Solution to Practice 1o

A5(a) The first thing we need to find is $\text{proj}_{\vec{d}}\vec{PQ}$, which we begin by calculating $\vec{PQ} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$. Then $\vec{d} \cdot \vec{PQ} = (-2)(-1) + (2)(-4) = -6$ and $||\vec{d}||^2 = (-2)^2 + 2^2 = 8$. So $\text{proj}_{\vec{d}}\vec{PQ} = (-6/8)\vec{d} = \begin{bmatrix} 3/2 \\ -3/2 \end{bmatrix}$. This means that $\vec{r} = \vec{p} + \text{proj}_{\vec{d}}\vec{PQ} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 3/2 \\ -3/2 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 5/2 \end{bmatrix}$, so the point on the line closest to Q is $R(5/2, 5/2)$. And to find the distance from Q to the line we first calculate $\text{perp}_{\vec{d}}\vec{PQ} = \vec{PQ} - \text{proj}_{\vec{d}}\vec{PQ} = \begin{bmatrix} -1 \\ -4 \end{bmatrix} - \begin{bmatrix} 3/2 \\ -3/2 \end{bmatrix} = \begin{bmatrix} -5/2 \\ -5/2 \end{bmatrix}$. So the distance from Q to the line is $||\text{perp}_{\vec{d}}\vec{PQ}|| = \sqrt{(-5/2)^2 + (-5/2)^2} = 5/\sqrt{2}$.

A5(b) The first thing we need to find is $\text{proj}_{\vec{d}}\vec{PQ}$, which we begin by calculating $\vec{PQ} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$. Then $\vec{d} \cdot \vec{PQ} = (1)(-1) + (-4)(-2) = 7$ and $||\vec{d}||^2 = 1^2 + (-4)^2 = 17$. So $\text{proj}_{\vec{d}}\vec{PQ} = (7/17)\vec{d} = \begin{bmatrix} 7/17 \\ -28/17 \end{bmatrix}$ This means that $\vec{r} = \vec{p} + \text{proj}_{\vec{d}}\vec{PQ} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} + \begin{bmatrix} 7/17 \\ -28/17 \end{bmatrix} = \begin{bmatrix} 58/17 \\ 91/17 \end{bmatrix}$, so the point on the line closest to Q is $R(58/17, 91/17)$. And to find the distance from Q to the line we first calculate $\text{perp}_{\vec{d}}\vec{PQ} = \vec{PQ} - \text{proj}_{\vec{d}}\vec{PQ} = \begin{bmatrix} -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 7/17 \\ -28/17 \end{bmatrix} = \begin{bmatrix} -24/17 \\ -6/17 \end{bmatrix}$. So the distance from Q to the line is $||\text{perp}_{\vec{d}}\vec{PQ}|| = \sqrt{(-24/17)^2 + (-6/17)^2} = 6/\sqrt{17}$.

A5(c) The first thing we need to find is $\text{proj}_{\vec{d}}\vec{PQ}$, which we begin by calculating $\vec{PQ} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$. Then $\vec{d} \cdot \vec{PQ} = (1)(-1) + (-2)(-2) + (1)(2) = 5$ and $||\vec{d}||^2 = 1^2 + (-2)^2 + 1^2 = 6$. So $\text{proj}_{\vec{d}}\vec{PQ} = (5/6)\vec{d} = \begin{bmatrix} 5/6 \\ -5/3 \\ 5/6 \end{bmatrix}$. This means that $\vec{r} = \vec{p} + \text{proj}_{\vec{d}}\vec{PQ} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 5/6 \\ -5/3 \\ 5/6 \end{bmatrix} = \begin{bmatrix} 17/6 \\ 1/3 \\ -1/6 \end{bmatrix}$, so the point on the line closest to Q is $R(17/6, 1/3, -1/6)$. And to find the distance from Q to the line we first calculate $\text{perp}_{\vec{d}}\vec{PQ} = \vec{PQ} - \text{proj}_{\vec{d}}\vec{PQ} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} - \begin{bmatrix} 5/6 \\ -5/3 \\ 5/6 \end{bmatrix} = \begin{bmatrix} -11/6 \\ -1/3 \\ 7/6 \end{bmatrix}$. So the distance from Q to the line is $||\text{perp}_{\vec{d}}\vec{PQ}|| = \sqrt{(-11/6)^2 + (-1/3)^2 + (7/6)^2} = \sqrt{29/6}$.

