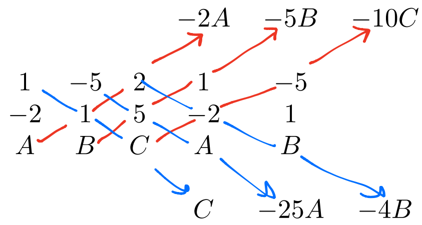
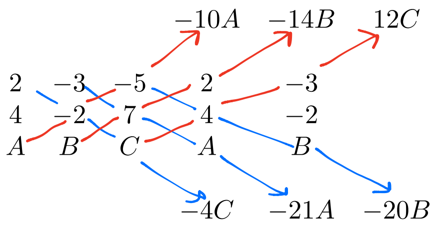
Solution to Practice 1p

A1(a)



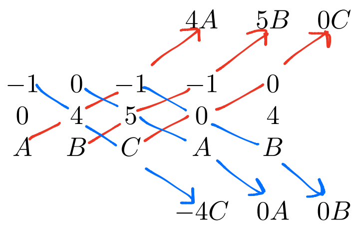
So $\begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix} \times \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} -25-2 \\ -4-5 \\ 1-10 \end{bmatrix} = \begin{bmatrix} -27 \\ -9 \\ -9 \end{bmatrix}$.

A1(b)



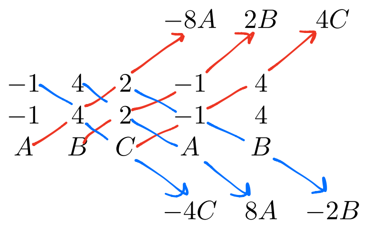
So $\begin{bmatrix} 2 \\ -3 \\ -5 \end{bmatrix} \times \begin{bmatrix} 4 \\ -2 \\ 7 \end{bmatrix} = \begin{bmatrix} -21-10 \\ -20-14 \\ -4+12 \end{bmatrix} = \begin{bmatrix} -31 \\ -34 \\ 8 \end{bmatrix}$.

A1(c)



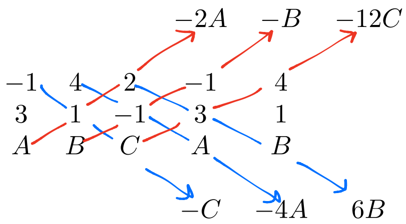
So $\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 4+0 \\ 5+0 \\ 0-4 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix}$.

A2(a)

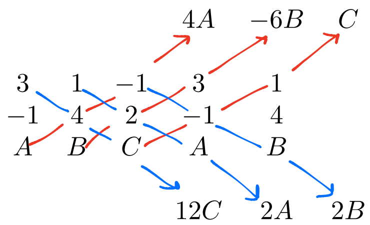


So $\vec{u} \times \vec{u} = \begin{bmatrix} 8-8 \\ -2+2 \\ -4+4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \vec{0}$.

A2(b)



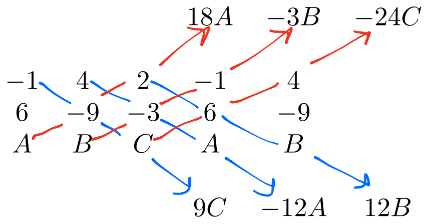
So $\vec{u} \times \vec{v} = \begin{bmatrix} -4-2 \\ 6-1 \\ -1-12 \end{bmatrix} = \begin{bmatrix} -6 \\ 5 \\ -13 \end{bmatrix}$. And



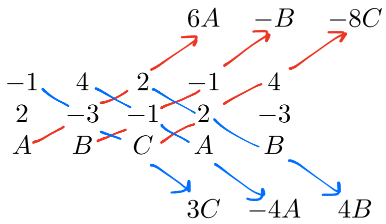
So $\vec{v} \times \vec{u} = \begin{bmatrix} 2+4 \\ 1-6 \\ 12+1 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 13 \end{bmatrix}$.

And so we see that $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$.

A2(c) To find $\vec{u} \times 3\vec{w}$, we look at

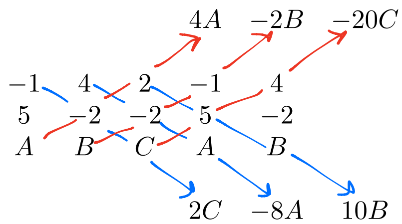


So $\vec{u} \times 3\vec{w} = \begin{bmatrix} -12+18 \\ 12-3 \\ 9-24 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ -15 \end{bmatrix}$. To find $\vec{u} \times \vec{w}$, we look at



So $3(\vec{u} \times \vec{w}) = 3 \begin{bmatrix} -4+6 \\ 4-1 \\ 3-8 \end{bmatrix} = 3 \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ -15 \end{bmatrix}$. And thus we see that $\vec{u} \times 3\vec{w} = 3(\vec{u} \times \vec{w})$.

