

### Solution to Practice 3g

**B1(a)** The domain of  $f_A$  is  $\mathbb{R}^3$ , because  $A$  has 3 columns. The codomain of  $f_A$  is  $\mathbb{R}^2$ , because  $A$  has 2 rows.

$$\mathbf{B1(b)} \quad f_A(3, 4, -5) = \begin{bmatrix} -1 & 4 & -2 \\ -5 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -3 + 16 + 10 \\ -15 + 12 - 5 \end{bmatrix} = \begin{bmatrix} 23 \\ -8 \end{bmatrix}$$

$$f_A(-2, 1, -4) = \begin{bmatrix} -1 & 4 & -2 \\ -5 & 3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 + 4 + 8 \\ 10 + 3 - 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 9 \end{bmatrix}$$

**B2(a)** The domain of  $f_A$  is  $\mathbb{R}^3$ , because  $A$  has 3 columns. The codomain of  $f_A$  is  $\mathbb{R}^4$ , because  $A$  has 4 rows.

$$\mathbf{B2(b)} \quad f_A(-4, 2, 1) = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 5 & 7 & 9 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 + 2 + 0 \\ 0 + 4 + 3 \\ -20 + 14 + 9 \\ -8 + 8 + 8 \end{bmatrix} = \begin{bmatrix} -6 \\ 7 \\ 3 \\ 8 \end{bmatrix}$$
$$f_A(3, -3, 2) = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 5 & 7 & 9 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 - 3 + 0 \\ 0 - 6 + 6 \\ 15 - 21 + 18 \\ 6 - 12 + 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 12 \\ 0 \end{bmatrix}$$