A5(d) The first thing we need to find is $\text{proj}_{\vec{d}}\vec{PQ}$, which we begin by calculating $\vec{PQ} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Then $\vec{d} \cdot \vec{PQ} = (1)(1) + (4)(2) + (1)(3) = 12$ and $||\vec{d}||^2 = 1^2 + 4^2 + 1^2 = 18$. So $\text{proj}_{\vec{d}}\vec{PQ} = (12/18)\vec{d} = \begin{bmatrix} 2/3 \\ 8/3 \\ 2/3 \end{bmatrix}$. This means that $\vec{r} = \vec{p} + \text{proj}_{\vec{d}}\vec{PQ} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 2/3 \\ 8/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 11/3 \\ -1/3 \end{bmatrix}$, so the point on the line closest to Q is $R(5/3, 11/3, -1/3)$. And to find the distance from Q to the line we first calculate $\text{perp}_{\vec{d}}\vec{PQ} = \vec{PQ} - \text{proj}_{\vec{d}}\vec{PQ} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 2/3 \\ 8/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 1/3 \\ -2/3 \\ 7/3 \end{bmatrix}$. So the distance from Q to the line is $||\text{perp}_{\vec{d}}\vec{PQ}|| = \sqrt{(1/3)^2 + (-2/3)^2 + (7/3)^2} = \sqrt{6}$.

A6(a) The first thing we need to do is pick a point P on the plane. I choose to use $P(0, -1, 1)$ Then $\vec{PQ} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$. Reading $\vec{n} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$ off the equation, we can now compute $\vec{n} \cdot \vec{PQ} = (3)(2) + (-1)(4) + (4)(0) = 2$ and $||\vec{n}||^2 = 3^2 + (-1)^2 + 4^2 = 26$. Then $\text{proj}_{\vec{n}}\vec{PQ} = (2/26)\vec{n} = \begin{bmatrix} -15/13 \\ 5/13 \\ -20/13 \end{bmatrix}$. So the distance from Q to the plane is $||\text{proj}_{\vec{n}}\vec{PQ}|| = \sqrt{(-15/13)^2 + (5/13)^+(-20/13)^2} = 2/\sqrt{26}$.

A6(b) The first thing we need to do is pick a point P on the plane. I choose to use $P(0, 0, 1)$ Then $\vec{PQ} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$. Reading $\vec{n} = \begin{bmatrix} 2 \\ -3 \\ -5 \end{bmatrix}$ off the equation, we can now compute $\vec{n} \cdot \vec{PQ} = (2)(-2) + (-3)(3) + (-5)(0) = -13$, and $||\vec{n}||^2 = 2^2 + (-3)^2 + (-5)^2 = 38$. Then $\text{proj}_{\vec{n}}\vec{PQ} = (-13/38)\vec{n} = \begin{bmatrix} -26/38 \\ -39/38 \\ 65/38 \end{bmatrix}$. So the distance from Q to the plane is $||\text{proj}_{\vec{n}}\vec{PQ}|| = \sqrt{(-26/38)^2 + (-39/38)^2 + (65/38)^2} = 13/\sqrt{38}$.

A6(c) The first thing we need to do is pick a point P on the plane. I choose to use $P(0,0,-5)$. Then $\vec{PQ} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$. Reading $\vec{n} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ off the equation, we can now compute $\vec{n} \cdot \vec{PQ} = (2)(0) + (0)(2) + (-1)(4) = -4$ and $||\vec{n}||^2 = 2^2 + 0^2 + (-1)^2 = 5$. Then $\text{proj}_{\vec{n}}\vec{PQ} = (-4/5)\vec{n} = \begin{bmatrix} -8/5 \\ 0 \\ 4/5 \end{bmatrix}$. So the distance from Q to the plane is $||\text{proj}_{\vec{n}}\vec{PQ}|| = \sqrt{(-8/5)^2 + 0^2 + (4/5)^2} = 4/\sqrt{5}$.

A6(d) The first thing we need to do is pick a point P on the plane. I choose to use $P(2,0,0)$. Then $\vec{PQ} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$. Reading $\vec{n} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$ off the equation, we can now compute $\vec{n} \cdot \vec{PQ} = (-3)(2) + (-1)(-1) + (1)(-1) = -6$ and $||\vec{n}||^2 = 2^2 + (-1)^2 + (-1)^2 = 6$. Then $\text{proj}_{\vec{n}}\vec{PQ} = (-6/6)\vec{n} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$. So the distance from Q to the plane is $||\text{proj}_{\vec{n}}\vec{PQ}|| = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}$.

