-1/2)R_2 \\ \end{array}$
$\sim \left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -5/2 & 1 & -1/2 & 0 \\ 0 & -3 & 3 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 + 3R_2 \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -5/2 & 1 & -1/2 & 0 \\ 0 & 0 & -9/2 & 2 & -3/2 & 1 \end{array}\right] \begin{array}{l} \\ \\ (-2/9)R_3 \end{array}$
$\sim \left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -5/2 & 1 & -1/2 & 0 \\ 0 & 0 & 1 & -4/9 & 1/3 & -2/9 \end{array}\right] \begin{array}{l} \\ R_2 + (5/2)R_3 \\ \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/9 & 1/3 & -5/9 \\ 0 & 0 & 1 & -4/9 & 1/3 & -2/9 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ \\ \end{array}$
$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 11/9 & -2/3 & 10/9 \\ 0 & 1 & 0 & -1/9 & 1/3 & -5/9 \\ 0 & 0 & 1 & -4/9 & 1/3 & -2/9 \end{array}\right]$

So we see that $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 5 \\ 1 & -1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 11/9 & -2/3 & 10/9 \\ -1/9 & 1/3 & -5/9 \\ -4/9 & 1/3 & -2/9 \end{bmatrix}$

B1(d) $\left[\begin{array}{rrr|rrr} 1 & 1 & -2 & 1 & 0 & 0 \\ 2 & 1 & 5 & 0 & 1 & 0 \\ 4 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 4R_1 \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 9 & -2 & 1 & 0 \\ 0 & -1 & 9 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_2 \end{array}$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 9 & -2 & 1 & 0 \\ 0 & 0 & 0 & -2 & -1 & 1 \end{array}\right]$$

We can see at this point that $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & 5 \\ 4 & 3 & 1 \end{bmatrix}$ does not have reduced row echelon form I, so it is not invertible.

B1(e) $$\left[\begin{array}{rrr|rrr} 2 & -1 & 3 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \; R_1 \updownarrow R_3 \quad \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & -1 & 3 & 1 & 0 & 0 \end{array}\right] \; \begin{array}{l} \\ R_2 - R_1 \\ R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 & -1 \\ 0 & -1 & 1 & 1 & 0 & -2 \end{array}\right] \; R_2 \updownarrow R_3 \quad \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & -2 \\ 0 & 2 & 1 & 0 & 1 & -1 \end{array}\right] \; -R_2$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & 2 \\ 0 & 2 & 1 & 0 & 1 & -1 \end{array}\right] \; R_3 - 2R_2 \quad \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & 2 \\ 0 & 0 & 3 & 2 & 1 & -5 \end{array}\right] \; (1/3)R_3$$

$$\sim \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 2/3 & 1/3 & -5/3 \end{array}\right] \; \begin{array}{l} R_1 - R_3 \\ R_2 + R_2 \\ \\ \end{array} \sim \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -2/3 & -1/3 & 8/3 \\ 0 & 1 & 0 & -1/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 2/3 & 1/3 & -5/3 \end{array}\right]$$

So we see that $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 2 \\ 1 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -2/3 & -1/3 & 8/3 \\ -1/3 & 1/3 & 1/3 \\ 2/3 & 1/3 & -5/3 \end{bmatrix}$

B1(f) $$\left[\begin{array}{rrrr|rrrr} 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & -2 & 3 & 5 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \; \begin{array}{l} \\ \\ R_3 - 2R_1 \\ R_4 - R_1 \end{array}$$

$$\sim \left[\begin{array}{rrrr|rrrr} 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \; \begin{array}{l} \\ \\ (1/3)R_3 \\ R_4 - R_2 \end{array}$$

$$\sim \left[\begin{array}{rrrr|rrrr} 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/3 & -2/3 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right] \; \begin{array}{l} R_1 - 2R_4 \\ \\ R_3 - (1/3)R_4 \\ \\ \end{array}$$

$$\sim \left[\begin{array}{rrrr|rrrr} 1 & -1 & 0 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 1/3 & 1/3 & -1/3 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right] \; \begin{array}{l} \\ R_2 - R_3 \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{rrrr|rrrr} 1 & -1 & 0 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & 0 & 1/3 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & 1 & 0 & -1/3 & 1/3 & 1/3 & -1/3 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right] \; \begin{array}{l} R_1 + R_2 \\ \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 10/3 & 8/3 & -1/3 & -5/3 \\ 0 & 1 & 0 & 0 & 1/3 & 2/3 & -1/3 & 1/3 \\ 0 & 0 & 1 & 0 & -1/3 & 1/3 & 1/3 & -1/3 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right]$$

