Lecture 3r

Rank Theorem

(pages 161-162)

We've been using the rank of a matrix in many of our results for this section, so it seemed like a good idea to go back and look all the things we now know about the rank of a matrix.

Summary: For an $m \times n$ matrix A:

$$\begin{aligned} \text{the rank of } A &= \text{the number of leading 1s in the reduced row echelon form of } A \\ &= \text{the number of non-zero rows in any row echelon form of } A \\ &= \dim \text{Row}(A) \\ &= \dim \text{Col}(A) \\ &= n - \dim \text{Null}(A) \end{aligned}$$

A fact that gets made obvious from this summary is that $\dim \text{Row}(A) = \dim \text{Col}(A)$. Note that this does not mean that $\text{Row}(A) = \text{Col}(A)$. In fact, they often aren't even subspaces of the same vector space.

The last fact comes from the result in Theorem 1.5.7 that says that $\dim \text{Null}(A) = n - \text{rank}(A)$. This fact, rewritten, is actually one of the more famous results of linear algebra.

Theorem 3.4.8 (Rank Theorem): If A is any $m \times n$ matrix, then

$$\text{rank}(A) + \text{nullity}(A) = n$$

I have also heard this referred to as the "rank-nullity theorem".