Solution to Practice 3p

$A, \vec{x}$**:** To see if $\vec{x}$ is in the rowspace of A, we need to see if $\vec{x}$ is in Span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} \right\}$. That is, we need to see if there are any solutions to the vector equation

$$t_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t_2 \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} + t_3 \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

To find the solutions to this vector equation/system of linear equations, we row reduce its augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & 2 & 3 & 0 \\ 1 & -2 & 2 & 3 \end{array}\right] R_3 - R_1 \quad \sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & 2 & 3 & 0 \\ 0 & -2 & -3 & 2 \end{array}\right] R_3 + R_2$$
$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

Since the third row is a bad row, we know that our vector equation does not have any solutions, and thus that $\vec{x}$ is not in the rowspace of A.

$A, \vec{y}$**:** To see if $\vec{y}$ is in the rowspace of A, we need to see if $\vec{y}$ is in Span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} \right\}$. That is, we need to see if there are any solutions to the vector equation

$$t_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t_2 \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} + t_3 \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

To find the solutions to this vector equation/system of linear equations, we row reduce its augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & 2 & 3 & 1 \\ 1 & -2 & 2 & 0 \end{array}\right] R_3 - R_1 \quad \sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & 2 & 3 & 1 \\ 0 & -2 & -3 & -1 \end{array}\right] R_3 + R_2$$
$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Our last matrix is in row echelon form, and since it has no bad rows, we know that our vector equation has a solution. Thus, $\vec{y}$ is in the rowspace of A.

$A, \vec{z}$**:** To see if $\vec{z}$ is in the rowspace of A, we need to see if $\vec{x}$ is in Span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} \right\}$.
That is, we need to see if there are any solutions to the vector equation

$$t_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t_2 \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} + t_3 \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

To find the solutions to this vector equation/system of linear equations, we row reduce its augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & -1 \\ 0 & 2 & 3 & 1 \\ 1 & -2 & 2 & 1 \end{array}\right] \begin{array}{c} \\ \\ R_3 - R_1 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & -1 \\ 0 & 2 & 3 & 1 \\ 0 & -2 & -3 & 2 \end{array}\right] \begin{array}{c} \\ \\ R_3 + R_2 \end{array}$$
$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & -1 \\ 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

Since the third row is a bad row, we know that our vector equation does not have any solutions, and thus that $\vec{z}$ is not in the rowspace of A.

$B, \vec{x}$**:** To see if $\vec{x}$ is in the rowspace of B, we need to see if $\vec{x}$ is in Span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -4 \\ -1 \end{bmatrix} \right\}$.
That is, we need to see if there are any solutions to the vector equation

$$t_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix} + t_2 \begin{bmatrix} 3 \\ 2 \\ -4 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 5 \\ 3 \end{bmatrix}$$

To find the solutions to this vector equation/system of linear equations, we row reduce its augmented matrix:

$$\left[\begin{array}{cc|c} 1 & 3 & 7 \\ 2 & 2 & 1 \\ 0 & -4 & 5 \\ -3 & -1 & 3 \end{array}\right] \begin{array}{c} \\ R_2 - 2R_1 \\ \\ R_3 + 3R_1 \end{array} \sim \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & -4 & -13 \\ 0 & -4 & 5 \\ 0 & 8 & 24 \end{array}\right] \begin{array}{c} \\ \\ R_3 - R_2 \\ R_4 + 2R_2 \end{array} \sim \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & -4 & -13 \\ 0 & 0 & 18 \\ 0 & 0 & -2 \end{array}\right]$$

Since the last row (and the second to last row) is a bad row, we know that our vector equation does not have any solutions, and thus that $\vec{x}$ is not in the rowspace of B.

$B, \vec{y}$: To see if $\vec{y}$ is in the rowspace of B, we need to see if $\vec{y}$ is in Span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -4 \\ -1 \end{bmatrix} \right\}$.

That is, we need to see if there are any solutions to the vector equation

$$t_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix} + t_2 \begin{bmatrix} 3 \\ 2 \\ -4 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \\ -2 \\ -5 \end{bmatrix}$$

To find the solutions to this vector equation/system of linear equations, we row reduce its augmented matrix:

$$\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 2 & 2 & -8 \\ 0 & -4 & -2 \\ -3 & -1 & -5 \end{array}\right] \begin{array}{c} \\ R_2 - 2R_1 \\ \\ R_3 + 3R_1 \end{array} \sim \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -4 & -10 \\ 0 & -4 & -2 \\ 0 & 8 & -2 \end{array}\right] \begin{array}{c} \\ \\ R_3 - R_2 \\ R_4 + 2R_2 \end{array} \sim \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & -4 & -10 \\ 0 & 0 & 8 \\ 0 & 0 & -22 \end{array}\right]$$

Since the last row (and the second to last row) is a bad row, we know that our vector equation does not have any solutions, and thus that $\vec{y}$ is not in the rowspace of B.

$B, \vec{z}$: To see if $\vec{z}$ is in the rowspace of B, we need to see if $\vec{z}$ is in Span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -4 \\ -1 \end{bmatrix} \right\}$.

