Solution to Practice 3a

B1(a) $\begin{bmatrix} 3 & -2 \\ -4 & 1 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 5 & 4 \\ 1 & 4 \\ -6 & -9 \end{bmatrix} = \begin{bmatrix} 3+5 & -2+4 \\ -4+1 & 1+4 \\ 3-6 & 7-9 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ -3 & 5 \\ -3 & -2 \end{bmatrix}$

B1(b) $(-5)\begin{bmatrix} 2 & 3 & -6 & -2 \\ -7 & 1 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -5(2) & -5(3) & -5(-6) & -5(-2) \\ -5(-7) & -5(1) & -5(0) & -5(5) \end{bmatrix} =$
$\begin{bmatrix} -10 & -15 & 30 & 10 \\ 35 & -5 & 0 & -25 \end{bmatrix}$

B1(c) $\begin{bmatrix} 4 & 2 & 3 \\ -2 & 1 & 5 \end{bmatrix} - 4\begin{bmatrix} -2 & -1 & 5 \\ 6 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 4-4(-2) & 2-4(-1) & 3-4(5) \\ -2-4(6) & 1-4(7) & 5-4(1) \end{bmatrix} =$
$\begin{bmatrix} 12 & 6 & -17 \\ -26 & -27 & 1 \end{bmatrix}$

Proof of Theorem 3.1.1 (7): Let A be an $n \times m$ matrix and let $s, t \in \mathbb{R}$ be scalars. Then

$$\begin{array}{rcll} (s(tA))_{ij} & = & s(tA)_{ij} & \text{defintion of scalar multiplication} \\ & = & s(t(A)_{ij}) & \text{definition of scalar multiplication} \\ & = & (st)(A)_{ij} & \text{associativity of multiplication of real numbers} \\ & = & ((st)A)_{ij} & \text{definition of scalar multiplication} \end{array}$$

And since $(s(tA))_{ij} = ((st)A)_{ij}$, by the definition of equality, we have that $s(tA) = (st)A$.