A7(a) The first thing we need to do is pick a point P on the hyperplane. I choose to use $P(0,0,0,0)$. Then $\vec{PQ} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. Reading $\vec{n} = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ off the equation, we can now compute $\vec{n} \cdot \vec{PQ} = (2)(1) + (-1)(0) + (1)(0) + (1)(1) = 3$ and $||\vec{n}||^2 = 2^2 + (-1)^2 + 1^2 + 1^2 = 7$. Then $\text{proj}_{\vec{n}}\vec{PQ} = (3/7)\vec{n} = \begin{bmatrix} 6/7 \\ -3/7 \\ 3/7 \\ 3/7 \end{bmatrix}$.
Finally, we can compute $\vec{r} = \vec{q} - \text{proj}_{\vec{n}}\vec{PQ} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 6/7 \\ -3/7 \\ 3/7 \\ 3/7 \end{bmatrix} = \begin{bmatrix} 1/7 \\ 3/7 \\ -3/7 \\ 4/7 \end{bmatrix}$.
Thus, the point on the hyperplane closest to Q is $R(1/7, 3/7, -3/7, 4/7)$.

A7(b) The first thing we need to do is pick a point P on the hyperplane. I choose to use $P(1,0,0,0)$. Then $\vec{PQ} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix}$. Reading

$\vec{n} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix}$ off the equation, we can now compute $\vec{n} \cdot \vec{PQ} = (1)(0) + (-2)(2) + (3)(1) + (0)(3) = -1$ and $||\vec{n}||^2 = 1^2 + (-2)^2 + 3^2 + 0^2 = 14$. Then $\text{proj}_{\vec{n}}\vec{PQ} = (-1/14)\vec{n} = \begin{bmatrix} -1/14 \\ -1/7 \\ 3/14 \\ 0 \end{bmatrix}$. Finally, we can compute $\vec{r} = \vec{q} - \text{proj}_{\vec{n}}\vec{PQ} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} -1/14 \\ -1/7 \\ 3/14 \\ 0 \end{bmatrix} = \begin{bmatrix} 15/14 \\ 13/7 \\ 17/14 \\ 3 \end{bmatrix}$. Thus, the point on the hyperplane closest to Q is $R(15/14, 13/7, 17/14, 3)$.

A7(c) The first thing we need to do is pick a point P on the hyperplane. I choose to use $P(0,0,0,0)$. Then $\vec{PQ} = \begin{bmatrix} 2 \\ 4 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 3 \\ 4 \end{bmatrix}$. Reading $\vec{n} = \begin{bmatrix} 3 \\ -1 \\ 4 \\ 1 \end{bmatrix}$ off the equation, we can now compute $\vec{n} \cdot \vec{PQ} = (3)(2) + (-1)(4) + (4)(3) + (1)(4) = 18$ and $||\vec{n}||^2 = 3^2 + (-1)^2 + 4^2 + 1^2 = 27$. Then $\text{proj}_{\vec{n}}\vec{PQ} = (18/27)\vec{n} = \begin{bmatrix} 2 \\ -2/3 \\ 8/3 \\ 2/3 \end{bmatrix}$. Finally, we can compute $\vec{r} = \vec{q} - \text{proj}_{\vec{n}}\vec{PQ} = \begin{bmatrix} 2 \\ 4 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 2 \\ -2/3 \\ 8/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 0 \\ 14/3 \\ 1/3 \\ 10/3 \end{bmatrix}$. Thus, the point on the hyperplane closest to Q is $R(0, 14/3, 1/3, 10/3)$.

A7(d) The first thing we need to do is pick a point P on the hyperplane. I choose to use $P(0,2,0,0)$. Then $\vec{PQ} = \begin{bmatrix} -1 \\ 3 \\ 2 \\ -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ -2 \end{bmatrix}$. Reading $\vec{n} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix}$ off the equation, we can now compute $\vec{n} \cdot \vec{PQ} = (1)(-1) + (2)(1) + (1)(2) + (-1)(-2) = 5$ and $||\vec{n}||^2 = 1^2 + 2^2 + 1^2 + (-1)^2 = 7$. Then $\text{proj}_{\vec{n}}\vec{PQ} =$

$(5/7)\vec{n} = \begin{bmatrix} 5/7 \\ 10/7 \\ 5/7 \\ -5/7 \end{bmatrix}$. Finally, we can compute $\vec{r} = \vec{q} - \text{proj}_{\vec{n}}\vec{PQ} = \begin{bmatrix} -1 \\ 3 \\ 2 \\ -2 \end{bmatrix} - \begin{bmatrix} 5/7 \\ 10/7 \\ 5/7 \\ -5/7 \end{bmatrix} = \begin{bmatrix} -12/7 \\ 11/7 \\ 9/7 \\ -9/7 \end{bmatrix}$. Thus, the point on the hyperplane closest to Q is $R(-12/7, 11/7, 9/7, -9/7)$