Solution to Practice 2b

Your answers may vary...

B1(a)

Plugging $x_3 = -2$ into the second equation gives us $x_2 + 3(-2) = 4 \Rightarrow x_2 = 10$. Plugging $x_3 = -2$ and $x_2 = 10$ into the first equation gives us $x_1 - 2(10) - (-2) = 5 \Rightarrow x_1 = 23$. So the general solution is $\begin{bmatrix} 23 \\ 10 \\ -2 \end{bmatrix}$.

B1(b)

We let $x_3 = s$, and then the second equation becomes $x_2 + 2s = -1 \Rightarrow x_2 = -1 - 2s$. Now we plug $x_3 = s$ and $x_2 = -1 - 2s$ into the first equation to get $x_1 - 3(-1 - 2s) + s = 1 \Rightarrow x_1 + 3 + 7s = 1 \Rightarrow x_1 = -2 - 7s$. So we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 - 7s \\ -1 - 2s \\ s \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -7 \\ -2 \\ 1 \end{bmatrix}, \quad s \in \mathbb{R}$$

Solution 1: $s = 0$ yields the solution $\begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

Solution 2: $s = 1$ yields the solution $\begin{bmatrix} -9 \\ -3 \\ 1 \end{bmatrix}$

Solution 3: $s = -1/2$ yields the solution $\begin{bmatrix} 3/2 \\ 0 \\ -1/2 \end{bmatrix}$

B1(c)

First, we set $x_4 = s$. Then the third equation becomes $x_3 + 3s = 3 \Rightarrow x_3 = 3 - 3s$. Now we plug $x_4 = s$ and $x_3 = 3 - 3s$ into the second equation, giving us $x_2 - (3 - 3s) + 2s = -1 \Rightarrow x_2 - 3 + 5s = -1$. So $x_2 = 2 - 5s$. Finally, we plug $x_4 = s$, $x_3 = 3 - 3s$, and $x_2 = 2 - 5s$ into the first equation, giving us $x_1 + 3(2 - 5s) - (3 - 3s) + 2s = -1 \Rightarrow x_1 + 3 - 10s = -1$, so $x_1 = -4 + 10s$. So we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -4 + 10s \\ 2 - 5s \\ 3 - 3s \\ s \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 3 \\ 0 \end{bmatrix} + s \begin{bmatrix} 10 \\ -5 \\ -3 \\ 1 \end{bmatrix}$$

Solution 1: $s = 0$ yields the solution $\begin{bmatrix} -4 \\ 2 \\ 3 \\ 0 \end{bmatrix}$

Solution 2: $s = 1$ yields the solution $\begin{bmatrix} 6 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

Solution 3: $s = 2$ yields the solution $\begin{bmatrix} 16 \\ -8 \\ -3 \\ 2 \end{bmatrix}$

B1(d)

First we set $x_4 = s$ and $x_5 = t$. Then the third equation becomes $x_3 + s - 2t = -3$, so $x_3 = -3 - s + 2t$. Using our new values for x_3, x_4, and x_5, the second equation becomes $x_2 - 2(-3-s+2t) - 2s + 3t = 4 \Rightarrow x_2 + 6 - t = 4$, so $x_2 = -2+t$. And now the first equation becomes $x_1 + 3(-2+t) - 2(-3-s+2t) - 2s + 2t = -2 \Rightarrow x_1 + t = -2$, so $x_1 = -2 - t$. So we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2-t \\ -2+t \\ -3-s+2t \\ s \\ t \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -3 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Solution 1: $s = t = 0$ yields the solution $\begin{bmatrix} -2 \\ -2 \\ -3 \\ 0 \\ 0 \end{bmatrix}$

Solution 2: $s = 1, t = 0$ yields the solution $\begin{bmatrix} -2 \\ -2 \\ -4 \\ 1 \\ 0 \end{bmatrix}$

Solution 3: $s = -1, t = 1$ yields the solution $\begin{bmatrix} -3 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$