Solution to Practice 1s

A6(a) First, let's find the direction vector $\vec{d}$ for the line, by taking the cross product of the normal vector $\vec{n}_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ of the first plane with the normal vector $\vec{n}_2 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ of the second plane:

$$\vec{n}_1 \times \vec{n}_2 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} (3)(1) - (-1)(-5) \\ (-1)(2) - (1)(1) \\ (1)(-5) - (3)(2) \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ -11 \end{bmatrix}$$

Now we need to find a point on the line. If we pick a point with $x_2 = 0$, then we are looking at a point that satisfies both $x_1+3(0)-x_3 = 5$ and $2x_1-5(0)+x_3 = 7$. From the first equation we see that $x_1 = 5 + x_3$. Plugging this into the second equation gives us $2(5 + x_3) + x_3 = 7 \Rightarrow 3x_3 + 10 = 7 \Rightarrow x_3 = -1$. Thus, $x_1 = 5 - 1 = 4$. So we see that $(4, 0, -1)$ is a point on the line of intersection of the two planes. Thus, a vector equation for the line of intersection of the planes is

$$\begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} -2 \\ -3 \\ -11 \end{bmatrix} \quad t \in \mathbb{R}$$

A6(b) First, let's find the direction vector $\vec{d}$ for the line, by taking the cross product of the normal vector $\vec{n}_1 = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$ of the first plane with the normal vector $\vec{n}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ of the second plane:

$$\vec{n}_1 \times \vec{n}_2 = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} (0)(2) - (-3)(1) \\ (-3)(0) - (2)(2) \\ (2)(1) - (0)(0) \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}$$

Now we need to find a point on the line. If we pick a point with $x_3 = 0$, then we are looking for a point that satisfies both $2x_1 - 3(0) = 7$ and $x_2 + 2(0) = 4$. The first equation tells us that $x_1 = 7/2$, and the second equation tells us that $x_2 = 4$. So the point $(7/2, 4, 0)$ is a point on the line of intersection of the two planes. Thus, a vector equation for the line of intersection of the planes is

$$\begin{bmatrix} 7/2 \\ 4 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix} \quad t \in \mathbb{R}$$