That is, we need to see if there are any solutions to the vector equation

$$t_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix} + t_2 \begin{bmatrix} 3 \\ 2 \\ -4 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 8 \\ -7 \end{bmatrix}$$

To find the solutions to this vector equation/system of linear equations, we row reduce its augmented matrix:

$$\left[\begin{array}{cc|c} 1 & 3 & -3 \\ 2 & 2 & 2 \\ 0 & -4 & 8 \\ -3 & -1 & -7 \end{array}\right] \begin{array}{c} \\ R_2 - 2R_1 \\ \\ R_3 + 3R_1 \end{array} \sim \left[\begin{array}{cc|c} 1 & 3 & -3 \\ 0 & -4 & 8 \\ 0 & -4 & 8 \\ 0 & 8 & -16 \end{array}\right] \begin{array}{c} \\ \\ R_3 - R_2 \\ R_4 + 2R_2 \end{array} \sim \left[\begin{array}{cc|c} 1 & 3 & -3 \\ 0 & -4 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Our last matrix is in row echelon form, and since it does not have any bad rows, we know that our vector equation has a solution. Thus, $\vec{z}$ is in the rowspace of B.

$C, \vec{x}$: To see if $\vec{x}$ is in the rowspace of C, we need to see if $\vec{x}$ is in

$\text{Span}\left\{\begin{bmatrix}1\\3\\2\end{bmatrix},\begin{bmatrix}3\\9\\0\end{bmatrix},\begin{bmatrix}-1\\-3\\10\end{bmatrix},\begin{bmatrix}2\\6\\-8\end{bmatrix}\right\}$. Since $\vec{x}$ is an element of $\left\{\begin{bmatrix}1\\3\\2\end{bmatrix},\begin{bmatrix}3\\9\\0\end{bmatrix},\begin{bmatrix}-1\\-3\\10\end{bmatrix},\begin{bmatrix}2\\6\\-8\end{bmatrix}\right\}$, we know that $\vec{x}$ is in its span, and thus that $\vec{x}$ is in the rowspace of C.

$C, \vec{y}$: To see if $\vec{y}$ is in the rowspace of C, we need to see if $\vec{y}$ is in $\text{Span}\left\{\begin{bmatrix}1\\3\\2\end{bmatrix},\begin{bmatrix}3\\9\\0\end{bmatrix},\begin{bmatrix}-1\\-3\\10\end{bmatrix},\begin{bmatrix}2\\6\\-8\end{bmatrix}\right\}$. That is, we need to see if there are any solutions to the vector equation

$$t_1\begin{bmatrix}1\\3\\2\end{bmatrix}+t_2\begin{bmatrix}3\\9\\0\end{bmatrix}+t_3\begin{bmatrix}-1\\-3\\10\end{bmatrix}+t_4\begin{bmatrix}2\\6\\-8\end{bmatrix}=\begin{bmatrix}-9\\3\\0\end{bmatrix}$$

To find the solutions to this vector equation/system of linear equations, we row reduce its augmented matrix:

$$\left[\begin{array}{cccc|c}1&3&-1&2&-9\\3&9&-3&6&3\\2&0&10&-8&0\end{array}\right]\begin{array}{l}\\R_2-3R_1\\R_3-2R_1\end{array}\sim\left[\begin{array}{cccc|c}1&3&-1&2&-9\\0&0&0&0&30\\0&-6&12&-14&18\end{array}\right]$$

The middle row of this matrix is a bad row, so we see that our vector equation has no solutions. Thus, $\vec{y}$ is not in the rowspace of C.

$C, \vec{z}$: To see if $\vec{z}$ is in the rowspace of C, we need to see if $\vec{z}$ is in $\text{Span}\left\{\begin{bmatrix}1\\3\\2\end{bmatrix},\begin{bmatrix}3\\9\\0\end{bmatrix},\begin{bmatrix}-1\\-3\\10\end{bmatrix},\begin{bmatrix}2\\6\\-8\end{bmatrix}\right\}$. That is, we need to see if there are any solutions to the vector equation

$$t_1\begin{bmatrix}1\\3\\2\end{bmatrix}+t_2\begin{bmatrix}3\\9\\0\end{bmatrix}+t_3\begin{bmatrix}-1\\-3\\10\end{bmatrix}+t_4\begin{bmatrix}2\\6\\-8\end{bmatrix}=\begin{bmatrix}4\\12\\4\end{bmatrix}$$

To find the solutions to this vector equation/system of linear equations, we row reduce its augmented matrix:

$$\left[\begin{array}{cccc|c}1&3&-1&2&4\\3&9&-3&6&12\\2&0&10&-8&4\end{array}\right]\begin{array}{l}\\R_2-3R_1\\R_3-2R_1\end{array}\sim\left[\begin{array}{cccc|c}1&3&-1&2&4\\0&0&0&0&0\\0&-6&12&-14&-4\end{array}\right]R_2\updownarrow R_3$$
$$\sim\left[\begin{array}{cccc|c}1&3&-1&2&4\\0&-6&12&-14&-4\\0&0&0&0&0\end{array}\right]$$

Our last matrix is in row echelon form, and since it doesn't have any bad rows, we know that our vector equation has a solution. Thus, $\vec{z}$ is in the rowspace of C.