Solution to Practice 1r

A4(a) First we need to find the normal vector for the plane: $\vec{n} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (3)(0) - (-1)(1) \\ (-1)(4) - (2)(0) \\ (2)(1) - (3)(4) \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ -10 \end{bmatrix}$. So we know that the scalar equation of the plane has the form $x_1 - 4x_2 - 10x_3 = d$. Then we plug in the point $(1, 4, 7)$ on the plane to get $(1)-4(4)-10(7) = -85$, and thus the scalar equation of the plane is $x_1 - 4x_2 - 10x_3 = -85$.

A4(b) First we need to find the normal vector for the plane: $\vec{n} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} (1)(2) - (0)(1) \\ (0)(-2) - (1)(2) \\ (1)(1) - (1)(-2) \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$. So we know that the scalar equation of the plane has the form $2x_1 - 2x_2 + 3x_3 = d$. Then we plug in the point $(2, 3, -1)$ on the plane to get $2(2) - 2(3) + 3(-1) = -5$, and thus the scalar equation of the plane is $2x_1 - 2x_2 + 3x_3 = -5$

A5(a) Before we can find the normal vector for the plane, we first need to find two linearly independent vectors *in* the plane. I will use the vectors $\vec{PQ}$ and $\vec{PR}$, but the vector $\vec{QR}$ could be used as well, as could $\vec{QP}$, $\vec{RP}$, or $\vec{RQ}$. Just don't try to use a $\vec{PQ}, \vec{QP}$ type pairing! (If you did this by accident, you would just end up with the cross product equal to $\vec{0}$. As $\vec{0}$ is never the normal vector for a plane, you would know you had made a mistake.)

So, we calculate $\vec{PQ} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}$ and $\vec{PR} = \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix}$.

Now, we can find the normal vector for the plane: $\vec{n} = \vec{PQ} \times \vec{PR} = \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix} \times \begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix} = \begin{bmatrix} (-4)(-6) - (-3)(5) \\ (-3)(0) - (2)(-6) \\ (2)(5) - (-4)(0) \end{bmatrix} = \begin{bmatrix} 39 \\ 12 \\ 10 \end{bmatrix}$. So we know that the scalar equation of the plane has the form $39x_1 + 12x_2 + 10x_3 = d$. Plugging in the point $P(2, 1, 5)$ we see that $d = 39(2)+12(1)+10(5) = 140$. So a scalar equation for the plane is $39x_1 + 12x_2 + 10x_3 = 140$. (Not sure of this answer when you got it? Remember that you can double check your equation for the plane by plugging in the three given points. If the equation works for all three points, then you can be certain that you've got the right equation.)

A5(b) Before we can find the normal vector for the plane, we first need to find two linearly independent vectors *in* the plane. I will use the vectors $\vec{PQ}$ and $\vec{PR}$:

$$\vec{PQ} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -5 \\ -1 \\ -2 \end{bmatrix} \text{ and } \vec{PR} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -5 \end{bmatrix}$$

Now we can find the normal vector for the plane: $\vec{n} = \vec{PQ} \times \vec{PR} = \begin{bmatrix} -5 \\ -1 \\ -2 \end{bmatrix} \times \begin{bmatrix} -2 \\ 3 \\ -5 \end{bmatrix} = \begin{bmatrix} (-1)(-5) - (-2)(3) \\ (-2)(-2) - (-5)(-5) \\ (-5)(3) - (-1)(-2) \end{bmatrix} = \begin{bmatrix} 11 \\ -21 \\ -17 \end{bmatrix}$. So we know that the scalar equation of the plane has the form $11x_1 - 21x_2 - 17x_3 = d$. Plugging in the point $P(3, 1, 4)$, we see that $d = 11(3) - 21(1) - 17(4) = -56$. So a scalar equation for the plane is $11x_1 - 21x_2 - 17x_3 = -56$.