So we see that $\begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & -2 & 3 & 5 \\ 1 & 0 & 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 10/3 & 8/3 & -1/3 & -5/3 \\ 1/3 & 2/3 & -1/3 & 1/3 \\ -1/3 & 1/3 & 1/3 & -1/3 \\ -1 & -1 & 0 & 1 \end{bmatrix}$

B1(g) $\left[\begin{array}{cccc|cccc} 0 & 2 & 2 & 5 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 3 & 0 & 0 & 1 & 0 \\ 3 & 6 & 0 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_1 \updownarrow R_3$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 5 & 1 & 0 & 0 & 0 \\ 3 & 6 & 0 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_4 - 3R_1$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 5 & 1 & 0 & 0 & 0 \\ 0 & -3 & -3 & -6 & 0 & 0 & -3 & 1 \end{array}\right] \quad \begin{array}{l} R_3 - 2R_2 \\ R_4 + 3R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & -2 & 0 & 0 \\ 0 & 0 & -3 & 3 & 0 & 3 & -3 & 1 \end{array}\right] \quad (-1/3)R_4$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 & 1 & -1/3 \end{array}\right] \quad R_3 \updownarrow R_4$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 & 1 & -1/3 \\ 0 & 0 & 2 & -1 & 1 & -2 & 0 & 0 \end{array}\right] \quad R_4 - 2R_3$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 & 1 & -1/3 \\ 0 & 0 & 0 & 1 & 1 & 0 & -2 & 2/3 \end{array}\right] \quad \begin{array}{l} R_1 - 3R_4 \\ R_2 - 3R_4 \\ R_3 + R_4 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 0 & -3 & 0 & 7 & -2 \\ 0 & 1 & 0 & 0 & -3 & 1 & 6 & -2 \\ 0 & 0 & 1 & 0 & 1 & -1 & -1 & 1/3 \\ 0 & 0 & 0 & 1 & 1 & 0 & -2 & 2/3 \end{array}\right] \quad R_1 - R_3$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & -4 & 1 & 8 & -7/3 \\ 0 & 1 & 0 & 0 & -3 & 1 & 6 & -2 \\ 0 & 0 & 1 & 0 & 1 & -1 & -1 & 1/3 \\ 0 & 0 & 0 & 1 & 1 & 0 & -2 & 2/3 \end{array}\right] \quad R_1 - 3R_2$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 5 & -2 & -10 & 11/3 \\ 0 & 1 & 0 & 0 & -3 & 1 & 6 & -2 \\ 0 & 0 & 1 & 0 & 1 & -1 & -1 & 1/3 \\ 0 & 0 & 0 & 1 & 1 & 0 & -2 & 2/3 \end{array}\right]$$

So we see that $\begin{bmatrix} 0 & 2 & 2 & 5 \\ 0 & 1 & 0 & 3 \\ 1 & 3 & 1 & 3 \\ 3 & 6 & 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & -2 & -10 & 11/3 \\ -3 & 1 & 6 & -2 \\ 1 & -1 & -1 & 1/3 \\ 1 & 0 & -2 & 2/3 \end{bmatrix}$

B1(h) $\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \updownarrow R_5 \\ R_2 \updownarrow R_4 \\ \\ \\ \\ \end{array}$

$$\sim \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \\ R_5 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 \end{array}\right]$$

And so we see that $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$

B1(i) $\left[\begin{array}{ccccc|ccccc} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \updownarrow R_5 \\ R_2 \updownarrow R_4 \\ \\ \\ \\ \end{array}$

$$\sim \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_5 \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0&0&1&0\\ 0&0&1&1&0&-1&0&1&0&0\\ 0&0&0&1&0&0&1&0&0&0\\ 0&0&0&0&1&1&0&0&0&0 \end{array}\right] R_3 - R_4$$

$$\sim \left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0&0&1&0\\ 0&0&1&0&0&-1&-1&1&0&0\\ 0&0&0&1&0&0&1&0&0&0\\ 0&0&0&0&1&1&0&0&0&0 \end{array}\right]$$

And so we see that $\begin{bmatrix} 0&0&0&0&1\\ 0&0&0&1&0\\ 0&0&1&1&1\\ 0&1&0&0&0\\ 1&0&0&0&0 \end{bmatrix}^{-1} = \begin{bmatrix} 0&0&0&0&1\\ 0&0&0&1&0\\ -1&-1&1&0&0\\ 0&1&0&0&0\\ 1&0&0&0&0 \end{bmatrix}$

B1(j) $\left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0&1&0\\ 1&1&1&1&1&0&0&0&0&1 \end{array}\right] R_5 - R_1$

$$\sim \left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0&1&0\\ 0&1&1&1&1&-1&0&0&0&1 \end{array}\right] R_5 - R_2$$

$$\sim \left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0&1&0\\ 0&0&1&1&1&-1&-1&0&0&1 \end{array}\right] R_5 - R_3$$

$$\sim \left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0&1&0\\ 0&0&0&1&1&-1&-1&-1&0&1 \end{array}\right] R_5 - R_4$$

$$\sim \left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0&1&0\\ 0&0&0&0&1&-1&-1&-1&-1&1 \end{array}\right]$$

And so we see that $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & -1 & 1 \end{bmatrix}$.