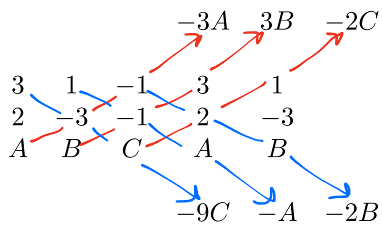
A2(d) First, we note that $\vec{v}+\vec{w} = \begin{bmatrix} 3+2 \\ 1-3 \\ -1-1 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -2 \end{bmatrix}$. So, to find $\vec{u}\times(\vec{v}+\vec{w})$ we look at



which shows us that $\vec{u} \times (\vec{v} + \vec{w}) = \begin{bmatrix} -8+4 \\ 10-2 \\ 2-20 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \\ -18 \end{bmatrix}$.

In part (b) we calculated that $\vec{u} \times \vec{v} = \begin{bmatrix} -6 \\ 5 \\ -13 \end{bmatrix}$ and in part (c) we calculated that $\vec{u}\times\vec{w} = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$. As such, we see that $\vec{u}\times\vec{v}+\vec{u}\times\vec{w} = \begin{bmatrix} -6 \\ 5 \\ -13 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix} = \begin{bmatrix} -6+2 \\ 5+3 \\ -13-5 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \\ -18 \end{bmatrix} = \vec{u}\times(\vec{v}+\vec{w})$.

A2(e) To find $\vec{v}\times\vec{w}$, we look at



which shows us that $\vec{v}\times\vec{w} = \begin{bmatrix} -1-3 \\ -2+3 \\ -9-2 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ -11 \end{bmatrix}$. Thus, $\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \\ -11 \end{bmatrix} = (-1)(-4)+(4)(1)+(2)(-11) = -14$.

In part (b) we calculated that $\vec{u}\times\vec{v} = \begin{bmatrix} -6 \\ 5 \\ -13 \end{bmatrix}$, so $\vec{w}\cdot(\vec{u}\times\vec{v}) = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} -6 \\ 5 \\ -13 \end{bmatrix} = (2)(-6)+(-3)(5)+(-1)(-13) = -14$.

And so we see that $\vec{u}\cdot(\vec{v}\times\vec{w}) = \vec{w}\cdot(\vec{u}\times\vec{v})$.

A2(f)

In part (e) we calculated that $\vec{u}\cdot(\vec{v}\times\vec{w}) = -14$. In part (c) we calculated that $\vec{u}\times\vec{w} = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$, so $-\vec{v}\cdot(\vec{u}\times\vec{w}) = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix} = (-3)(2)+(-1)(3)+(1)(-5) = -14$. Thus, we see that $\vec{u}\cdot(\vec{v}\times\vec{w}) = -\vec{v}\cdot(\vec{u}\times\vec{w})$.

A9 Instead of using the definition and looking at each component, we can use Theorem 1 to show this:

$$
\begin{aligned}
(\vec{u}-\vec{v})\times(\vec{u}+\vec{v}) &= ((\vec{u}-\vec{v})\times\vec{u})+((\vec{u}-\vec{v})\times\vec{v}) && \text{line 3}\\
&= (-u\times(\vec{u}-\vec{v}))+(-\vec{v}\times(\vec{u}-\vec{v})) && \text{line 1}\\
&= (-u\times\vec{u})+(-u\times-\vec{v})+(-\vec{v}\times\vec{u})+(-\vec{v}\times-\vec{v}) && \text{line 3}\\
&= \vec{0}+(-u\times-\vec{v})+(-\vec{v}\times\vec{u})+\vec{0} && \text{line 2}\\
&= (-\vec{v}\times\vec{u})+(-\vec{v}\times\vec{u}) && \text{line 1}\\
&= -\vec{v}\times(\vec{u}+\vec{u}) && \text{line 3}\\
&= (\vec{u}+\vec{u})\times\vec{v} && \text{line 1}\\
&= 2\vec{u}\times\vec{v}
\end{aligned}
$$