Solution to Practice 4i

(a)

$C_{11} = (-1)^{1+1}\text{det} \begin{bmatrix} 2 \end{bmatrix} = 2$

$C_{12} = (-1)^{1+2}\text{det} \begin{bmatrix} 7 \end{bmatrix} = -7$

$C_{21} = (-1)^{2+1}\text{det} \begin{bmatrix} 4 \end{bmatrix} = -4$

$C_{22} = (-1)^{2+2}\text{det} \begin{bmatrix} -3 \end{bmatrix} = -3$

So the cofactor matrix is $\begin{bmatrix} 2 & -7 \\ -4 & -3 \end{bmatrix}$.

(b)

$C_{11} = (-1)^{1+1}\text{det} \begin{bmatrix} 3 \end{bmatrix} = 3$

$C_{12} = (-1)^{1+2}\text{det} \begin{bmatrix} -2 \end{bmatrix} = 2$

$C_{21} = (-1)^{2+1}\text{det} \begin{bmatrix} 4 \end{bmatrix} = -4$

$C_{22} = (-1)^{2+2}\text{det} \begin{bmatrix} 5 \end{bmatrix} = 5$

So the cofactor matrix is $\begin{bmatrix} 3 & 2 \\ -4 & 5 \end{bmatrix}$.

(c)

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 0 \\ 5 & 4 \end{vmatrix} = 12 - 0 = 12$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} -3 & 0 \\ 2 & 4 \end{vmatrix} = -(-12 - 0) = 12$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} -3 & 3 \\ 2 & 5 \end{vmatrix} = -15 - 6 = -21$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = -(8 + 15) = -23$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 0 & -3 \\ 2 & 4 \end{vmatrix} = 0 + 6 = 6$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 0 & 2 \\ 2 & 5 \end{vmatrix} = -(0 - 4) = 4$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -3 \\ 3 & 0 \end{vmatrix} = 0 + 9 = 9$$

$C_{32} = (-1)^{3+2} \begin{vmatrix} 0 & -3 \\ -3 & 0 \end{vmatrix} = -(0-9) = 9$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 0 & 2 \\ -3 & 3 \end{vmatrix} = 0+6 = 6$

So the cofactor matrix is $\begin{bmatrix} 12 & 12 & -21 \\ -23 & 6 & 4 \\ 9 & 9 & 6 \end{bmatrix}$.

(d)

$C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1-0 = 1$

$C_{12} = (-1)^{1+2} \begin{vmatrix} -2 & 0 \\ 3 & 1 \end{vmatrix} = -(-2-0) = 2$

$C_{13} = (-1)^{1+3} \begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix} = 2-3 = -1$

$C_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = -(0+2) = -2$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1-6 = -5$

$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} = -(-1-0) = 1$

$C_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} = 0-2 = -2$

$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ -2 & 0 \end{vmatrix} = -(0+4) = -4$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} = 1+0 = 1$

So the cofactor matrix is $\begin{bmatrix} 1 & 2 & -1 \\ -2 & -5 & 1 \\ -2 & -4 & 1 \end{bmatrix}$.

(e)

$C_{11} = (-1)^{1+1} \begin{vmatrix} 4 & -2 \\ -7 & 2 \end{vmatrix} = 8-14 = -6$

$C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & -2 \\ 0 & 2 \end{vmatrix} = 0$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 4 \\ 0 & -7 \end{vmatrix} = 0$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 2 \\ -7 & 2 \end{vmatrix} = -(4 + 14) = -18$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 6 & 2 \\ 0 & 2 \end{vmatrix} = 12 - 0 = 12$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 6 & 2 \\ 0 & -7 \end{vmatrix} = -(-42 - 0) = 42$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 2 \\ 4 & -2 \end{vmatrix} = -4 - 8 = -12$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 6 & 2 \\ 0 & -2 \end{vmatrix} = -(-12 - 0) = 12$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 6 & 2 \\ 0 & 4 \end{vmatrix} = 24 - 0 = 24$$

So the cofactor matrix is $\begin{bmatrix} -6 & 0 & 0 \\ -18 & 12 & 42 \\ -12 & 12 & 24 \end{bmatrix}$.