Solution to Practice 3s

D1 $((AB)^T)^{-1} = (B^T A^T)^{-1} = (A^T)^{-1}(B^T)^{-1} = (A^{-1})^T(B^{-1})^T$

D2(a) Since $A^3 = I$, we have that $A(A^2) = I$, so $A^2 = A^{-1}$.

D2(b) Since $B^5 + B^3 + B = I$, we have that $B(B^4 + B^2 + I) = I$, so $B^4 + B^2 + I = B^{-1}$.

D3 Suppose that AB is invertible, and let $C = (AB)^{-1}$. Then $(AB)C = I$, so $A(BC) = I$, and thus $BC = A^{-1}$, and this means that A is invertible. We also have $C(AB) = I$, so $(CA)B = I$, and thus $CA = B^{-1}$, and this means that B is invertible.