B3(a) $\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$ $R_1 \updownarrow R_2$ $\sim \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 2 & 5 & 1 & 0 \end{array}\right]$ $R_2 - 2R_1$
$\sim \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{array}\right]$ $R_1 - 2R_2$ $\sim \left[\begin{array}{cc|cc} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -2 \end{array}\right]$

So $A^{-1} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$.

$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$ $R_2 - 3R_1$ $\sim \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right]$ $R_1 - 2R_2$
$\sim \left[\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$

So $B^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$.

B3(b) $AB = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} (2)(1)+(5)(3) & (2)(2)+(5)(7) \\ (1)(1)+(2)(3) & (1)(2)+(2)(7) \end{bmatrix} = \begin{bmatrix} 17 & 39 \\ 7 & 16 \end{bmatrix}$. We find $(AB)^{-1}$ as follows:

$\left[\begin{array}{cc|cc} 17 & 39 & 1 & 0 \\ 7 & 16 & 0 & 1 \end{array}\right]$ $(1/17)R_1$ $\sim \left[\begin{array}{cc|cc} 1 & 39/17 & 1/17 & 0 \\ 7 & 16 & 0 & 1 \end{array}\right]$ $R_2 - 7R_1$
$\sim \left[\begin{array}{cc|cc} 1 & 39/17 & 1/17 & 0 \\ 0 & -1/17 & -7/17 & 1 \end{array}\right]$ $R_1 + 39R_2$ $\sim \left[\begin{array}{cc|cc} 1 & 0 & -16 & 39 \\ 0 & -1/17 & -7/17 & 1 \end{array}\right]$ $-17R_2$
$\sim \left[\begin{array}{cc|cc} 1 & 0 & -16 & 39 \\ 0 & 1 & 7 & -17 \end{array}\right]$

So we see that $(AB)^{-1} = \begin{bmatrix} -16 & 39 \\ 7 & -17 \end{bmatrix}$. We can confirm this by noting that $B^{-1}A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} (7)(-2)+(-2)(1) & (7)(5)+(-2)(-2) \\ (-3)(-2)+(1)(1) & (-3)(5)+(1)(-2) \end{bmatrix} = \begin{bmatrix} -16 & 39 \\ 7 & -17 \end{bmatrix}$ as well.

B3(c) $5A = \begin{bmatrix} 10 & 25 \\ 5 & 10 \end{bmatrix}$, and we find $(5A)^{-1}$ as follows:

$\left[\begin{array}{cc|cc} 10 & 25 & 1 & 0 \\ 5 & 10 & 0 & 1 \end{array}\right]$ $(1/10)R_1$ $\sim \left[\begin{array}{cc|cc} 1 & 5/2 & 1/10 & 0 \\ 5 & 10 & 0 & 1 \end{array}\right]$ $R_2 - 5R_1$
$\sim \left[\begin{array}{cc|cc} 1 & 5/2 & 1/10 & 0 \\ 0 & -5/2 & -1/2 & 1 \end{array}\right]$ $R_1 + R_2$ $\sim \left[\begin{array}{cc|cc} 1 & 0 & -2/5 & 1 \\ 0 & -5/2 & -1/2 & 1 \end{array}\right]$ $(-2/5)R_2$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & -2/5 & 1 \\ 0 & 1 & 1/5 & -2/5 \end{array}\right]$$

So we see that $(5A)^{-1} = \begin{bmatrix} -2/5 & 1 \\ 1/5 & -2/5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix} = \frac{1}{5}A^{-1}$.

B3(d) $A^T = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}$, and we find $(A^T)^{-1}$ as follows:

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right] \ (1/2)R_1 \sim \left[\begin{array}{cc|cc} 1 & 1/2 & 1/2 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right] \ R_2 - 5R_1$$

$$\sim \left[\begin{array}{cc|cc} 1 & 1/2 & 1/2 & 0 \\ 0 & -1/2 & -5/2 & 1 \end{array}\right] \ R_1 + R_2 \sim \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -1/2 & -5/2 & 1 \end{array}\right] \ (-2)R_2$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 5 & -2 \end{array}\right].$$

So $(A^T)^{-1} = \begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix}$, and we see that

$$\begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} (2)(-2)+(1)(5) & (2)(1)+(1)(-2) \\ (5)(-2)+(2)(5) & (5)(1)+(2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$