Solution to Practice 1m

A7(a) First we compute $\vec{n} \cdot \vec{p} = (2)(-1) + (4)(2) + (-1)(-3) = 9$. Then the equation for the plane is $2x_1 + 4x_2 - x_3 = 9$.

A7(b) First we compute $\vec{n} \cdot \vec{p} = (3)(2) + (0)(5) + (5)(4) = 26$. Then the equation for the plane is $3x_1 + 5x_3 = 26$.

A7(c) First we compute $\vec{n} \cdot \vec{p} = (3)(1) + (-4)(-1) + (1)(1) = 8$. Then the equation for the plane is $3x_1 - 4x_2 + x_3 = 8$.

A7(d) First we compute $\vec{n} \cdot \vec{p} = (-4)(2) + (-2)(1) + (-2)(1) = -12$. Then the equation for the plane is $-4x_1 - 2x_2 - 2x_3 = -12$.

A8(a) First we compute $\vec{n} \cdot \vec{p} = (3)(1) + (1)(1) + (4)(-1) = 0$. Then the equation for the plane is $3x_1 + 1x_2 + 4x_3 = 0$.

A8(b) First we compute $\vec{n} \cdot \vec{p} = (0)(2) + (1)(-2) + (3)(0) + (3)(1) = 1$. Then the equation for the plane is $x_2 + 3x_3 + 3x_4 = 1$.

A8(c) First we compute $\vec{n} \cdot \vec{p} = \vec{n} \cdot \vec{0} = 0$. Then the equation for the plane is $x_1 - 4x_2 + 5x_3 - 2x_4 = 0$.

A8(d) First we compute $\vec{n} \cdot \vec{p} = (0)(1) + (1)(0) + (2)(1) + (-1)(2) + (1)(1) = 1$ Then the equation for the plane is $x_2 + 2x_3 - x_4 + x_5 = 1$

A9(a) $\vec{n} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

A9(b) $\vec{n} = \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$

A9(c) $\vec{n} = \begin{bmatrix} -4 \\ 3 \\ -5 \end{bmatrix}$

A9(d) $\vec{n} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -3 \end{bmatrix}$

A9(e) $\vec{n} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}$

A10(a) We can use the normal vector for the given plane, $\vec{n} = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$, as the normal vector for our plane. So next we compute $\vec{n} \cdot \vec{p} = (2)(1) + (-3)(-3) + (5)(-1) = 6$. Then the equation for the plane is $2x_1 - 3x_2 + 5x_3 = 6$. Worth noting is that this new plane has the same equation as the given plane, except that the constant term is different. However, the calculation we perform to find the new constant is the same as if we plug the given point P into the left side of the plane equation. It doesn't change the calculations, but it cuts down on some of the words. See the next two assignment solutions for this method.

A10(b) Since $(-2) = -2$, our plane has equation $x_2 = -2$.

A10(c) Since $1 - 2 + 3(1) = 2$, our plane has equation $x_1 - x_2 + 3x_3 = 2$.

D6 Let $\vec{u} \in \mathbb{R}^n$, and let $S = \{\vec{v} \mid \vec{v} \cdot \vec{u} = 0\}$. Then $\vec{0} \in S$, since $\vec{0} \cdot \vec{u} = 0$. Thus, S is non-empty. So now let $\vec{w}, \vec{v} \in S$ and $t \in \mathbb{R}$. Then

$$\begin{aligned} (\vec{w} + \vec{v}) \cdot \vec{u} &= \vec{u} \cdot (\vec{w} + \vec{v}) && \text{Theorem 1, line 2} \\ &= \vec{u} \cdot \vec{w} + \vec{u} \cdot \vec{v} && \text{Theorem 1, line 3} \\ &= 0 + 0 && \text{because } \vec{w}, \vec{v} \in S \\ &= 0 \end{aligned}$$

so $\vec{w} + \vec{v} \in S$. Thus, S is closed under addition. Lastly,

$$\begin{aligned} (t\vec{w}) \cdot \vec{u} &= t(\vec{w} \cdot \vec{u}) && \text{Theorem 1, line (4)} \\ &= t(0) && \text{because } \vec{w} \in S \\ &= 0 \end{aligned}$$

so $t\vec{w} \in S$. Thus S is closed under scalar multiplication. And since S is non-empty, closed under addition, and closed under scalar multiplication, we see that S is a subset of $\mathbb{R}^n$.