Solution to Practice 4k

B3(a) The coefficient matrix is $A = \begin{bmatrix} 2 & -7 \\ 5 & 4 \end{bmatrix}$, and det $A = 8 + 35 = 43$.

$N_1 = \begin{bmatrix} 3 & -7 \\ -17 & 4 \end{bmatrix}$, so det $N_1 = 12 - 119 = -107$.

$N_2 = \begin{bmatrix} 2 & 3 \\ 5 & -17 \end{bmatrix}$, so det $N_2 = -34 - 15 = -49$.

Thus,

$$x_1 = \frac{\det N_1}{\det A} = -\frac{107}{43} \quad \text{and} \quad x_2 = \frac{\det N_2}{\det A} = -\frac{49}{43}$$

B3(b) The coefficient matrix is $A = \begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix}$, and det $A = 9 - 5 = 4$

$N_1 = \begin{bmatrix} -2 & 5 \\ -3 & 3 \end{bmatrix}$, so det $N_1 = -6 + 15 = 9$.

$N_2 = \begin{bmatrix} 3 & -2 \\ 1 & -3 \end{bmatrix}$, so det $N_2 = -9 + 2 = -7$.

Thus,

$$x_1 = \frac{\det N_1}{\det A} = \frac{9}{4} \quad \text{and} \quad x_2 = \frac{\det N_2}{\det A} = -\frac{7}{4}$$

B3(c) The coefficient matrix is $A = \begin{bmatrix} 1 & -5 & -2 \\ 2 & 0 & 3 \\ 4 & 1 & -1 \end{bmatrix}$, so

det A (expanding along the second row)

$$= -2\begin{vmatrix} -5 & -2 \\ 1 & -1 \end{vmatrix} + 0 - 3\begin{vmatrix} 1 & -5 \\ 4 & 1 \end{vmatrix}$$

$$= -2(5 + 2) - 3(1 + 20) = -14 - 63 = -77$$

$N_1 = \begin{bmatrix} -2 & -5 & -2 \\ 3 & 0 & 3 \\ 1 & 1 & -1 \end{bmatrix}$, so det N_1 (expanding along the second row)

$$= -3\begin{vmatrix} -5 & -2 \\ 1 & -1 \end{vmatrix} + 0 - 3\begin{vmatrix} -2 & -5 \\ 1 & 1 \end{vmatrix} \quad .$$

$$= -3(5+2) - 3(-2+5) = -21 - 9 = -30$$

$N_2 = \begin{bmatrix} 1 & -2 & -2 \\ 2 & 3 & 3 \\ 4 & 1 & -1 \end{bmatrix}$, so

det N_2 (expanding along the first row)

$$= \begin{vmatrix} 3 & 3 \\ 1 & -1 \end{vmatrix} - (-2)\begin{vmatrix} 2 & 3 \\ 4 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$$

$$= (-3-3) + 2(-2-12) - 2(2-12) = -6 - 28 + 20 = -14$$

$N_3 = \begin{bmatrix} 1 & -5 & -2 \\ 2 & 0 & 3 \\ 4 & 1 & 1 \end{bmatrix}$, so

det N_3 (expanding along the second row)

$$= -2\begin{vmatrix} -5 & -2 \\ 1 & 1 \end{vmatrix} + 0 - 3\begin{vmatrix} 1 & -5 \\ 4 & 1 \end{vmatrix}$$

$$= -2(-5+2) - 3(1+20) = 6 - 63 = -57$$

Thus,

$$x_1 = \frac{\det N_1}{\det A} = \frac{30}{77}, \quad x_2 = \frac{\det N_2}{\det A} = \frac{14}{77}, \quad x_3 = \frac{\det N_3}{\det A} = \frac{57}{77}$$

B3(d) The coefficient matrix is $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ -2 & 1 & 0 \end{bmatrix}$, so

det A (expanding along the third row)

$$= -2\begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} + 0$$

$$= -2(0+2) - (3-6) = -4 + 3 = -1$$

$N_1 = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -1 & 3 \\ 0 & 1 & 0 \end{bmatrix}$, so

$$\det N_1 \quad \text{(expanding along the third row)}$$

$$= 0 - \begin{vmatrix} -2 & 2 \\ 5 & 3 \end{vmatrix} + 0$$

$$= -(-6 - 10) = 16$$

$$N_2 = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 5 & 3 \\ -2 & 0 & 0 \end{bmatrix}, \text{ so}$$

$$\det N_2 \quad \text{(expanding along the third row)}$$

$$= -2 \begin{vmatrix} -2 & 2 \\ 5 & 3 \end{vmatrix} + 0 + 0$$

$$= -2(-6 - 10) = 32$$

$$N_3 = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 5 \\ -2 & 1 & 0 \end{bmatrix}, \text{ so}$$

$$\det N_3 \quad \text{(expanding along the third row)}$$

$$= -2 \begin{vmatrix} 0 & -2 \\ -1 & 5 \end{vmatrix} - \begin{vmatrix} 1 & -2 \\ 3 & 5 \end{vmatrix} + 0$$

$$= -2(0 - 2) - (5 + 6) = 4 - 11 = -7$$

Thus,

$$x_1 = \frac{\det N_1}{\det A} = -16, \quad x_2 = \frac{\det N_2}{\det A} = -32, \quad x_3 = \frac{\det N_3}{\det A} = 7$$