Solution to Practice 2i

B1(a) In Lecture 2h, we found that the reduced row echelon form was $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$.

Since this matrix has two leading 1s, the rank is 2.

B1(b) In Lecture 2h, we found that the reduced row echelon form was $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Since this matrix has three leading 1s, the rank is 3.

B1(c) In Lecture 2h, we found that the reduced row echelon form was $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

Since this matrix has two leading 1s, the rank is 2.

B1(d) In Lecture 2h, we found that the reduced row echelon form was $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

Since this matrix has three leading 1s, the rank is 3.

B1(e) In Lecture 2h, we found that the reduced row echelon form was $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

Since this matrix has three leading 1s, the rank is 3.

B1(f) In Lecture 2h, we found that the reduced row echelon form was $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Since this matrix has four leading 1s, the rank is 4.

B1(g) In Lecture 2h, we found that the reduced row echelon form was $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Since this matrix has three leading 1s, the rank is 3.

D3(a) For this case, you can not determine from the information given whether or not the system is consistent. For example, both of the following matrices satisfy the given conditions:

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The matrix on the left is an example of an inconsistent system, while the example on the right is a consistent system. But, IF the system is consistent, then we know that the number of parameters equals $n - \text{rank}$, which in this case is 3.

D3(b) We know that a system satisfying these conditions must be consistent, because the rank of the augmented matrix cannot be more than the number of rows, i.e. it cannot be more than m. But the rank of the augmented matrix also cannot be less than the rank of the coefficient matrix, which in this case happens to be the same as m. So, we know that the rank of the coefficient matrix equals the rank of the augmented matrix, so by Theorem 2 the system is consistent. Moreover, by Theorem 2, we know that the number of parameters in the general solution is $n - \text{rank} = 6 - 3 = 3$.

D3(c) Again, this is a situation where we cannot know for certain whether or not the system is consistent. Consider the following two matrices, both of which satisfy the given conditions:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The matrix on the left corresponds to an inconsistent system, while the matrix on the right corresponds to a consistent system. But, IF the system is consistent, then we know that the number of parameters equals $n - \text{rank} = 4 - 4 = 0$, so if the system is consistent, then it has a unique solution.