Lecture 1k

Length and Dot Product in $\mathbb{R}^2$ and $\mathbb{R}^3$

(pages 28-31)

If we go back to visualizing a vector as a directed line segment from the origin to the point affiliated with the vector, then we can compute the length of a vector as follows:

Definition If $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$, then its **length** is defined to be

$$||\vec{x}|| = \sqrt{x_1^2 + x_2^2}$$

Definition If $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$, then its **length** is defined to be

$$||\vec{x}|| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

The first application of these definitions is that now we can calculate the distance between two points P and Q, by calculating the length of $\vec{PQ}$.

Example: Find the distance between $P(2, 3)$ and $Q(-1, 5)$.

Since $\vec{PQ} = \vec{q} - \vec{p} = \begin{bmatrix} -1 \\ 5 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, we have that the distance between P and Q is $||\vec{PQ}|| = \sqrt{(-3)^2 + (2)^2} = \sqrt{13}$.

While we never define "multiplication of vectors", we do define the following:

Definition If $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \in R^2$, we define the dot product of $\vec{x}$ and $\vec{y}$ by

$$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2$$

Definition If $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \in R^3$, we define the dot product of $\vec{x}$ and $\vec{y}$ by

$$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + x_3 y_3$$

Example: $\begin{bmatrix} 1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 2 \end{bmatrix} = (1)(-4) + (3)(2) = 2$