Solution to Practice 3k

Exercise 5 We show that $M \circ L$ is linear as follows:

$$\begin{array}{rcll} (M \circ L)(s\vec{x}+\vec{y}) &=& M(L(s\vec{x}+\vec{y})) & \text{(definition of function composition)} \\ &=& M(sL(\vec{x})+L(\vec{y})) & \text{(because } L \text{ is linear)} \\ &=& sM(L(\vec{x}))+M(L(\vec{y})) & \text{(because } M \text{ is linear)} \\ &=& s(M\circ L)(\vec{x})+(M\circ L)(\vec{y}) & \text{(definition of function composition)} \end{array}$$

B5(a) The domain of S is $\mathbb{R}^2$ and the codomain of S is $\mathbb{R}^3$. The domain of T is $\mathbb{R}^2$ and the codomain of T is $\mathbb{R}^3$.

B5(b) The matrix that represents $(T+S)$ is $[T]+[S] = \begin{bmatrix} 3 & 1 \\ 1 & -2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & -2 \\ 3 & 1 \end{bmatrix} =$
$\begin{bmatrix} 5 & 2 \\ 2 & -4 \\ 4 & 1 \end{bmatrix}$

The matrix that represents $(-S+2T)$ is $-[S]+2[T] = -\begin{bmatrix} 2 & 1 \\ 1 & -2 \\ 3 & 1 \end{bmatrix} + 2\begin{bmatrix} 3 & 1 \\ 1 & -2 \\ 1 & 0 \end{bmatrix} =$
$\begin{bmatrix} -2+6 & -1+2 \\ -1+2 & 2-4 \\ -3+2 & -1+0 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & -2 \\ -1 & -1 \end{bmatrix}$

B6(a) The domain of S is $\mathbb{R}^2$ and the codomain of S is $\mathbb{R}^4$. The domain of T is $\mathbb{R}^4$ and the codomain of T is $\mathbb{R}^2$.

B6(b) The matrix that represents $S \circ T$ is $[S][T] = \begin{bmatrix} 4 & -3 \\ 1 & 1 \\ 5 & -3 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 & 2 & 3 \\ -2 & 1 & 3 & 0 \end{bmatrix} =$
$\begin{bmatrix} 16+6 & -3 & 8-9 & 12 \\ 4-2 & 1 & 2+3 & 3 \\ 20+6 & -3 & 10-9 & 15 \\ -8 & 0 & -4 & -6 \end{bmatrix} = \begin{bmatrix} 22 & -3 & -1 & 12 \\ 2 & 1 & 5 & 3 \\ 26 & -3 & 1 & 15 \\ -8 & 0 & -4 & -6 \end{bmatrix}.$

The matrix that represents $T \circ S$ is $[T][S] = \begin{bmatrix} 4 & 0 & 2 & 3 \\ -2 & 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ 1 & 1 \\ 5 & -3 \\ -2 & 0 \end{bmatrix} =$
$\begin{bmatrix} 16+0+10-6 & -12+0-6+0 \\ -8+1+15+0 & \\ 6+1-9+0 & \end{bmatrix} = \begin{bmatrix} 20 & -18 \\ 8 & -2 \end{bmatrix}.$

B7(a) This composition is defined. The domain is the domain of M, which is $\mathbb{R}^4$, and the codomain is the codomain of L, which is $\mathbb{R}^3$.

B7(b) This composition is not defined, because the codomain of L ($\mathbb{R}^3$) is not the same as the domain of M ($\mathbb{R}^4$).

B7(c) This composition is not defined, because the codomain of N ($\mathbb{R}^4$) is not the same as the domain of L ($\mathbb{R}^2$).

B7(d) This composition is defined. The domain is the domain of L, which is $\mathbb{R}^2$, and the codomain is the codomain of N, which is $\mathbb{R}^4$.

B7(e) This composition is defined. The domain is the domain of N, which is $\mathbb{R}^3$, and the codomain is the codomain of M, which is $\mathbb{R}^2$.

B7(f) This composition is not defined, because the codomain of M ($\mathbb{R}^2$) is not the same as the domain of N ($\mathbb{R}^3$).